

Setup: (Suitaly functional) Six-functor formalism is assignment Spaces Categories of coefficients
for cohomology

X 1-> E(X) top spaces analytic careties Schemes friangelated or e.g. g. coh. sheaves, l-shic stears, wixed Hodge models, D-wolder, 10 outegoires motions, steams of (motions) spectra, Six-functors & -/ Itom on each C(X) $f^*: \mathcal{C}(y) \rightleftharpoons \mathcal{C}(x): f_*$ } for switable maps fix -> 4 f: (€(X) € (4):p' + a long tot of axious encoding materal/ properties we'll look at some of these & how they me applied. - box to get cohomology out of the Comoline?
- foundation of duality & cramples pt II - whole a an low/ GFT: frame formula & hh! cong.

function - short directioning & goon up. then - how to weekend the six grantous? mothy zure le wow.

\$ 2 Example: fopologisal spaces & cheat whomology Spaces = loc. gst. Husdarff space & cts works X mis D⁵(X) = banded derived askyony of stemes of de gos on X

See Towney S

C) f² -1 Rf * adving pullbak & pashformand lactures (C)f2 -1 Rf. pushfound or/ compact support. for i was x you america, j. Fix extension by zero.

for your and direct whom by zero. Lever product & internal Home ⊗ -1 1/om cohomology: $H^{q}(X, \mathcal{F}) = \mathbb{R}^{q} \Gamma(X, \mathcal{F})$ der ed global sertions Col. w/qd support: $H_c^q(x, \Xi) = R^q f_i(\Xi)$ more examples O-modules (topolytral & Lyconic) mixed Holge modules also raboration functions C-adir shems / as morphisms between colnect sheares instuces, instruce stable homotopy equirant topology, (also stocks) paratited spector recartly a lot of most in (p-abol analytic sedings

§ 3 (copour logs from six functors for p: X => pt resour cohomology etc hom 518 fuctors: cohomology: p. p. 1 E ((pt) - in example above or col. gry How Equi) (1) Pap 1 (x, Z) = H (x, Z) coh. w/ apt. Support: p: p # 1 innediately generalize horno logy. P! P. I to fox -> 4. Bord - Move landlegy: p. p 4 pletick Unions functionalities revoked from adjunctions; f + -1 f + ~ ~ unit wap PS L7 g. 5 -> g. f. f. 5 = p. 15 in hoes H. (Y, 3) -> H. (x, p. 5) aun legous: f! -1 f: -> count unap p: p: 2 = q: f: f: q: 11 -> q. q: 14 indus H. (x, z) -> H. (Y, z) Cup product from for thoug inomidal: for $f: \times \rightarrow y$ mis $f^{2}: \mathcal{C}(x) \rightarrow \mathcal{C}(x)$ is strong manoidal function,

i. $f^{*}(M \otimes N) \simeq f^{*}(M) \otimes f^{*}(N)$ -> for lax monoider i.e. have materal drast, for (M) Of. (N) -> for (M & N) $m \neq A \in CAG(C(X)), \quad p \in A \in CAG(C(Y))$ So pa 1 13 CAY li example above get up product on 14°(X, Z) = @ Wan (1, pap 1/2 (2))

54 applientia: Commenting duality let's start w/ simple adjunction, for f: X - spt (Hom (f. f. 1 tn], 1) = Hom (4, f. f. 12-47) the (x)

the (x)

