ERRATUM: THE GENERALIZED DE RHAM-WITT COMPLEX OVER A FIELD IS A COMPLEX OF ZERO-CYCLES

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At the end of the proof of [Ru07, Th. 3.6], page 148 , line 4, I use a residue theorem [Ru07, Th. 2.19] to conclude. But there is a gap as Deligne pointed out to me, since the theorem [Ru07, Th. 2.19] is stated for smooth projective curves over a base field k, whereas in the proof of [Ru07, Th. 3.6] I use it for normal projective curves. In this erratum I show that after a slight modification of the definition of the residue, which replaces [Ru07, Def. 2.15], together with the results [HuKu94, Th. 1, Th. 4] the residue theorem in fact holds for regular projective curves. Furthermore the calculation of the residue in the course of the proof of [Ru07, Th. 3.6] is adjusted. These modifications are enough to conclude exactly as in [Ru07] and in particular Theorem 3.6 of [Ru07] and all other results are true and remain unchanged.

I am deeply grateful to Pierre Deligne for pointing out this mistake.

Let k be a field of characteristic exponent $p \neq 2$, S a finite truncation set, C a regular curve over k (i.e. a one dimensional regular, integral, separated scheme of finite type over k), $K = k(C)$ the function field of C and $P \in C$ a closed point.

The following construction is from [HuKu94], page 88.

Denote $k(K^{p^n})$ by K_n, $n \in \mathbb{N}$. Viewing K as a constant sheaf on C and $\mathcal{O}_C, K_n \subset K$ as subsheaves we define $C_n = \mathbf{Spec}(\mathcal{O}_C \cap K_n)$. Thus we obtain maps

$$C = C_0 \to C_1 \to C_2 \to \cdots .$$

Since $k(\mathcal{O}_C^{p^n}) \subset (\mathcal{O}_C \cap K_n) \subset \mathcal{O}_C$ it is clear that these maps induce isomorphisms of the underlying topological spaces and that each C_n is a separated integral scheme of finite type over k with function field K_n. Denote by P_n the image of P in C_n. Then for almost all n, C_n is smooth over k and $k(P_n)$ is the separable closure of k in $k(P)$, by [HuKu94, Th. 1, Th. 4]. Now [Ru07, Def. 2.15] must be replaced by the following definition (we use the notation of the article).

Definition-Proposition 1 (cf. [Ku86], 17.4.)**.** Let n be a natural number such that for all $n' \geq n$ (in the above notation) $C_{n'}$ is smooth over k and $k(P_{n'})$ is the separable closure of k in $k(P)$. We write $\kappa_n = k(P_n)$ and $K_n = k(C_n) = k(K^{p^n})$. Finally we denote by $\widehat{K}_n$ the completion of K_n in P_n. Now the choice of a local parameter t in P_n determines a unique continuous isomorphism $\kappa_n((t)) \xrightarrow{\simeq} \widehat{K}_n$ of fields over k (this is an isomorphism over k, since $\kappa_n \supset k$ is separable) and we have a natural inclusion $\iota : K_n \hookrightarrow \kappa_n((t))$. Take $\omega \in \mathbb{W}_S\Omega^q_K$, then we define the *residue of* ω *in* P to be

$$\mathrm{Res}_{P,S}(\omega) = \mathrm{Res}_P(\omega) = \mathrm{Tr}_{\kappa_n/k}\Big(\mathrm{Res}^q_{t,S}\big(\iota(\mathrm{Tr}_{K/K_n}(\omega))\big)\Big) \in \mathbb{W}_S\Omega^{q-1}_k, \tag{1}$$

where the $\mathrm{Res}^q_{t,S}$ on the right hand side, is the residue on $\mathbb{W}_S\Omega^q_{\kappa_n((t))}$ from [Ru07, Def. 2.11]. The residue is well defined, i.e. independent of the choice of the local parameter t and the number n.

The proof is exactly the one from [Ru07, Def. 2.15], except that we have to mention, that by the choice of n, K_n is separable over k and thus by [Ku86, 5.10. Th.] $K_{n+1} = k(K_n^p) \subset K_n$ is purely inseparable of degree p.

[Ru07, Rem 2.16] remains unchanged, in [Ru07, Rem 2.17] write C_n instead of $C^{(p^n)}$, [Ru07, Prop. 2.18] remains unchanged. [Ru07, Th. 2.19] now becomes

Theorem 2. *Let C be a regular projective curve over k with function field K. Then*

$$\sum_{P\in C} \mathrm{Res}_P(\omega) = 0, \text{ for all } \omega \in \mathbb{W}_S\Omega^q_K,\ q \geq 1.$$

(Notice, that $\mathrm{Res}_P(\omega) = 0$, if ω has no pole in P, thus the sum is finite.)

The proof remains the same, except that at the beginning we insert the following sentence: Since $\mathrm{Res}_P(\omega)$ is non-zero for only a finite number of points we can assume by [HuKu94, Th. 1, Th. 4] and [Ru07, Rem. 2.16] that C is smooth over k and the points P with $\mathrm{Res}_P(\omega) \neq 0$ are étale over k. In the original proof a line like this appears on page 139, line (-12) to (-10), this one has to be cancelled.

We want to take the opportunity to correct a misprint. The formula on page 139, line (-3) should be

$$\mathrm{Res}_P(\omega) = \sum_j \mathrm{Res}_{Q_j}(\omega_j) \quad \text{in } \mathrm{W}_n(\bar{k}),$$

where the Q_j's are the preimages of P in $C \times_k \bar{k}$.

Now in the proof of [Ru07, Th. 3.6] the first paragraph remains unchanged and the beginning of the second, line (-15) to line (-9) on page 146, has to be replaced with the following (we use notation of the article):

Take $P \in \nu^{-1}(y_n = 0) \cap \Sigma$ and denote $\kappa = k(P)$. Write K for the function field of $\widetilde{C}$ and $K_i = k(K^{p^i})$, $i \geq 0$. Furthermore denote $\mathbf{Spec}(\mathcal{O}_{\widetilde{C}} \cap K_i)$ by $\widetilde{C}_i$ and let P_i be the image of P in $\widetilde{C}_i$. Choose $l \geq 0$, such that for all $l' \geq l$ $\widetilde{C}_{l'}$ is smooth over k and $\kappa_{l'} := k(P_{l'}) \supset k$ is separable. Let e_P be the ramification index of P over P_l and $f_P = [\kappa : \kappa_l]$ and write

$$e_P = p^r, \quad f_P = p^s, \quad [K : K_l] = p^j.$$

Then by [Ku86, 5.10 Th., a)] and [Se68, I, §4, Prop. 10]

$$j = r + s \geq l. \tag{3.6.1}$$

(In the article we wrongly wrote an equal sign here.)

Now in the following calculation, line (-8) on page 146 to line (-8) on page 147, replace $F^j(P)$ by P_l, K_j by K_l and κ_j by κ_l, but the j's appearing in the powers of p (such as $p^{j(n-1)+r}$ etc.) stay the same. Then the whole proof of the formula in (3.6.3) goes through, since $x^{p^j} \in K_l$ for $x \in K$.

Finally at the end of the proof of [Ru07, Th. 3.6], page 148 , line 4, one now refers to Theorem 2 instead of [Ru07, Th. 2.19].

The rest of the paper remains unchanged.

References

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