Localization of Admissible Locally Analytic Representations

Colloque Tournant du GDR Théorie de Lie Algébrique et Géométrique

Université de Rennes 1

Andrés Sarrazola Alzate

Università degli studi di Padova

March 25th, 2021

Università degli Studi di Padova

Université de Rennes 1

Historical Setting

Arithmetic Version

Arithmetic Differential operators on Admissible Blow-ups

G_0-equivariance of Formal Models of Flag Varieties

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots
  $d = \dim(X) = |\Lambda^+|$.

- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots
  $d = \dim(X) = |\Lambda^+|$.

- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.

- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots
  $d = \dim(X) = |\Lambda^+|$.

- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Historical Setting

- G complex semi-simple algebraic group.
- $T \subseteq B \subseteq G$ a Borel sub-group and a maximal torus of G.
- $X := G/B$ the **flag variety**.
- $\Lambda^+ \subseteq \mathfrak{t}_{\mathbb{C}}^*$ positives roots $d = \dim(X) = |\Lambda^+|$.

- $\mathfrak{g}_{\mathbb{C}} := \mathrm{Lie}(G)$.
- $\mathfrak{t}_{\mathbb{C}} := \mathrm{Lie}(T)$ and $\lambda \in \mathfrak{t}_{\mathbb{C}}^*$.
- $\mathfrak{z} \subseteq \mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ the center.

$\rightsquigarrow \rho$ **Weyl character**

We have two constructions:

- Via the isomorphism of Harish-Chandra we have a maximal ideal $\mathfrak{m}_\lambda \subseteq \mathfrak{z}$.
- A sheaf $\mathcal{D}_\lambda$ of λ-twisted differential operators.

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

Intrinsic definition ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l\in\mathbb{N}} F_l\mathcal{D}_X$$

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

Intrinsic definition ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l\in\mathbb{N}} F_l\mathcal{D}_X$$

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

Intrinsic definition ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l \in \mathbb{N}} F_l\mathcal{D}_X$$

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

Intrinsic definition ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l\in\mathbb{N}} F_l\mathcal{D}_X$$

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

<u>Intrinsic definition</u> ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l \in \mathbb{N}} F_l\mathcal{D}_X$$

Sheaf of (algebraic) differential operators

We start by considering the **tangent sheaf**

$$\mathcal{T}_X := \{\theta \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \theta(fg) = \theta(f)g + f\theta(g)\}.$$

By identifying

$$\begin{array}{ccc} \mathcal{O}_X & \hookrightarrow & \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \\ f & \mapsto & [g \mapsto fg] \end{array}$$

We can define

$$\mathcal{D}_X := \{\mathbb{C}\text{-(sub)algebra generated by } \mathcal{O}_X \text{ and } \mathcal{T}_X\} \subseteq \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X).$$

Intrinsic definition ($l \in \mathbb{N}$ and $F_0\mathcal{D}_X = \mathcal{O}_X$)

$$F_l\mathcal{D}_X := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{O}_X) \mid \forall f \in \mathcal{O}_X,\ [P, f] \in F_{l-1}\mathcal{D}_X\},$$

$$\mathcal{D}_X = \cup_{l \in \mathbb{N}} F_l\mathcal{D}_X$$

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{cccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ \text{(locally)} & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha} \in \mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{rccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ (\text{locally}) & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{cccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ (\text{locally}) & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha} \in \mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{rccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ \text{(locally)} & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{cccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ (\text{locally}) & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{cccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ (\text{locally}) & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{cccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ \text{(locally)} & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Finiteness properties

If U is a chart with coordinate system $\{x_i, \partial_{x_i}\}$, then

$$\mathcal{D}_U = \bigoplus_{\underline{\alpha}\in\mathbb{N}^d} \mathcal{O}_U \underline{\partial}^{\underline{\alpha}}, \qquad \underline{\partial}^{\underline{\alpha}} := \partial_{x_1}^{\alpha_1}\cdots\partial_{x_d}^{\alpha_d}.$$

Furthermore, the obvious morphism

$$\mathcal{T}_X \xrightarrow{\sigma} \mathrm{gr}_1(\mathcal{D}_X) \to \mathrm{gr}(\mathcal{D}_X)$$

induces a canonical identification

$$\begin{array}{lccc} & \mathrm{Sym}(\mathcal{T}_X) & \to & \mathrm{gr}(\mathcal{D}_X) \\ \text{(locally)} & \partial_{x_i} & \mapsto & \xi_i := \sigma(\partial_{x_i}) \end{array}$$

and we have

$$\mathrm{gr}(\mathcal{D}_U) = \mathcal{O}_U[\xi_1, \cdots, \xi_d].$$

- $\mathcal{D}_X$ has noetherian sections over affine open subsets.
- $\mathcal{D}_X$ is a coherent sheaf of rings.

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}_X^1).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}_X^1).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}^1_X).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Sheaves of (homogeneous) twisted differential operators

We will need the following generalization of the pair $(\mathcal{D}_X, \mathcal{O}_X \xrightarrow{\iota_X} \mathcal{D}_X)$.

A pair $(\mathcal{A}, \iota)$ is a **sheaf of twisted differential operators** on X if:

- $\iota : \mathcal{O}_X \to \mathcal{A}$ is a morphism of $\mathbb{C}$-algebras with unit,
- X admits a cover by open sets U such that $(\mathcal{A}|_U, \iota|_U) \simeq (\mathcal{D}_U, \iota_U)$.

We have a bijection

$$\textbf{IsoClass(t.d.o)} \simeq H^1(X, \mathcal{Z}_X^1).$$

In this presentation we will consider the following subcategory

$(\mathcal{A}, \iota)$ is a **homogeneous sheaf of t.d.o** if

- $\mathcal{A}$ is endowed with an algebraic G-action preserving mult.,
- Differentiating the G-action induces a G-equivariant morphism

$$\Phi : \mathcal{U}(\mathfrak{g}) \to \Gamma(X, \mathcal{A}).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda) \text{ invertible.}}$

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{matrix} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow & s \otimes P \otimes s^\vee \end{matrix}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\boxed{\mathcal{D}_X \text{ is a sheaf of twisted differential operators on } X.}$

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathcal{L}(\lambda)$ invertible.

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{array}{ccc} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow & s \otimes P \otimes s^\vee \end{array}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\mathcal{D}_X$ is a sheaf of twisted differential operators on X.

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda) \text{ invertible.}}$

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{matrix} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow\!\shortmid & s \otimes P \otimes s^\vee \end{matrix}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\boxed{\mathcal{D}_X \text{ is a sheaf of twisted differential operators on } X.}$

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda) \text{ invertible.}}$

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{matrix} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow & s \otimes P \otimes s^\vee \end{matrix}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\boxed{\mathcal{D}_X \text{ is a sheaf of twisted differential operators on } X.}$

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda) \text{ invertible.}}$

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{array}{ccc} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \hookleftarrow & s \otimes P \otimes s^\vee \end{array}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\boxed{\mathcal{D}_X \text{ is a sheaf of twisted differential operators on } X.}$

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathcal{L}(\lambda)$ invertible.

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{matrix} \cup_{l\in\mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow\!\shortmid & s \otimes P \otimes s^\vee \end{matrix}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\mathcal{D}_X$ is a sheaf of twisted differential operators on X.

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\text{alg. gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Technical example

- $\lambda \in \mathrm{Hom}(T, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda) \text{ invertible.}}$

In this case ($l \in \mathbb{N}$ and $F_0\mathcal{D}_\lambda = \mathcal{O}_X$)

$$F_l\mathcal{D}_\lambda := \{P \in \mathcal{E}nd_{\mathbb{C}}(\mathcal{L}(\lambda)) \mid \forall f \in \mathcal{L}(\lambda),\ [P, f] \in F_{l-1}\mathcal{D}_\lambda\}$$

and

$$\mathcal{D}_\lambda := \begin{matrix} \cup_{l \in \mathbb{N}} F_l\mathcal{D}_\lambda & = & \mathcal{L}(\lambda) \otimes_{\mathcal{O}_X} \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}(\lambda)^\vee \\ [t \mapsto P(\langle s^\vee, t\rangle)s] & \leftarrow\!\shortmid & s \otimes P \otimes s^\vee \end{matrix}$$

is the sheaf of (twisted) differential operators acting over $\mathcal{L}(\lambda)$.

$\boxed{\mathcal{D}_X \text{ is a sheaf of twisted differential operators on } X.}$

We have an inclusion

$$X^*(T) := \mathrm{Hom}_{\mathrm{alg.\ gps}}(T, \mathbb{G}_m) \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.** Endowed with commuting (G, T)-actions.

$X := G/B$ **the flag variety**

$$\boxed{\xi : \widetilde{X} \to X}$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.**
Endowed with commutating (G, T)-actions.

$X := G/B$ **the flag variety**

$$\xi : \widetilde{X} \to X$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.** Endowed with commutating (G, T)-actions.

$X := G/B$ **the flag variety**

$$\xi : \widetilde{X} \to X$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.**
Endowed with commutating (G, T)-actions.

$X := G/B$ **the flag variety**

$$\boxed{\xi : \widetilde{X} \to X}$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.** Endowed with commutating (G,T)-actions.

$X := G/B$ **the flag variety**

$$\xi : \widetilde{X} \to X$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.** Endowed with commutating (G, T)-actions.

$X := G/B$ **the flag variety**

$$\boxed{\xi : \widetilde{X} \to X}$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}_X^1).$$

Differential operators attached to non-algebraic characters

We consider

- $\mathbf{N} \subseteq B$ unipotent radical of B.

$\widetilde{X} := G/\mathbf{N}$ **the affine basic space.**
Endowed with commutating (G, T)-actions.