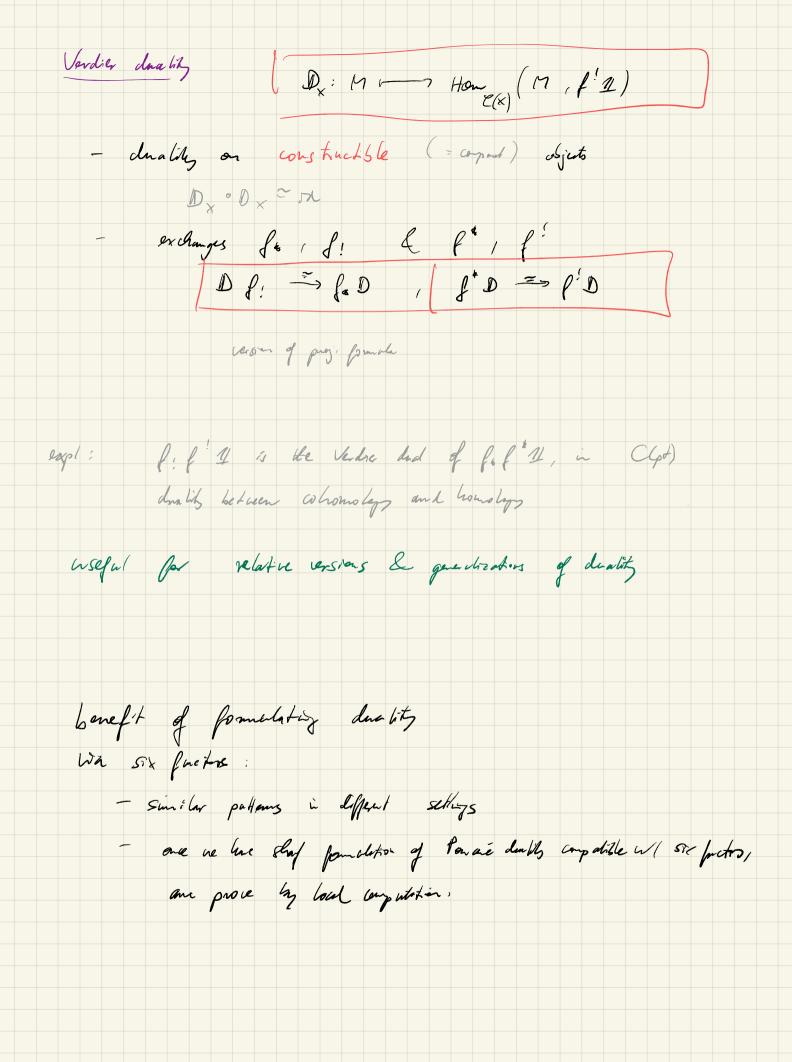
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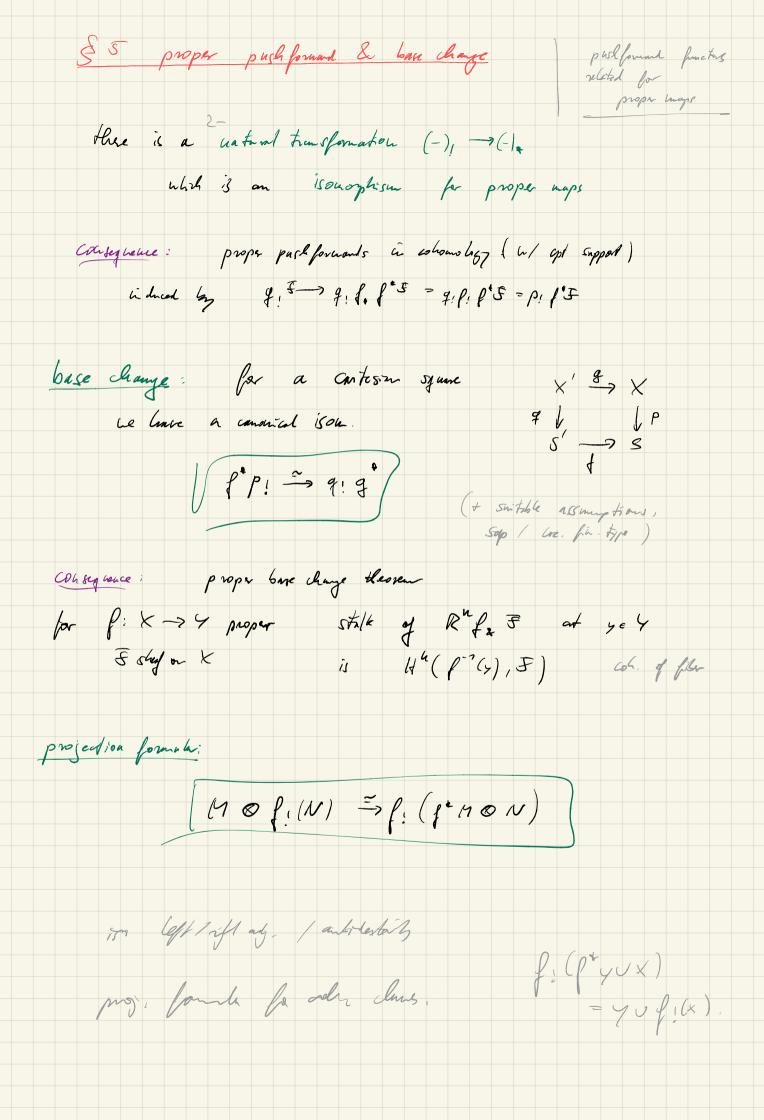
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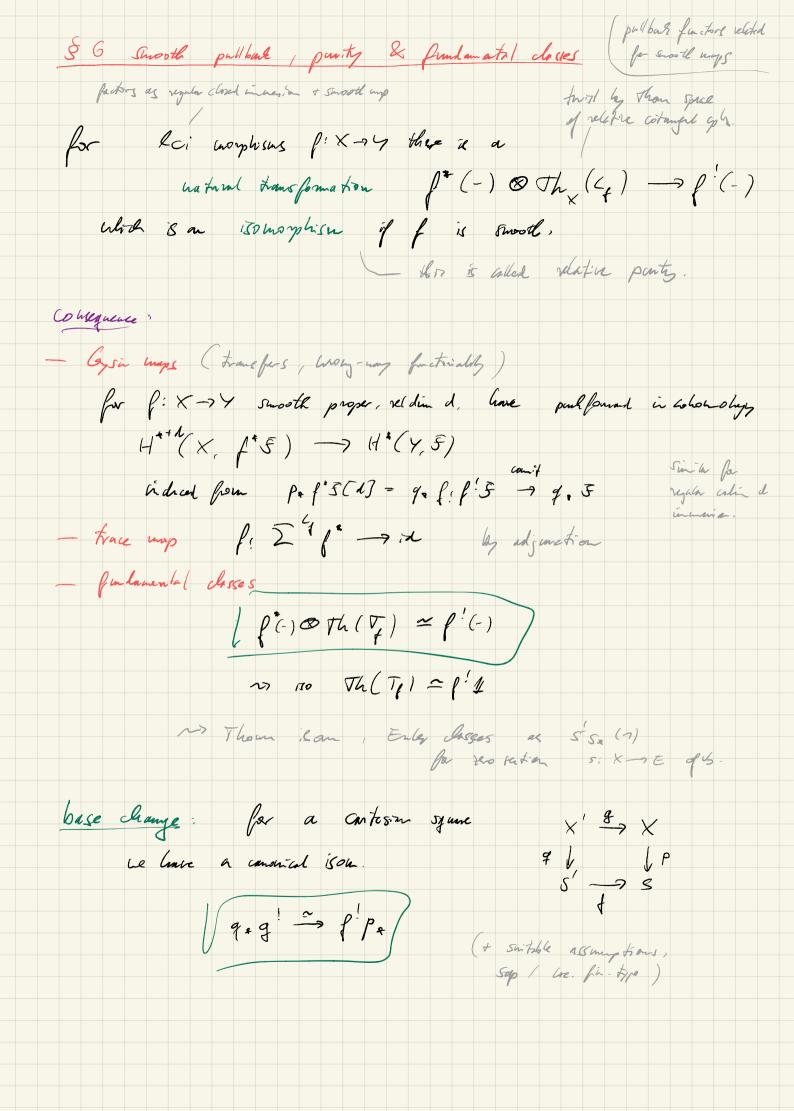
the (x)

