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Title : Weierstrass sections for some truncated parabolic subalgebras

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Abstract

Let $\mathfrak{g}$ be a reductive Lie algebra over the field of complex numbers $\mathbb{C}$ and let $Y(\mathfrak{g})$ be the algebra of invariant polynomial functions on its dual space $\mathfrak{g}^*$. By a classical result of Chevalley, $Y(\mathfrak{g})$ is a polynomial algebra over $\mathbb{C}$, that is, the algebra $Y(\mathfrak{g})$ is generated by a finite number of homogeneous algebraically independent invariant functions. Moreover by a result of Kostant, there exists in $\mathfrak{g}^*$ an affine subspace $\mathcal{S}$, called the Kostant section or Kostant slice, such that restriction of functions to $\mathcal{S}$ induces an algebra isomorphism between $Y(\mathfrak{g})$ and the algebra of polynomial functions on $\mathcal{S}$. This subspace $\mathcal{S}$ is given by a principal $\mathfrak{sl}_2$-triple in $\mathfrak{g}$. In my talk I will extend the notion of Kostant section, called here Weierstrass section, following the Russian school, to non reductive Lie algebras, namely to some (truncated) parabolic subalgebras $\mathfrak{p}$ in a simple Lie algebra $\mathfrak{g}$. Of course a Weierstrass section requires the polynomiality of $Y(\mathfrak{p})$ but the latter is not sufficient. As for the Kostant section, our Weierstrass section is given by an analogue of a principal $\mathfrak{sl}_2$-triple (which does not exist in general in $\mathfrak{p}$), that is, by a so-called adapted pair for $\mathfrak{p}$.