$X := G/B$ **the flag variety**

$$\boxed{\xi : \widetilde{X} \to X}$$

- ξ is a locally trivial T-torsor,
- $\widetilde{\mathcal{D}} := (\xi_* \mathcal{D}_{\widetilde{X}})^T$. If $U \subseteq X$ trivialises ξ, then $\widetilde{\mathcal{D}}|_U \simeq \mathcal{D}_U \otimes_{\mathbb{C}} \mathcal{U}(\mathfrak{t})$,
- for $\lambda \in \mathfrak{t}^* := \mathrm{Hom}_{\mathbb{C}-mods}(\mathfrak{t}, \mathbb{C})$, we can define

$$\mathcal{D}_\lambda := \widetilde{\mathcal{D}} \otimes_{\mathcal{U}(\mathfrak{t})} \mathbb{C}_\lambda,$$

and we get an injection

$$\mathfrak{t}^* \hookrightarrow H^1(X, \mathcal{Z}^1_X).$$

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

<ins>The localization theorem decomposes in two parts:</ins>

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}.$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}.$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)}. \qquad \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}_{\mathbb{C}}^*$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

Localization of $\mathfrak{g}_{\mathbb{C}}$-modules

The localization theorem decomposes in two parts:

The canonical action of G over X gives a canonical isomorphism of algebras

$$\boxed{\mathcal{U}_\lambda := \mathcal{U}(\mathfrak{g}_{\mathbb{C}})/\mathfrak{m}_\lambda \simeq H^0(X, \mathcal{D}_\lambda).}$$

If $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{C}}$ is a regular and dominant character, then

$$\{\mathcal{D}_\lambda\text{-modules}\} \xrightarrow{H^0(X,\bullet)} . \qquad \{\mathcal{U}_\lambda\text{-modules}\}$$

is equivalence of categories.

References

1. A. Beilinson & J. Bernstein, Localisation de $\mathfrak{g}$-modules. Comptes Rendus, 292 (1981), 15–18. MathSciNet MATH.

2. R. Bezrukavnikov, I. Mirkovic & D. Rumynin, Localization of modules for a semisimple Lie algebra in prime characteristic. Annals of Mathematics, 2008, p. 945-991.

3. W. Borho & J.L. brylinski, Differential operators on homogeneous spaces. II: relative enveloping algebras. Bulletin de la Société Mathématique de France, 117.2 (1989), 167-210.

4. D. Miličić, Localization and representation theory of reductive Lie groups. Preprint (April 1993), http://www. math. utah. edu/milicic/Eprints/book.pdf.

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

• $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

• $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),

$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,

$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

• $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ $(e \leq p-1\ !)$,
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p - 1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \partial^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ ($e \leq p-1$!),
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^\dagger_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^\dagger_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \partial^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Context

In **mixed characteristic** an important progress can be found in the work of Huyghe-Schmidt.

In this situation:

$\mathfrak{o}$ is the ring of integers of a finite extension L of $\mathbb{Q}_p$ $(e \leq p-1\ !)$,
$\mathbb{G}$ is a split connected reductif group over $\mathfrak{o}$,
$\mathbb{T} \subseteq \mathbb{B} \subseteq \mathbb{G}$ a Borel subgroup containing a split maximal torus.

$X := \mathbb{G}/\mathbb{B}$ the $\mathfrak{o}$-flag scheme

$\mathfrak{X}$ the formal completion along its special fiber.

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}$ the sheaf of **Berthelot's differential operators**.

If $(U, \partial_i) \subseteq X$ is a coordinated affine open subset and $\partial_i^{[\nu_i]} = \frac{\partial_i^{\nu_i}}{\nu_i!}$

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(U) = \left\{ \sum_{\underline{\nu}}^{\infty} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} \mid \exists c > 0,\ \eta < 1 \text{ s.t } |a_{\underline{\nu}}| < c\eta^{|\underline{\nu}|},\ a_{\underline{\nu}} \in \mathcal{O}_{\mathfrak{X},L}(U) \right\}$$

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_{\lambda}$ p-adic completion

$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$

$D^{\dagger}(\mathbb{G})_{\lambda} = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$

$$D^{(m)}(\mathbb{G})_{\lambda} := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_{\lambda}) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_{\lambda}$ p-adic completion

$$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$$

$$D^{\dagger}(\mathbb{G})_{\lambda} = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$$

$$D^{(m)}(\mathbb{G})_{\lambda} := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_{\lambda}) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_{\lambda}$ p-adic completion

$$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$$

$$D^{\dagger}(\mathbb{G})_{\lambda} = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$$

$$D^{(m)}(\mathbb{G})_{\lambda} := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_{\lambda}) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_\lambda$ p-adic completion

$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$

$D^{\dagger}(\mathbb{G})_\lambda = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$

$$D^{(m)}(\mathbb{G})_\lambda := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_\lambda) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m) \quad \rightsquigarrow \quad$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^\dagger_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^\vee.$$

$D^{(m)}(\mathbb{G})$ arithmetic distribution algebra

$\widehat{D}^{(m)}(\mathbb{G})_\lambda$
p-adic completion

$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$

$D^\dagger(\mathbb{G})_\lambda = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$

$$D^{(m)}(\mathbb{G})_\lambda := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_\lambda) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ arithmetic distribution algebra

$\widehat{D}^{(m)}(\mathbb{G})_\lambda$ p-adic completion

$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$

$D^{\dagger}(\mathbb{G})_\lambda = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$

$$D^{(m)}(\mathbb{G})_\lambda := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_\lambda) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$

$\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^\dagger_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^\vee.$$

$D^{(m)}(\mathbb{G})$ arithmetic distribution algebra

$\widehat{D}^{(m)}(\mathbb{G})_\lambda$ p-adic completion

$$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$$

$$D^\dagger(\mathbb{G})_\lambda = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}$$

$$D^{(m)}(\mathbb{G})_\lambda := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_\lambda) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m) \quad \rightsquigarrow \quad \boxed{\mathscr{L}(\lambda) \text{ invertible}}$

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_{\lambda}$ p-adic completion

$$\boxed{D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)}$$

$$\boxed{D^{\dagger}(\mathbb{G})_{\lambda} = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}}$$

$$D^{(m)}(\mathbb{G})_{\lambda} := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_{\lambda}) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\mathscr{L}(\lambda)$ invertible

- $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^{\dagger}_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^{\vee}.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_{\lambda}$ p-adic completion

$$D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)$$

$$D^{\dagger}(\mathbb{G})_{\lambda} = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda, L}$$

$$D^{(m)}(\mathbb{G})_{\lambda} := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_{\lambda}) \cap D^{(m)}(\mathbb{G}) \right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^{\dagger}_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Arithmetic Version of Beilinson-Bernstein's Localization (Algebraic Case)

$\lambda \in \mathrm{Hom}(\mathbb{T}, \mathbb{G}_m)$ $\rightsquigarrow$ $\boxed{\mathscr{L}(\lambda) \text{ invertible}}$

- $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$ sheaf of (arithmetic) λ-twisted differential operators

$$\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda) := \mathscr{L}(\lambda) \otimes \mathscr{D}^\dagger_{\mathfrak{X}} \otimes \mathscr{L}(\lambda)^\vee.$$

$D^{(m)}(\mathbb{G})$ **arithmetic distribution algebra**

$\widehat{D}^{(m)}(\mathbb{G})_\lambda$ p-adic completion

$$\boxed{D^{(m)}(\mathbb{G})_L = \mathcal{U}(\mathfrak{g}_L)}$$

$$\boxed{D^\dagger(\mathbb{G})_\lambda = \varinjlim \widehat{D}^{(m)}(\mathbb{G})_{\lambda,L}}$$

$$D^{(m)}(\mathbb{G})_\lambda := D^{(m)}(\mathbb{G}) / \left(\mathrm{Ker}(\chi_\lambda) \cap D^{(m)}(\mathbb{G})\right).$$

Theorem. [HS] The formal flag $\mathfrak{o}$-scheme $\mathfrak{X}$ is $\mathscr{D}^\dagger_{\mathfrak{X}}(\lambda)$-affine for every algebraic character λ such that $\lambda + \rho$ is dominant and regular.

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1 !} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4 !}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \quad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1 !} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4 !}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1 !} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4 !}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $GL_{2,\mathbb{Z}_p}$

$$\mathbb{G} = GL_{2,\mathbb{Z}_p} \quad \rightsquigarrow \quad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(GL_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1 !} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4 !}$$

$$\boxed{D^{(m)}(GL_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $\mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p} \qquad \rightsquigarrow \qquad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}!\frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}!\binom{h_1}{\nu_2} \cdot q_{\nu_3}!\binom{h_2}{\nu_3} \cdot q_{\nu_4}!\frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(\mathrm{GL}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Example $GL_{2,\mathbb{Z}_p}$

$$\mathbb{G} = GL_{2,\mathbb{Z}_p} \qquad \rightsquigarrow \qquad \mathfrak{gl}_{2,\mathbb{Z}_p}$$

We have

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathfrak{gl}_{2,\mathbb{Z}_p} = \mathbb{Z}_p \cdot e \oplus \mathbb{Z}_p \cdot h_1 \oplus \mathbb{Z}_p \cdot h_2 \oplus \mathbb{Z}_p \cdot f$$

- $D^{(m)}(GL_{2,\mathbb{Z}_p}) \subseteq \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is the $\mathbb{Z}_p$-subalgebra generated by

$$\boxed{\nu = q_\nu p^m + r_\nu} \qquad q_{\nu_1}! \frac{e^{\nu_1}}{\nu_1!} \cdot q_{\nu_2}! \binom{h_1}{\nu_2} \cdot q_{\nu_3}! \binom{h_2}{\nu_3} \cdot q_{\nu_4}! \frac{f^{\nu_4}}{\nu_4!}$$

$$\boxed{D^{(m)}(GL_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathcal{U}(\mathfrak{gl}_{2,\mathbb{Z}_p}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b+dx}{a+cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay+c}{by+d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad h_1 \to -x\partial_x$ $\quad h_2 \to x\partial_x$ $\quad f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b + dx}{a + cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay + c}{by + d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad$ $h_1 \to -x\partial_x$ $\quad$ $h_2 \to x\partial_x$ $\quad$ $f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b+dx}{a+cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay+c}{by+d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad h_1 \to -x\partial_x$ $\quad h_2 \to x\partial_x$ $\quad f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b + dx}{a + cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay + c}{by + d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad$ $h_1 \to -x\partial_x$ $\quad$ $h_2 \to x\partial_x$ $\quad$ $f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b+dx}{a+cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay+c}{by+d}$$

gives

$$\boxed{\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})}$$

$e \mapsto \partial_x$ $\quad$ $h_1 \mapsto -x\partial_x$ $\quad$ $h_2 \mapsto x\partial_x$ $\quad$ $f \mapsto x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b + dx}{a + cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay + c}{by + d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad h_1 \to -x\partial_x$ $\quad h_2 \to x\partial_x$ $\quad f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b+dx}{a+cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay+c}{by+d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \to \partial_x$ $\quad$ $h_1 \to -x\partial_x$ $\quad$ $h_2 \to x\partial_x$ $\quad$ $f \to x^2\partial_x$