the (x)

the content of Back- (boxe honology this becomes more interesting, for of smooth, proper $\begin{cases}
\frac{1}{2} & \frac$ f smooth: 486 If '1 = wx [d] (severally p' M = f' 1 of M) overtation steel, local system for once to tion covering conterty us the lie bundle cox = 1 52) SO RHS is Hom (4, f. Wx [d-u]) = 4d-n(x, wx) if i addition X orientable (Wx = 72) this is Hd-h(x) Poincare devaliby $H^{n}(x)^{\nu} = H^{d-n}(x)$ for gpt. over table refl, field couples, or durative complexes, & the take column.







S7 Example: (ques:) coherent sheares (X & Sch/R mm? D(Och(x)) Grotlalier, Hatshome Residnes & Onating, but no f! can be fixed by solid modeles in Comer Schoolse condensed moth, duality: cohert setting of : X -> 9 proper morphism unotherm fix-din sheres (Rf. F, G) => Rf. R How (F, P'G) E bounded above up by of Ox hand. granical. whom E bounded below I am co-treet cohom. resole Sene duality for f: X -> Spec & , X/2 smooth, proj. din d CAS - H7(X,5)~ G = Cgocc 2 P: S = S2 d [d] RHI = $GA^{d-q}(\overline{5}, \Sigma_{x/2}^{d})$ dealising sleep. adjunction imp fift - 7 it inless trace imp & d(x, 2 1/2) -> &, iso by X smooth prope preming of 1= & findemental class / colone form f (X -> 4 Sugoth, w. din 1) brose generally (·(-) ≈ 52/ (-) /