The Arithmetic Projective Line

$$X = \mathbb{P}^1_{\mathbb{Z}_p} = \mathrm{Spec}(\mathbb{Z}_p[x]) \cup \mathrm{Spec}(\mathbb{Z}_p[y])$$

The canonical right action

$$x \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{b+dx}{a+cx} \qquad y \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{ay+c}{by+d}$$

gives

$$\varphi : \mathfrak{gl}_{2,\mathbb{Z}_p} \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{T}_{\mathbb{P}^1_{\mathbb{Z}_p}}) \to H^0(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}})$$

$e \mapsto \partial_x$ $\quad$ $h_1 \mapsto -x\partial_x$ $\quad$ $h_2 \mapsto x\partial_x$ $\quad$ $f \mapsto x^2\partial_x$

Arithmetic Differential Operators on the Projective Line

For every $m \in \mathbb{N}$, we can consider the sheaf $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}$ $(\subseteq \mathcal{D}_{\mathbb{P}^1_{\mathbb{Z}_p}} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$.

$$\sum_{\nu}^{<\infty} a_\nu[x] \frac{q_\nu^{(m)}!}{\nu!} \partial_x^\nu \qquad \sum_{\nu}^{<\infty} a'_\nu[y] \frac{q_\nu^{(m)}!}{\nu!} \partial_y^\nu$$

The relation

$$\boxed{\binom{x\partial_x}{\nu} = x^\nu \frac{\partial_x^\nu}{\nu!}}$$

allows to complete the diagram

$$\begin{array}{ccccc} & \mathfrak{gl}_{2,\mathbb{Z}_p} & \xrightarrow{\varphi} & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \\ & \downarrow & & \downarrow & \\ q_\nu^{(m)}!\binom{h_i}{\nu} \in & D^{(m)}(GL_{2,\mathbb{Z}_p}) & \dashrightarrow & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \ni x^\nu \frac{q_\nu^{(m)}!}{\nu!} \partial_x^\nu \end{array}$$

Arithmetic Differential Operators on the Projective Line

For every $m \in \mathbb{N}$, we can consider the sheaf $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}$ $(\subseteq \mathcal{D}_{\mathbb{P}^1_{\mathbb{Z}_p}} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$.

$$\sum_{\nu}^{<\infty} a_\nu[x] \frac{q_\nu^{(m)}!}{\nu!} \partial_x^\nu \qquad \sum_{\nu}^{<\infty} a'_\nu[y] \frac{q_\nu^{(m)}!}{\nu!} \partial_y^\nu$$

The relation

$$\boxed{\binom{x\partial_x}{\nu} = x^\nu \frac{\partial_x^\nu}{\nu!}}$$

allows to complete the diagram

$$\begin{array}{ccccc} & \mathfrak{gl}_{2,\mathbb{Z}_p} & \xrightarrow{\varphi} & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \\ & \downarrow & & \downarrow & \\ q_\nu^{(m)}!\binom{h_i}{\nu} \in & D^{(m)}(GL_{2,\mathbb{Z}_p}) & \dashrightarrow & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \ni x^\nu \frac{q_\nu^{(m)}!}{\nu!}\partial_x^\nu \end{array}$$

Arithmetic Differential Operators on the Projective Line

For every $m \in \mathbb{N}$, we can consider the sheaf $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}$ $(\subseteq \mathcal{D}_{\mathbb{P}^1_{\mathbb{Z}_p}} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$.

$$\sum_{\nu}^{<\infty} a_v[x] \frac{q_v^{(m)}!}{v!} \partial_x^v \qquad\qquad \sum_{\nu}^{<\infty} a'_v[y] \frac{q_v^{(m)}!}{v!} \partial_y^v$$

The relation

$$\boxed{\binom{x\partial_x}{\nu} = x^{\nu} \frac{\partial_x^{\nu}}{\nu!}}$$

allows to complete the diagram

$$\begin{array}{ccccc} & \mathfrak{gl}_{2,\mathbb{Z}_p} & \xrightarrow{\varphi} & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \\ & \downarrow & & \downarrow & \\ q_\nu^{(m)}!\binom{h_i}{\nu} \in & D^{(m)}(GL_{2,\mathbb{Z}_p}) & \dashrightarrow & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \ni x^{\nu} \frac{q_\nu^{(m)}!}{\nu!} \partial_x^{\nu} \end{array}$$

Arithmetic Differential Operators on the Projective Line

For every $m \in \mathbb{N}$, we can consider the sheaf $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}$ $(\subseteq \mathcal{D}_{\mathbb{P}^1_{\mathbb{Z}_p}} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$.

$$\sum_{\nu}^{<\infty} a_v[x] \frac{q_v^{(m)}!}{v!} \partial_x^v \qquad \sum_{\nu}^{<\infty} a'_v[y] \frac{q_v^{(m)}!}{v!} \partial_y^v$$

The relation

$$\boxed{\binom{x\partial_x}{\nu} = x^\nu \frac{\partial_x^\nu}{\nu!}}$$

allows to complete the diagram

$$\begin{array}{ccccc} & \mathfrak{gl}_{2,\mathbb{Z}_p} & \xrightarrow{\varphi} & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \\ & \downarrow & & \downarrow & \\ q_\nu^{(m)}!\binom{h_i}{\nu} \in & D^{(m)}(GL_{2,\mathbb{Z}_p}) & \dashrightarrow & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \ni x^\nu \frac{q_\nu^{(m)}!}{\nu!} \partial_x^\nu \end{array}$$

Arithmetic Differential Operators on the Projective Line

For every $m \in \mathbb{N}$, we can consider the sheaf $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}$ $(\subseteq \mathcal{D}_{\mathbb{P}^1_{\mathbb{Z}_p}} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$.

$$\sum_{\nu}^{<\infty} a_v[x] \frac{q_v^{(m)}!}{v!} \partial_x^v \qquad \sum_{\nu}^{<\infty} a'_v[y] \frac{q_v^{(m)}!}{v!} \partial_y^v$$

The relation

$$\boxed{\binom{x\partial_x}{\nu} = x^\nu \frac{\partial_x^\nu}{\nu!}}$$

allows to complete the diagram

$$\begin{array}{ccccc} & \mathfrak{gl}_{2,\mathbb{Z}_p} & \xrightarrow{\varphi} & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(0)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \\ & \downarrow & & \downarrow & \\ q_\nu^{(m)}!\binom{h_i}{\nu} \in & D^{(m)}(GL_{2,\mathbb{Z}_p}) & \dashrightarrow & H^0\left(\mathbb{P}^1_{\mathbb{Z}_p}, \mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathbb{Z}_p}}\right) & \ni x^\nu \frac{q_\nu^{(m)}!}{\nu!}\partial_x^\nu \end{array}$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in \mathbb{N}} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$\xi : \widetilde{X} \to X$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\mathrm{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

How to generalize for non-algebraic characters the Arithmetic Beilinson-Bernstein Localization?

- **For technical reasons we need to restrict our constructions to $\mathbb{Z}_p$.**

We consider

$\mathbf{N} \subseteq \mathbb{B}$ unipotent radical of $\mathbb{B}$

$\widetilde{X} := \mathbb{G}/\mathbf{N}$ the affine basic space

$X := \mathbb{G}/\mathbb{B}$ the flag scheme

$$\xi : \widetilde{X} \to X$$

We will also consider the distribution algebra

$$\text{Dist}(\mathbb{T}) = \varinjlim_{m \in N} D^{(m)}(\mathbb{T})$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(SL_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} T & \\ & T^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[T^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-T)^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for Dist($\mathbb{T}$) $\subseteq$ Dist($\mathbb{T}$) $\otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{bijection}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(\mathrm{SL}_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for Dist($\mathbb{T}$) $\subseteq$ Dist($\mathbb{T}$) $\otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T})\otimes_{\mathbb{Z}_p}\mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t})\otimes_{\mathbb{Z}_p}\mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{\text{bijection}}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(\mathrm{SL}_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for Dist($\mathbb{T}$) $\subseteq$ Dist($\mathbb{T}$) $\otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{\text{bijection}}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(SL_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} T & \\ & T^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[T^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-T)^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for $\mathrm{Dist}(\mathbb{T}) \subseteq \mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T})\otimes_{\mathbb{Z}_p}\mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t})\otimes_{\mathbb{Z}_p}\mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{\text{bijection}}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(\mathrm{SL}_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for $\mathrm{Dist}(\mathbb{T}) \subseteq \mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$\overset{\text{bijection}}{\Longrightarrow}$ $$\boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(SL_{2,\mathbb{Z}_p} \supseteq) \; \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for $\mathrm{Dist}(\mathbb{T}) \subseteq \mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$\overset{bijection}{\Longrightarrow}$ $\boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(\mathrm{SL}_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for $\mathrm{Dist}(\mathbb{T}) \subseteq \mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{\text{bijection}}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

Distribution Algebra

Let us exemplify the avatar Dist($\mathbb{T}$) when $\mathbb{T}$ is the maximal torus

$$(\mathrm{SL}_{2,\mathbb{Z}_p} \supseteq)\ \mathbb{T} = \left\{ \begin{pmatrix} \mathrm{T} & \\ & \mathrm{T}^{-1} \end{pmatrix} \right\} = \mathrm{Spec}(\mathbb{Z}_p[\mathrm{T}^{\pm 1}]) = \mathbb{G}_m$$

$\forall n \in \mathbb{N}$, we can consider $\delta_n((1-\mathrm{T})^i) = \delta_{n,i}$ (Kronecher's delta).

$\{\delta_n\}_{n\in\mathbb{N}}$ is a $\mathbb{Z}_p$-basis for Dist($\mathbb{T}$) $\subseteq$ Dist($\mathbb{T}$) $\otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ s.t

$$\delta_n = \binom{\delta_1}{n}.$$

In particular

$$\boxed{\mathrm{Dist}(\mathbb{T}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = \mathbb{Q}_p[\delta_1] = \mathcal{U}(\mathfrak{t}) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}$$

Now, $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p)$ induces $\lambda \in \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Q}_p)$

$$\lambda(\delta_n) = \binom{\lambda(\delta_1)}{n} \in \mathbb{Z}_p$$

$$\overset{\text{bijection}}{\Longrightarrow} \quad \boxed{\mathrm{Hom}_{\mathbb{Z}_p\text{-mod}}(\mathfrak{t}, \mathbb{Z}_p) \xrightarrow{\simeq} \mathrm{Hom}_{\mathbb{Z}_p\text{-alg}}(\mathrm{Dist}(\mathbb{T}), \mathbb{Z}_p)}$$

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$**-torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \mathrm{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial $\mathbb{T}$-torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \text{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p.$

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$**-torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \mathrm{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$-**torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \text{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$**-torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \text{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$-**torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \mathrm{Dist}(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial $\mathbb{T}$-torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in$ Dist$(\mathbb{T})^*$.

We consider $\Longrightarrow$ $\mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p$.

(Arithmetic) Relative Enveloping Algebras of Finite Level

ξ is a **locally trivial** $\mathbb{T}$-**torsor** $\rightsquigarrow$ Rel. enveloping alg. of finite level

$$\widetilde{\mathcal{D}}^{(m)} := \left(\xi_* \mathcal{D}^{(m)}_{\widetilde{X}}\right)^{\mathbb{T}}$$

It is a $D^{(m)}(\mathbb{T})$-module and gives a family of **t.d.o** on X.

$\lambda \in \mathfrak{t}^*$ $\rightsquigarrow$ $\lambda \in \text{Dist}(\mathbb{T})^*$.

$$\text{We consider} \Longrightarrow \quad \mathcal{D}^{(m)}_{X,\lambda} := \widetilde{\mathcal{D}}^{(m)} \otimes_{D^{(m)}(\mathbb{T}),\lambda} \mathbb{Z}_p.$$

Important Properties

- Let $\mathcal{S}$ the set of affine open subsets of X that trivialize ξ

Let $U \in \mathcal{S}$.

$\widetilde{\mathcal{D}}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U \otimes_{\mathbb{Z}_p} D^{(m)}(\mathbb{T})$

$\Gamma(\mathbb{T}, \mathcal{D}^{(m)})^{\mathbb{T}} = D^{(m)}(\mathbb{T})!$

$\mathcal{D}_{X,\lambda}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U.$

The sheaf $\mathcal{D}_{X,\lambda}^{(m)}$ is an **integral model** of $\mathcal{D}_\lambda$.

Important Properties

- Let $\mathcal{S}$ the set of affine open subsets of X that trivialize ξ

Let $U \in \mathcal{S}$.

$$\widetilde{\mathcal{D}}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U \otimes_{\mathbb{Z}_p} D^{(m)}(\mathbb{T})$$

$$\Gamma(\mathbb{T}, \mathcal{D}^{(m)})^{\mathbb{T}} = D^{(m)}(\mathbb{T})!$$

$$\mathcal{D}_{X,\lambda}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U.$$

The sheaf $\mathcal{D}_{X,\lambda}^{(m)}$ is an **integral model** of $\mathcal{D}_\lambda$.

Important Properties

- Let $\mathcal{S}$ the set of affine open subsets of X that trivialize ξ

Let $U \in \mathcal{S}$.

$$\widetilde{\mathcal{D}}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U \otimes_{\mathbb{Z}_p} D^{(m)}(\mathbb{T})$$

$$\Gamma(\mathbb{T}, \mathcal{D}^{(m)})^{\mathbb{T}} = D^{(m)}(\mathbb{T})!$$

$$\mathcal{D}_{X,\lambda}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U.$$

The sheaf $\mathcal{D}_{X,\lambda}^{(m)}$ is an **integral model** of $\mathcal{D}_\lambda$.

Important Properties

- Let $\mathcal{S}$ the set of affine open subsets of X that trivialize ξ

Let $U \in \mathcal{S}$.

$$\widetilde{\mathcal{D}}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U \otimes_{\mathbb{Z}_p} D^{(m)}(\mathbb{T})$$

$$\Gamma(\mathbb{T}, \mathcal{D}^{(m)})^{\mathbb{T}} = D^{(m)}(\mathbb{T})!$$

$$\mathcal{D}_{X,\lambda}^{(m)}|_U \simeq \mathcal{D}_X^{(m)}|_U.$$

The sheaf $\mathcal{D}_{X,\lambda}^{(m)}$ is an **integral model** of $\mathcal{D}_\lambda$.

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda} / p^{i+1} \mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem. [S]
Let $\lambda \in \mathrm{Dist}(\mathbb{T})^*$ be a character of $\mathrm{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\mathrm{Mod}_{\mathrm{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \overset{H^0(\mathfrak{X},\bullet)}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\boxed{\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).}$$

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda}/p^{i+1}\mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem.[S]
Let $\lambda \in \text{Dist}(\mathbb{T})^*$ be a character of $\text{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\text{Mod}_{\text{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \overset{H^0(\mathfrak{X},\bullet)}{\rightsquigarrow} \text{Mod}_{\text{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).$$

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda}/p^{i+1}\mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem.[S]
Let $\lambda \in \mathrm{Dist}(\mathbb{T})^*$ be a character of $\mathrm{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\mathrm{Mod}_{\mathrm{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \xrightarrow[\rightsquigarrow]{H^0(\mathfrak{X},\bullet)} \mathrm{Mod}_{\mathrm{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).$$

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda} / p^{i+1} \mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem.[S]
Let $\lambda \in \mathrm{Dist}(\mathbb{T})^*$ be a character of $\mathrm{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\mathrm{Mod}_{\mathbf{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \quad \overset{H^0(\mathfrak{X},\bullet)}{\rightsquigarrow} \quad \mathrm{Mod}_{\mathbf{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).$$

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda} / p^{i+1}\mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem.[S]
Let $\lambda \in \mathrm{Dist}(\mathbb{T})^*$ be a character of $\mathrm{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\mathrm{Mod}_{\mathbf{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \quad \overset{H^0(\mathfrak{X},\bullet)}{\rightsquigarrow} \quad \mathrm{Mod}_{\mathbf{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).$$

Arithmetic Beilinson-Bernstein Theorem

As in the algebraic case, we consider

$$\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} := \varinjlim_{m\in\mathbb{N}} \left(\underbrace{\left(\varprojlim_{i\in\mathbb{N}} \mathcal{D}^{(m)}_{X,\lambda} / p^{i+1} \mathcal{D}^{(m)}_{X,\lambda} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p}_{\hat{\mathscr{D}}^{(m)}_{\mathfrak{X},\lambda,\mathbb{Q}_p}} \right).$$

Theorem.[S]
Let $\lambda \in \mathrm{Dist}(\mathbb{T})^*$ be a character of $\mathrm{Dist}(\mathbb{T})$ s.t $\lambda + \rho \in \mathfrak{t}^*_{\mathbb{Q}_p}$ is a dominant and regular character. Then

$$\mathrm{Mod}_{\mathbf{coh}}(\mathscr{D}^{\dagger}_{\mathfrak{X},\lambda}) \quad \overset{H^0(\mathfrak{X},\bullet)}{\rightsquigarrow} \quad \mathrm{Mod}_{\mathbf{fp}}(D^{\dagger}(\mathbb{G})_{\lambda})$$

- The inverse functor is determined by the localization functor

$$\boxed{\mathscr{L}oc^{\dagger}_{\mathfrak{X},\lambda}(\bullet) := \mathscr{D}^{\dagger}_{\mathfrak{X},\lambda} \otimes_{D^{\dagger}(\mathbb{G})_{\lambda}} (\bullet).}$$

References

1. P. Berthelot, $\mathcal{D}$-modules arithmétiques I. Opérateurs différentiels de niveau fini. Ann. Sci. École Norm. Sup.(4), 29(2), 185-272.

2. C. Huyghe, $\mathcal{D}^{\dagger}$-affinité de l'espace projectif. Compositio Mathematica, 1997, vol. 108, no 3, p. 277-318.

3. C. Huyghe, Un théorème de Beilinson-Bernstein pour les $\mathcal{D}$-modules arithmétiques. Bull. Soc. math. France 137 (2), 2009, p. 159–183.

4. C. Huyghe & T. Schmidt, $\mathcal{D}$-modules arithmétiques, distributions et localisation. Rendiconti del Seminario Matematico della Università di Padova 139, 1–76 (2018).

5. C. Huyghe & T. Schmidt, $\mathcal{D}$-modules arithmétiques sur la variété de drapeaux.To appear in Journal für die Reine und Angewandte Mathematik. doi 10.1515/ crelle-2017-0021.

6. A. Sarrazola Alzate, A Beilinson-Bernstein theorem for twisted arithmetic differential operators on the formal flag variety. arXiv preprint arXiv:1811.11567v4. Preprint 2019.

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\boxed{\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]}$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\rightarrow} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\boxed{\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d} \cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]}$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\rightarrow} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\boxed{\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]}$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \qquad \rightsquigarrow \qquad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d} \cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \qquad \rightsquigarrow \qquad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\boxed{\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]}$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \overset{t\mapsto p^k}{\rightarrow} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \quad \rightsquigarrow \quad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Arithmetic Differential Operators with Congruence Level

Consider the order filtration $\{\mathcal{D}^{(m)}_{X,d}\}_{d\in\mathbb{N}}$ and the asso. Rees ring

$$\boxed{\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) := \bigoplus_{d\in\mathbb{N}} \mathcal{D}^{(m)}_{X,d}\cdot t^d \subseteq \mathcal{D}^{(m)}_X[t]}$$

Its specialization in p^k

$$\mathcal{D}^{(m,k)}_X := \mathrm{Im}\left(\mathrm{R}\left(\mathcal{D}^{(m)}_X\right) \stackrel{t\mapsto p^k}{\to} \mathcal{D}^{(m)}_X\right)$$

it is the sheaf of **differential operators with congruence level** k.

$$gr(\mathcal{D}^{(m)}_X) = \mathrm{Sym}^{(m)}(\mathcal{T}_X) \qquad \rightsquigarrow \qquad \mathcal{D}^{(m,k)}_{X,d} = \sum_{l=0}^{d} p^{kl}\mathcal{D}^{(m)}_{X,l}$$

$$gr(\mathcal{D}^{(m,k)}_X) = \mathrm{Sym}^{(m)}(p^k\mathcal{T}_X).$$

Historical Setting

Arithmetic Version

Arithmetic Differential operators on Admissible Blow-ups

G_0-equivariance of Formal Models of Flag Varieties

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \text{ such that } p^k \in \mathcal{I}$$

A blow-up $\mathrm{pr} : Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let $\mathrm{pr} : Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \mathrm{pr}^* \mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\mathrm{pr}^{-1}\mathcal{O}_X} \mathrm{pr}^{-1} \mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1 \partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1 \partial_2$$

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \text{ such that } p^k \in \mathcal{I}$$

A blow-up $\mathrm{pr} : Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let $\mathrm{pr} : Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \mathrm{pr}^*\mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\mathrm{pr}^{-1}\mathcal{O}_X} \mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1\partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1\partial_2$$

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \text{ such that } p^k \in \mathcal{I}$$

A blow-up $\mathrm{pr} : Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let $\mathrm{pr} : Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \mathrm{pr}^* \mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\mathrm{pr}^{-1}\mathcal{O}_X} \mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1\partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1\partial_2$$

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \text{ such that } p^k \in \mathcal{I}$$

A blow-up pr : $Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let pr : $Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \text{pr}^* \mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\text{pr}^{-1}\mathcal{O}_X} \text{pr}^{-1}\mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\text{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1\partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1\partial_2$$

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \quad \text{such that} \quad p^k \in \mathcal{I}$$

A blow-up $\mathrm{pr} : Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let $\mathrm{pr} : Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \mathrm{pr}^* \mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\mathrm{pr}^{-1}\mathcal{O}_X} \mathrm{pr}^{-1} \mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1 \partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1 \partial_2$$

Admissible Blow-ups

$$\mathcal{I} \subseteq \mathcal{O}_X \ \text{ such that } \ p^k \in \mathcal{I}$$

A blow-up $\mathrm{pr} : Y \to X$ along $V(\mathcal{I})$ is called an **admissible blow-up**.

- **Congruence level** of Y:

$$k_Y := \min_{\mathcal{I}} \min\{k \in \mathbb{N} \mid p^k \in \mathcal{I}\}$$

Theorem.[HS] Let $\mathrm{pr} : Y \to X$ be an admissible blow-up and $k \geq k_Y$. Then

$$\mathcal{D}_Y^{(m,k)} = \mathrm{pr}^* \mathcal{D}_X^{(m,k)} = \mathcal{O}_Y \otimes_{\mathrm{pr}^{-1}\mathcal{O}_X} \mathrm{pr}^{-1} \mathcal{D}_X^{(m,k)}$$

is endowed with a mult. structure extending $\mathrm{pr}^{-1}\mathcal{D}_X^{(m,k)}$.

$$(f_1 \otimes \partial_1) \cdot (f_2 \otimes \partial_2) = f_1 \partial_1(f_2) \otimes \partial_2 + f_1 f_2 \otimes \partial_1 \partial_2$$

Twisted Differential Operators on Formal Models of Flags Varieties

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\mathcal{L}(\lambda)_{/\mathcal{Y}}$

$\lambda \in X(\mathbb{T}) \quad \rightsquigarrow$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1} \mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

$\text{pr} : \mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\mathcal{L}(\lambda)_{/\mathcal{Y}}$

$\lambda \in X(\mathbb{T}) \quad \rightsquigarrow$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1}\mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

$\text{pr} : \mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\mathcal{L}(\lambda)_{/\mathcal{Y}}$

$\lambda \in X(\mathbb{T}) \rightsquigarrow$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1} \mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

pr : $\mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\lambda \in X(\mathbb{T})$ $\rightsquigarrow$ $\boxed{\mathcal{L}(\lambda)_{/\mathcal{Y}}}$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1}\mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

pr : $\mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\lambda \in X(\mathbb{T}) \quad \rightsquigarrow \quad \boxed{\mathcal{L}(\lambda)_{/\mathcal{Y}}}$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1}\mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

$\text{pr} : \mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\lambda \in X(\mathbb{T}) \quad \rightsquigarrow \quad \boxed{\mathcal{L}(\lambda)_{/\mathcal{Y}}}$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1}\mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Twisted Differential Operators on Formal Models of Flags Varieties

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ formal completion of an admissible blow-up.

$\lambda \in X(\mathbb{T}) \quad \rightsquigarrow \quad \boxed{\mathcal{L}(\lambda)_{/\mathcal{Y}}}$

If $k \geq k_Y$ we have a sheaf of **differential operators** on $\mathcal{Y}$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k} := \varinjlim_{m\in\mathbb{N}} \left(\varprojlim_{j\in\mathbb{N}} \mathcal{D}^{(m,k)}_{Y} / p^{j+1} \mathcal{D}^{(m,k)}_{Y} \right) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$$

- Let us fix $k \geq k_Y$.

We can define a sheaf of λ-twisted differential operators acting over $\mathscr{L}(\lambda)$

$$\mathscr{D}^{\dagger}_{\mathcal{Y},k}(\lambda) := \mathscr{L}(\lambda) \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{D}^{\dagger}_{\mathcal{Y},k} \otimes_{\mathcal{O}_{\mathcal{Y}}} \mathscr{L}(\lambda)^{\vee}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Beilinson-Bernstein with Congruence Level

Theorem. [S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{X},k,\lambda}) \underset{\mathscr{L}oc^{\dagger}}{\overset{H^0(\mathfrak{X},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{fp}(D^{\dagger}(\mathbb{G}(k))_{\lambda})$$

Congruence groups

- $\mathbb{G}(k)$ denotes the k-th congruence subgroup.

Example

$\mathbb{G} = \mathrm{GL}_{2,\mathbb{Z}_p}$

$$\mathbb{G}(k)(\mathbb{Z}_p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a-1, b, c, d-1 \in p^k\mathbb{Z}_p \right\}$$

Localization Theorem over an Admissible Blow-up

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$\boxed{H^0(\mathcal{Y}, \bullet) = H^0(\mathfrak{X}, \bullet) \circ \mathrm{pr}_*}$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $pr_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{pr_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$H^0(\mathcal{Y},\bullet) = H^0(\mathfrak{X},\bullet) \circ pr_*$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$H^0(\mathcal{Y},\bullet) = H^0(\mathfrak{X},\bullet) \circ \mathrm{pr}_*$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$H^0(\mathcal{Y}, \bullet) = H^0(\mathfrak{X}, \bullet) \circ \mathrm{pr}_*$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$H^0(\mathcal{Y},\bullet) = H^0(\mathfrak{X},\bullet) \circ \mathrm{pr}_*$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod_{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod_{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$H^0(\mathcal{Y}, \bullet) = H^0(\mathfrak{X}, \bullet) \circ \mathrm{pr}_*$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod_{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod_{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$\boxed{H^0(\mathcal{Y}, \bullet) = H^0(\mathfrak{X}, \bullet) \circ \mathrm{pr}_*}$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$\boxed{H^0(\mathcal{Y},\bullet) = H^0(\mathfrak{X},\bullet) \circ \mathrm{pr}_*}$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

pr : $\mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$\boxed{H^0(\mathcal{Y},\bullet) = H^0(\mathfrak{X},\bullet) \circ \mathrm{pr}_*}$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

Localization Theorem over an Admissible Blow-up

$\mathrm{pr} : \mathcal{Y} \to \mathfrak{X}$ a (formal) admissible blow-up.

Theorem[S]

We have $\mathrm{pr}_* \mathscr{D}^\dagger_{\mathcal{Y},k,\lambda} = \mathscr{D}^\dagger_{\mathfrak{X},k,\lambda}$. Moreover

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \overset{\mathrm{pr}_*}{\rightsquigarrow} \mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathfrak{X},k,\lambda})$$

$$\boxed{H^0(\mathcal{Y}, \bullet) = H^0(\mathfrak{X}, \bullet) \circ \mathrm{pr}_*}$$

Corollary

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho$ is dominant and regular. Then

$$\mathrm{Mod}_{\mathrm{Coh}}(\mathscr{D}^\dagger_{\mathcal{Y},k,\lambda}) \underset{\mathscr{L}oc^\dagger}{\overset{H^0(\mathcal{Y},\bullet)}{\rightleftarrows}} \mathrm{Mod}_{\mathrm{fp}}(D^\dagger(\mathbb{G}(k))_\lambda)$$

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules
$\Updownarrow$ $(\bullet)'_b$	
$D(G_0, \mathbb{Q}_p)_\lambda$	

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules
$\Updownarrow \ (\bullet)'_b$	
$D(G_0, \mathbb{Q}_p)_\lambda$	

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules

$\Updownarrow$ $(\bullet)'_b$

$D(G_0, \mathbb{Q}_p)_\lambda$

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules

$\Updownarrow$ $(\bullet)'_b$

$$\boxed{D(G_0, \mathbb{Q}_p)_\lambda}$$

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules

$\Updownarrow$ $(\bullet)'_b$

$D(G_0, \mathbb{Q}_p)_\lambda$

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

G_0-equivariance of Formal Models of Flag Varieties

We start by considering the compact p-adic group $G_0 := \mathbb{G}(\mathbb{Z}_p)$.

Locally analytic G_0-representations (with central char. λ)	*Coadmissible* G_0-equivariant arithmetic $\mathscr{D}_\lambda$-modules
$\Updownarrow$ $(\bullet)'_b$	
$D(G_0, \mathbb{Q}_p)_\lambda$	

Key point: to build a **weak Fréchet-Stein** structure over $D(G_0, \mathbb{Q}_p)$.

Weak Fréchet-Stein Structure

The work carried by Huyghe-Schmidt gives us

$$D^{\dagger}(\mathbb{G}(k))_{\lambda} \xrightarrow{\simeq} \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})_{\lambda}$$

where $\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ}) := \mathrm{Hom}^{\mathrm{cont}}_{\mathbb{Q}_p}(\mathcal{O}_{\mathbb{G}(k)^{\circ}}(\mathbb{G}(k)^{\circ}), \mathbb{Q}_p)$.

Let $D(\mathbb{G}(k)^{\circ}, G_0) := (\mathcal{C}^{\mathrm{cont}}(G_0, \mathbb{Q}_p)_{\mathbb{G}(k)^{\circ}-\mathrm{an}})'_b$ such that

$$D(G_0, L) \xrightarrow{\simeq} \varprojlim_{k\in\mathbb{N}} D(\mathbb{G}(k)^{\circ}, G_0)$$

defines a weak Fréchet-Stein algebra structure over $D(G_0, \mathbb{Q}_p)$. Moreover

$$D(\mathbb{G}(k)^{\circ}, G_0) \overset{\mathrm{Rings}}{=} \bigoplus_{g\in G_0/G_k} \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})\delta_g.$$

$G_k := \mathbb{G}(k)(\mathbb{Z}_p)$ and δ_g is the Dirac distribution supported at g.

Weak Fréchet-Stein Structure

The work carried by Huyghe-Schmidt gives us

$$\boxed{D^{\dagger}(\mathbb{G}(k))_{\lambda} \xrightarrow{\simeq} \mathcal{D}^{an}(\mathbb{G}(k)^{\circ})_{\lambda}}$$

where $\mathcal{D}^{an}(\mathbb{G}(k)^{\circ}) := \mathrm{Hom}^{cont}_{\mathbb{Q}_p}(\mathcal{O}_{\mathbb{G}(k)^{\circ}}(\mathbb{G}(k)^{\circ}), \mathbb{Q}_p)$.

Let $D(\mathbb{G}(k)^{\circ}, G_0) := (\mathcal{C}^{cont}(G_0, \mathbb{Q}_p)_{\mathbb{G}(k)^{\circ}-an})'_b$ such that

$$\boxed{D(G_0, L) \xrightarrow{\simeq} \varprojlim_{k\in\mathbb{N}} D(\mathbb{G}(k)^{\circ}, G_0)}$$

defines a weak Fréchet-Stein algebra structure over $D(G_0, \mathbb{Q}_p)$. Moreover

$$\boxed{D(\mathbb{G}(k)^{\circ}, G_0) \overset{\text{Rings}}{=} \bigoplus_{g\in G_0/G_k} \mathcal{D}^{an}(\mathbb{G}(k)^{\circ})\delta_g.}$$

$G_k := \mathbb{G}(k)(\mathbb{Z}_p)$ and δ_g is the Dirac distribution supported at g.

Weak Fréchet-Stein Structure

The work carried by Huyghe-Schmidt gives us

$$\boxed{D^{\dagger}(\mathbb{G}(k))_{\lambda} \xrightarrow{\simeq} \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})_{\lambda}}$$

where $\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ}) := \mathrm{Hom}^{\mathrm{cont}}_{\mathbb{Q}_p}(\mathcal{O}_{\mathbb{G}(k)^{\circ}}(\mathbb{G}(k)^{\circ}), \mathbb{Q}_p)$.

Let $D(\mathbb{G}(k)^{\circ}, G_0) := (\mathcal{C}^{\mathrm{cont}}(G_0, \mathbb{Q}_p)_{\mathbb{G}(k)^{\circ}-\mathrm{an}})'_b$ such that

$$\boxed{D(G_0, L) \xrightarrow{\simeq} \varprojlim_{k\in\mathbb{N}} D(\mathbb{G}(k)^{\circ}, G_0)}$$

defines a <u>weak Fréchet-Stein</u> algebra structure over $D(G_0, \mathbb{Q}_p)$. Moreover

$$\boxed{D(\mathbb{G}(k)^{\circ}, G_0) \overset{\mathrm{Rings}}{=} \bigoplus_{g\in G_0/G_k} \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})\delta_g .}$$

$G_k := \mathbb{G}(k)(\mathbb{Z}_p)$ and δ_g is the Dirac distribution supported at g.

Weak Fréchet-Stein Structure

The work carried by Huyghe-Schmidt gives us

$$\boxed{D^{\dagger}(\mathbb{G}(k))_{\lambda} \xrightarrow{\simeq} \mathcal{D}^{an}(\mathbb{G}(k)^{\circ})_{\lambda}}$$

where $\mathcal{D}^{an}(\mathbb{G}(k)^{\circ}) := \mathrm{Hom}^{cont}_{\mathbb{Q}_p}(\mathcal{O}_{\mathbb{G}(k)^{\circ}}(\mathbb{G}(k)^{\circ}), \mathbb{Q}_p)$.

Let $D(\mathbb{G}(k)^{\circ}, G_0) := (\mathcal{C}^{cont}(G_0, \mathbb{Q}_p)_{\mathbb{G}(k)^{\circ}-an})'_b$ such that

$$\boxed{D(G_0, L) \xrightarrow{\simeq} \varprojlim_{k\in\mathbb{N}} D(\mathbb{G}(k)^{\circ}, G_0)}$$

defines a <u>weak Fréchet-Stein</u> algebra structure over $D(G_0, \mathbb{Q}_p)$. Moreover

$$\boxed{D(\mathbb{G}(k)^{\circ}, G_0) \overset{\text{Rings}}{=} \bigoplus_{g\in G_0/G_k} \mathcal{D}^{an}(\mathbb{G}(k)^{\circ})\delta_g.}$$

$G_k := \mathbb{G}(k)(\mathbb{Z}_p)$ and δ_g is the Dirac distribution supported at g.

Strongly G_0-equivariant Arithmetic $\mathscr{D}_\lambda$-modules

On the geometric side, if pr : $\mathfrak{Y} \to \mathfrak{X}$ is admissible and G_0-equivariant we have a left G_0-action

$$T_g : \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \to (\rho_g)_* \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \qquad T_{hg} = (\rho_g)_* T_h \circ T_g$$

$h, g \in G_0$ and $\rho_g : \mathfrak{Y} \to \mathfrak{Y}$ is the comorphism induced by the action.

A **coherent** $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$**-module** $\mathscr{M}$ **is strongly** G_0**-equivariant**, if there exists a family $(\varphi_g)_{g\in G_0}$ of isomorphisms

$$\varphi_g : \mathscr{M} \to (\rho_g)_* \mathscr{M}$$

of sheaves of $\mathbb{Q}_p$-vect. spaces satisfying the following properties (†):

- $\forall h, g \in G_0$, we have $\varphi_{hg} = (\rho_g)_* \varphi_h \circ \varphi_g$.
- Locally $\varphi_g(P \cdot m) = T_g(P) \cdot \varphi_g(m)$.
- If $g \in G_{k+1}$, then $\varphi_g =$ multiplication by $\delta_g \in \mathcal{D}^{an}(\mathbb{G}(k)^\circ)_\lambda$.

Strongly G_0-equivariant Arithmetic $\mathscr{D}_\lambda$-modules

On the geometric side, if $\mathrm{pr} : \mathfrak{Y} \to \mathfrak{X}$ is admissible and G_0-equivariant we have a left G_0-action

$$T_g : \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \to (\rho_g)_* \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \qquad T_{hg} = (\rho_g)_* T_h \circ T_g$$

$h, g \in G_0$ and $\rho_g : \mathfrak{Y} \to \mathfrak{Y}$ is the comorphism induced by the action.

A **coherent** $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$**-module** $\mathscr{M}$ **is strongly** G_0**-equivariant**, if there exists a family $(\varphi_g)_{g\in G_0}$ of isomorphisms

$$\varphi_g : \mathscr{M} \to (\rho_g)_* \mathscr{M}$$

of sheaves of $\mathbb{Q}_p$-vect. spaces satisfying the following properties (†):

- $\forall h, g \in G_0$, we have $\varphi_{hg} = (\rho_g)_* \varphi_h \circ \varphi_g$.
- Locally $\varphi_g(P \cdot m) = T_g(P) \cdot \varphi_g(m)$.
- If $g \in G_{k+1}$, then $\varphi_g =$ multiplication by $\delta_g \in \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})_{\lambda}$.

Strongly G_0-equivariant Arithmetic $\mathscr{D}_\lambda$-modules

On the geometric side, if $\mathrm{pr} : \mathfrak{Y} \to \mathfrak{X}$ is admissible and G_0-equivariant we have a left G_0-action

$$T_g : \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \to (\rho_g)_* \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \qquad T_{hg} = (\rho_g)_* T_h \circ T_g$$

$h, g \in G_0$ and $\rho_g : \mathfrak{Y} \to \mathfrak{Y}$ is the comorphism induced by the action.

A **coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-module $\mathscr{M}$ is strongly G_0-equivariant**, if there exists a family $(\varphi_g)_{g \in G_0}$ of isomorphisms

$$\varphi_g : \mathscr{M} \to (\rho_g)_* \mathscr{M}$$

of sheaves of $\mathbb{Q}_p$-vect. spaces satisfying the following properties (†):

- $\forall h, g \in G_0$, we have $\varphi_{hg} = (\rho_g)_* \varphi_h \circ \varphi_g$.
- Locally $\varphi_g(P \cdot m) = T_g(P) \cdot \varphi_g(m)$.
- If $g \in G_{k+1}$, then $\varphi_g =$ multiplication by $\delta_g \in \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^{\circ})_\lambda$.

Strongly G_0-equivariant Arithmetic $\mathscr{D}_\lambda$-modules

On the geometric side, if $\mathrm{pr} : \mathfrak{Y} \to \mathfrak{X}$ is admissible and G_0-equivariant we have a left G_0-action

$$T_g : \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \to (\rho_g)_* \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \qquad T_{hg} = (\rho_g)_* T_h \circ T_g$$

$h, g \in G_0$ and $\rho_g : \mathfrak{Y} \to \mathfrak{Y}$ is the comorphism induced by the action.

A **coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module $\mathscr{M}$ is strongly G_0-equivariant**, if there exists a family $(\varphi_g)_{g \in G_0}$ of isomorphisms

$$\varphi_g : \mathscr{M} \to (\rho_g)_* \mathscr{M}$$

of sheaves of $\mathbb{Q}_p$-vect. spaces satisfying the following properties (†):

- $\forall h, g \in G_0$, we have $\varphi_{hg} = (\rho_g)_* \varphi_h \circ \varphi_g$.
- Locally $\varphi_g(P \cdot m) = T_g(P) \cdot \varphi_g(m)$.
- If $g \in G_{k+1}$, then $\varphi_g =$ multiplication by $\delta_g \in \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda$.

Strongly G_0-equivariant Arithmetic $\mathscr{D}_\lambda$-modules

On the geometric side, if $\mathrm{pr}:\mathfrak{Y}\to\mathfrak{X}$ is admissible and G_0-equivariant we have a left G_0-action

$$T_g : \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \to (\rho_g)_*\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \qquad T_{hg} = (\rho_g)_* T_h \circ T_g$$

$h, g \in G_0$ and $\rho_g : \mathfrak{Y} \to \mathfrak{Y}$ is the comorphism induced by the action.

A **coherent** $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$**-module** $\mathscr{M}$ **is strongly** G_0**-equivariant**, if there exists a family $(\varphi_g)_{g\in G_0}$ of isomorphisms

$$\varphi_g : \mathscr{M} \to (\rho_g)_*\mathscr{M}$$

of sheaves of $\mathbb{Q}_p$-vect. spaces satisfying the following properties (†):

- $\forall h, g \in G_0$, we have $\varphi_{hg} = (\rho_g)_*\varphi_h \circ \varphi_g$.
- Locally $\varphi_g(P\cdot m) = T_g(P)\cdot\varphi_g(m)$.
- If $g \in G_{k+1}$, then $\varphi_g =$ multiplication by $\delta_g \in \mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda$.

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$\begin{array}{c} D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods} \\ \text{of finite} \\ \text{presentations} \end{array} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^{\dagger}_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$\begin{array}{c} D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods} \\ \text{of finite} \\ \text{presentations} \end{array} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods of finite presentations} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^{\dagger}_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods of finite presentations} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods of finite presentations} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \mathrm{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods of finite presentations} \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \mathrm{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$$

First Equivalence

Let us denote

- $\mathcal{C}_{G_0,\lambda} := \{\text{Coadmissible } D(G_0,\mathbb{Q}_p)\text{-modules}\} \cap \text{Mod}(D(G_0,\mathbb{Q}_p)_\lambda)$.
- $\text{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$; category of strongly G_0-equivariant coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-modules.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$\begin{array}{c} D(\mathbb{G}(k)^\circ, G_0)_\lambda\text{-mods} \\ \text{of finite} \\ \text{presentations} \end{array} \quad \underset{H^0(\mathfrak{Y},\bullet)}{\overset{\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)}{\rightleftarrows}} \quad \text{Coh}(\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda), G_0)$$

Coadmissible G_0-equivariant $\mathscr{D}_\lambda$-modules

Still on the geometric side, Let $\underline{\mathcal{F}}_{\mathfrak{X}}$ be the set of couples $(\mathfrak{Y}, k)$ such that $\mathfrak{Y}$ is an admissible blow-up $\mathfrak{X}$ and $k \geq k_Y$.

Let $g \in G_0$. Pour every $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ there exists $(\mathfrak{Y} \cdot g, k_{\mathfrak{Y} \cdot g}) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ endowed with an isomorphism $\rho_g : \mathfrak{Y} \to \mathfrak{Y} \cdot g$ such that $k_{\mathfrak{Y}} = k_{\mathfrak{Y} \cdot g}$.

A family $\mathscr{M} := (\mathscr{M}_{\mathfrak{Y},k})_{(\mathfrak{Y},k) \in \underline{\mathcal{F}}_{\mathfrak{X}}}$ of coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules is a **coadmissible** G_0-equivariant $\mathscr{D}(\lambda)$-**module** over $\underline{\mathcal{F}}_{\mathfrak{X}}$, if for every $g \in G_0$, there is

$$\varphi_g : \mathscr{M}_{\mathfrak{Y} \cdot g, k} \to (\rho_g)_* \mathscr{M}_{\mathfrak{Y},k}$$

satisfying (†) and such that if $(\mathfrak{Y}', k') \succeq (\mathfrak{Y}, k)$ with $\pi : \mathfrak{Y}' \to \mathfrak{Y}$, then there exists a transition morphism $\pi_* \mathscr{M}_{\mathfrak{Y}',k'} \to \mathscr{M}_{\mathfrak{Y},k}$.

Those morphisms allow us to form the proj. limit

$$\Gamma(\mathscr{M}) := \varprojlim_{(\mathfrak{Y},k) \in \underline{\mathcal{F}}_{\mathfrak{X}}} H^0(\mathfrak{Y}, \mathscr{M}_{\mathfrak{Y},k}).$$

Coadmissible G_0-equivariant $\mathscr{D}_\lambda$-modules

Still on the geometric side, Let $\underline{\mathcal{F}}_{\mathfrak{X}}$ be the set of couples $(\mathfrak{Y}, k)$ such that $\mathfrak{Y}$ is an admissible blow-up $\mathfrak{X}$ and $k \geq k_Y$.

Let $g \in G_0$. Pour every $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ there exists $(\mathfrak{Y} \cdot g, k_{\mathfrak{Y}\cdot g}) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ endowed with an isomorphism $\rho_g : \mathfrak{Y} \to \mathfrak{Y} \cdot g$ such that $k_{\mathfrak{Y}} = k_{\mathfrak{Y}\cdot g}$.

A family $\mathscr{M} := (\mathscr{M}_{\mathfrak{Y},k})_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}$ of coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules is a **coadmissible** G_0-equivariant $\mathscr{D}(\lambda)$-**module** over $\underline{\mathcal{F}}_{\mathfrak{X}}$, if for every $g \in G_0$, there is

$$\varphi_g : \mathscr{M}_{\mathfrak{Y}.g,k} \to (\rho_g)_* \mathscr{M}_{\mathfrak{Y},k}$$

satisfying (†) and such that if $(\mathfrak{Y}', k') \succeq (\mathfrak{Y}, k)$ with $\pi : \mathfrak{Y}' \to \mathfrak{Y}$, then there exists a transition morphism $\pi_* \mathscr{M}_{\mathfrak{Y}',k'} \to \mathscr{M}_{\mathfrak{Y},k}$.

Those morphisms allow us to form the proj. limit

$$\Gamma(\mathscr{M}) := \varprojlim_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}} H^0(\mathfrak{Y}, \mathscr{M}_{\mathfrak{Y},k}).$$

Coadmissible G_0-equivariant $\mathscr{D}_\lambda$-modules

Still on the geometric side, Let $\underline{\mathcal{F}}_{\mathfrak{X}}$ be the set of couples $(\mathfrak{Y}, k)$ such that $\mathfrak{Y}$ is an admissible blow-up $\mathfrak{X}$ and $k \geq k_Y$.

Let $g \in G_0$. Pour every $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ there exists $(\mathfrak{Y} \cdot g, k_{\mathfrak{Y}\cdot g}) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ endowed with an isomorphism $\rho_g : \mathfrak{Y} \to \mathfrak{Y} \cdot g$ such that $k_{\mathfrak{Y}} = k_{\mathfrak{Y}\cdot g}$.

A family $\mathscr{M} := (\mathscr{M}_{\mathfrak{Y},k})_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}$ of coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules is a **coadmissible** G_0-equivariant $\mathscr{D}(\lambda)$-**module** over $\underline{\mathcal{F}}_{\mathfrak{X}}$, if for every $g \in G_0$, there is

$$\varphi_g : \mathscr{M}_{\mathfrak{Y}.g,k} \to (\rho_g)_* \mathscr{M}_{\mathfrak{Y},k}$$

satisfying (†) and such that if $(\mathfrak{Y}', k') \succeq (\mathfrak{Y}, k)$ with $\pi : \mathfrak{Y}' \to \mathfrak{Y}$, then there exists a transition morphism $\pi_* \mathscr{M}_{\mathfrak{Y}',k'} \to \mathscr{M}_{\mathfrak{Y},k}$.

Those morphisms allow us to form the proj. limit

$$\Gamma(\mathscr{M}) := \varprojlim_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}} H^0(\mathfrak{Y}, \mathscr{M}_{\mathfrak{Y},k}).$$

Coadmissible G_0-equivariant $\mathscr{D}_\lambda$-modules

Still on the geometric side, Let $\underline{\mathcal{F}}_{\mathfrak{X}}$ be the set of couples $(\mathfrak{Y}, k)$ such that $\mathfrak{Y}$ is an admissible blow-up $\mathfrak{X}$ and $k \geq k_Y$.

Let $g \in G_0$. Pour every $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ there exists $(\mathfrak{Y} \cdot g, k_{\mathfrak{Y}\cdot g}) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ endowed with an isomorphism $\rho_g : \mathfrak{Y} \to \mathfrak{Y} \cdot g$ such that $k_{\mathfrak{Y}} = k_{\mathfrak{Y}\cdot g}$.

A family $\mathscr{M} := (\mathscr{M}_{\mathfrak{Y},k})_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}$ of coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules is a **coadmissible** G_0-equivariant $\mathscr{D}(\lambda)$-**module** over $\underline{\mathcal{F}}_{\mathfrak{X}}$, if for every $g \in G_0$, there is

$$\varphi_g : \mathscr{M}_{\mathfrak{Y}.g,k} \to (\rho_g)_* \mathscr{M}_{\mathfrak{Y},k}$$

satisfying $(\dagger)$ and such that if $(\mathfrak{Y}', k') \succeq (\mathfrak{Y}, k)$ with $\pi : \mathfrak{Y}' \to \mathfrak{Y}$, then there exists a transition morphism $\pi_* \mathscr{M}_{\mathfrak{Y}',k'} \to \mathscr{M}_{\mathfrak{Y},k}$.

Those morphisms allow us to form the proj. limit

$$\Gamma(\mathscr{M}) := \varprojlim_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}} H^0(\mathfrak{Y}, \mathscr{M}_{\mathfrak{Y},k}).$$

Coadmissible G_0-equivariant $\mathscr{D}_\lambda$-modules

Still on the geometric side, Let $\underline{\mathcal{F}}_{\mathfrak{X}}$ be the set of couples $(\mathfrak{Y}, k)$ such that $\mathfrak{Y}$ is an admissible blow-up $\mathfrak{X}$ and $k \geq k_Y$.

Let $g \in G_0$. Pour every $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ there exists $(\mathfrak{Y} \cdot g, k_{\mathfrak{Y}\cdot g}) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ endowed with an isomorphism $\rho_g : \mathfrak{Y} \to \mathfrak{Y} \cdot g$ such that $k_{\mathfrak{Y}} = k_{\mathfrak{Y}\cdot g}$.

A family $\mathscr{M} := (\mathscr{M}_{\mathfrak{Y},k})_{(\mathfrak{Y},k)\in \underline{\mathcal{F}}_{\mathfrak{X}}}$ of coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-modules is a **coadmissible** G_0-equivariant $\mathscr{D}(\lambda)$-**module** over $\underline{\mathcal{F}}_{\mathfrak{X}}$, if for every $g \in G_0$, there is

$$\varphi_g : \mathscr{M}_{\mathfrak{Y}.g,k} \to (\rho_g)_* \mathscr{M}_{\mathfrak{Y},k}$$

satisfying $(\dagger)$ and such that if $(\mathfrak{Y}', k') \succeq (\mathfrak{Y}, k)$ with $\pi : \mathfrak{Y}' \to \mathfrak{Y}$, then there exists a transition morphism $\pi_* \mathscr{M}_{\mathfrak{Y}',k'} \to \mathscr{M}_{\mathfrak{Y},k}$.

Those morphisms allow us to form the proj. limit

$$\Gamma(\mathscr{M}) := \varprojlim_{(\mathfrak{Y},k)\in \underline{\mathcal{F}}_{\mathfrak{X}}} H^0(\mathfrak{Y}, \mathscr{M}_{\mathfrak{Y},k}).$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$M \in \mathcal{C}_{G_0,\lambda}$ $\quad (\bullet)'_b \quad$ $V \in \mathrm{Rep}^{adm}(G_0)$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathrm{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \quad \overset{(\bullet)'_b}{\rightsquigarrow} \quad V \in \mathrm{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathrm{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \qquad \overset{(\bullet)'_b}{\rightsquigarrow} \qquad V \in \mathrm{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathrm{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^{\dagger}_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^{\dagger}_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_{\lambda}(M) := \left(\mathscr{L}oc^{\dagger}_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \qquad \overset{(\bullet)'_b}{\rightsquigarrow} \qquad V \in \mathrm{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathrm{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathrm{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

The Localization Functor

- $\text{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \quad \overset{(\bullet)'_b}{\rightsquigarrow} \quad V \in \text{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathbf{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathbf{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \qquad \overset{(\bullet)'_b}{\rightsquigarrow} \qquad V \in \mathrm{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathbf{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathbf{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.$$

The Localization Functor

- $\mathrm{Rep}^{adm}(G_0)$: admissible locally analytic representations.

$$M \in \mathcal{C}_{G_0,\lambda} \qquad \overset{(\bullet)'_b}{\rightsquigarrow} \qquad V \in \mathrm{Rep}^{adm}(G_0)$$

- $M_k := (V_{\mathbb{G}(k)^\circ-\mathbf{an}})'$ is a $D(\mathbb{G}(k)^\circ, G_0)$-module of finite presentation.

- For every element $(\mathfrak{Y}, k) \in \underline{\mathcal{F}}_{\mathfrak{X}}$ we get a coherent $\mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda)$-module

$$\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k) := \mathscr{D}^\dagger_{\mathfrak{Y},k}(\lambda) \otimes_{\mathcal{D}^{\mathbf{an}}(\mathbb{G}(k)^\circ)_\lambda} M_k$$

$$\boxed{\mathscr{L}oc^{G_0}_\lambda(M) := \left(\mathscr{L}oc^\dagger_{\mathfrak{Y},k}(\lambda)(M_k)\right)_{(\mathfrak{Y},k)\in\underline{\mathcal{F}}_{\mathfrak{X}}}.}$$

Second Equivalence

- Let $\mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$ be the category of coadmissible G_0-equivariant $\mathscr{D}(\lambda)$-modules over $\underline{\mathcal{F}}_{\mathfrak{X}}$.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$\mathcal{C}_{G_0,\lambda} \underset{\Gamma(\bullet)}{\overset{\mathscr{L}oc_{\lambda}^{G_0}(\bullet)}{\rightleftarrows}} \mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$$

Second Equivalence

- Let $\mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$ be the category of coadmissible G_0-equivariant $\mathscr{D}(\lambda)$-modules over $\underline{\mathcal{F}}_{\mathfrak{X}}$.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$\mathcal{C}_{G_0,\lambda} \underset{\Gamma(\bullet)}{\overset{\mathscr{L}oc_\lambda^{G_0}(\bullet)}{\rightleftarrows}} \mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$$

Second Equivalence

- Let $\mathcal{C}^{G_0}_{\mathfrak{X},\lambda}$ be the category of coadmissible G_0-equivariant $\mathscr{D}(\lambda)$-modules over $\underline{\mathcal{F}}_{\mathfrak{X}}$.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}^*_L$ is dominant and regular. Then

$$\mathcal{C}_{G_0,\lambda} \underset{\Gamma(\bullet)}{\overset{\mathscr{L}oc^{G_0}_{\lambda}(\bullet)}{\rightleftarrows}} \mathcal{C}^{G_0}_{\mathfrak{X},\lambda}$$

Second Equivalence

- Let $\mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$ be the category of coadmissible G_0-equivariant $\mathscr{D}(\lambda)$-modules over $\underline{\mathcal{F}}_{\mathfrak{X}}$.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$\mathcal{C}_{G_0,\lambda} \underset{\Gamma(\bullet)}{\overset{\mathscr{L}oc_\lambda^{G_0}(\bullet)}{\rightleftarrows}} \mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$$

Second Equivalence

- Let $\mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$ be the category of coadmissible G_0-equivariant $\mathscr{D}(\lambda)$-modules over $\underline{\mathcal{F}}_{\mathfrak{X}}$.

Theorem[S]

Let $\lambda \in X(\mathbb{T})$ such that $\lambda + \rho \in \mathfrak{t}_L^*$ is dominant and regular. Then

$$\mathcal{C}_{G_0,\lambda} \underset{\Gamma(\bullet)}{\overset{\mathscr{L}oc_{\lambda}^{G_0}(\bullet)}{\rightleftarrows}} \mathcal{C}_{\mathfrak{X},\lambda}^{G_0}$$

$G = \mathbb{G}(\mathbb{Q}_p)$-equivariance structures

Actually, we dispose of a (non-compact) version of the previous equivalence for the group $G = \mathbb{G}(\mathbb{Q}_p)$.

$$\begin{array}{ccccc} D(G,\mathbb{Q}_p) & \mathcal{C}_{G,\lambda} & \longrightarrow & \mathcal{C}_{\lambda}^{G} & \\ \uparrow & \downarrow & & \downarrow & \text{Forgetful funct.} \\ D(G_0,\mathbb{Q}_p) & \mathcal{C}_{G_0,\lambda} & \longrightarrow & \mathcal{C}_{\mathfrak{X},\lambda}^{G_0}. & \end{array}$$

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in \mathcal{C}^{\mathrm{la}}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b) f(g) \ b \in B, \ g \in G\}$$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))'_b$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B) \otimes_{\mathcal{U}(\mathfrak{b})} \mathcal{U}(\mathfrak{g})} \left(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \right)$$

and

$$\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^{s} (\rho_{g_i})_* \mathscr{D}^{\dagger}_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, ..., g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in C^{la}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b) f(g) \; b \in B, \; g \in G\}$$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))'_b$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B) \otimes_{\mathcal{U}(\mathfrak{b})} \mathcal{U}(\mathfrak{g})} \Big(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \Big)$$

and

$$\mathscr{L}oc^\dagger(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^s (\rho_{g_i})_* \mathscr{D}^\dagger_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^\dagger(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, \ldots, g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in \mathcal{C}^{\mathrm{la}}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b)f(g) \ b \in B, \ g \in G\}$$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))_b'$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B) \otimes_{\mathcal{U}(\mathfrak{b})} \mathcal{U}(\mathfrak{g})} \left(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \right)$$

and

$$\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^{s} (\rho_{g_i})_* \mathscr{D}^{\dagger}_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, ..., g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in \mathcal{C}^{\mathrm{la}}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b)f(g)\ b \in B,\ g \in G\}$$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))_b'$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B)\otimes_{\mathcal{U}(\mathfrak{b})}\mathcal{U}(\mathfrak{g})} \left(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \right)$$

and

$$\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^{s} (\rho_{g_i})_* \mathscr{D}^{\dagger}_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, ..., g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in \mathcal{C}^{\mathrm{la}}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b)f(g)\ b \in B,\ g \in G\}$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))'_b$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B)\otimes_{\mathcal{U}(\mathfrak{b})}\mathcal{U}(\mathfrak{g})} \left(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \right)$$

and

$$\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^{s} (\rho_{g_i})_* \mathscr{D}^{\dagger}_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, \ldots, g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

Localization of principal series representations

$$G = \mathbb{G}(\mathbb{Q}_p) \quad B = \mathbb{B}(\mathbb{Q}_p) \quad T = \mathbb{T}(\mathbb{Q}_p)$$

Let $\lambda : T \to \mathbb{Q}_p$ be an analytic character.

$$\mathrm{Ind}_B^G(\lambda^{-1}) := \{f \in \mathcal{C}^{\mathrm{la}}(G, \mathbb{Q}_p) \mid f(gb) = \lambda(b)f(g)\ b \in B,\ g \in G\}$$

The coadmissible $D(G, \mathbb{Q}_p)$-module $\mathbb{M}(\lambda) := (\mathrm{Ind}_B^G(\lambda))'_b$ satisfies

$$\mathbb{M}(\lambda) = D(G) \otimes_{D(B) \otimes_{\mathcal{U}(\mathfrak{b})} \mathcal{U}(\mathfrak{g})} \left(\underbrace{\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\mathfrak{b})} \mathbb{Q}_{p,d\lambda}}_{M(\lambda)} \right)$$

and

$$\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k) = \oplus_{i=1}^{s} (\rho_{g_i})_* \mathscr{D}^{\dagger}_{\mathfrak{X},\mathfrak{k},\lambda} \otimes (\mathrm{sp}_{\mathfrak{X}})_* \iota^* \mathrm{Loc}(M(\lambda))$$

Example.[HPSS] If $\lambda = -2\rho$ then $\mathscr{L}oc^{\dagger}(\mathbb{M}(\lambda)_k)$ is a sum of a skyscraper sheaf placed at finitely many points $g_1 o, \ldots, g_s o \in \mathfrak{X}$ and $o = \mathrm{sp}_{\mathfrak{X}} \iota^{-1}(\mathbb{B})$.

References

1. M. Emerton, Locally Analytic Vectors in Representations of Locally p-adic Analytic Groups. American Mathematical Soc., 2017.
2. C. Huyghe, T. Schmidt & m. strauch, Arithmetic structures for differential operators on formal schemes. Nagoya Mathematical Journal: 1-48, 2019.
3. C. Huyghe, D. Patel, T. Schmidt & M. Strauch, $\mathscr{D}^\dagger$-affinity of formal models of flag varieties. arXiv:1501.05837v2 (to appear in Mathematical Research Letters). Preprint 2017.
4. D. Patel, T. Schmidt & M. Strauch, Locally analytic representations of GL(2,L) via semistable models of $\mathbb{P}^1$. Journal of the Institute of Mathematics of Jussieu, p. 1-63, 2017.
5. A. Sarrazola-Alzate, G-equivariance of formal models of flag varieties.To appear in Rendiconti del Seminario Matematico della Università di Padova (appeared online 2020).
6. P. Schneider & J. Teitelbaum, Algebras of p-adic distributions and admissible representations. Inventiones mathematicae, vol. 153, no 1, p. 145-196, 2003.

¡Muchas gracias!