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Integer and Nonlinear Optimization
Exercise 6

Problem 1
Consider the sets $P = \{\underline{x} \in \mathbb{R}^2_+ : x_1 - x_2 \geq -1, 2x_1 + 6x_2 \leq 15, x_1 - x_2 \leq 3, 2x_1 + 4x_2 \leq 7\}$ and $S = P \cap \mathbb{Z}^2_+$. Use the generic relaxation cutting algorithm (Algorithm 4.5) to solve the problem with objective function $\max \underline{c}\,\underline{x} = x_1 + 6x_2$, using a graphical procedure for the generation of cuts as in Example 4.6.

Problem 2
Def. 4.7.

(i) The valid inequalities $\underline{\pi}\,\underline{x} \leq \pi_0$ and $\underline{\gamma}\underline{x} \leq \gamma_0$ are said to be *equivalent* if $(\underline{\gamma}, \gamma_0) = \lambda(\underline{\pi}, \pi_0)$ for some $\lambda > 0$.

(ii) If they are not equivalent and

$$\exists\, \mu > 0 : \quad \mu\,\underline{\pi} \leq \underline{\gamma} \quad \textit{and} \quad \mu\,\pi_0 \geq \gamma_0$$

then $\{\underline{x} \in \mathbb{R}^n_+, \underline{\gamma}\,\underline{x} \leq \gamma_0\} \subset \{\underline{x} \in \mathbb{R}^n_+ : \underline{\pi}\,\underline{x} \leq \pi_0\}$. In this case we say that $(\underline{\gamma}, \gamma_0)$ *dominates* (*is stronger than*) $(\underline{\pi}, \pi_0)$ or that $(\underline{\pi}, \pi_0)$ *is dominated by* $(\underline{\lambda}, \lambda_0)$.

(iii) A *maximal valid inequality* is one that is not dominated by any other valid inequality.

Let $S = \{\underline{x} \in \{0,1\}^n : \sum_{j \in N} a_j x_j \leq b\}$ with $a_j > 0 \forall j \in N, b > 0$ and $N \subseteq \{1, \ldots, n\}$. Show that a valid inequality $\sum_{j \in N} \pi_j x_j \leq \pi_0$ with $\pi_0 > 0$ and $\pi_j < 0$ for $j \in T \subseteq N, T \neq \emptyset$ is dominated by the valid inequality $\sum_{j \in N} \max\{\pi_j, 0\} x_j \leq \pi_0$.

Problem 3
Prove Theorem 4.9.:
Theorem 4.9:
Let $(\underline{\pi}\,\pi_0)$ be any valid inequality for $P = \{\underline{x} \in \mathbb{R}^n_+ : A\underline{x} \leq \underline{b}\}$. Then $(\underline{\pi}, \pi_0)$ is either equivalent to or dominated by an inequality of the form $\underline{u}A\underline{x} \leq \underline{u}\,\underline{b}, \underline{u} \in \mathbb{R}^m_+$, if any of the following conditions hold:

(1) $P \neq \emptyset$ (in this case no more than $\min(m, n)$ components of $\underline{u}$ need to be positive)

(2) $\{\underline{u} \in \mathbb{R}^m_+ : \underline{u}A \geq \underline{\pi}\} \neq \emptyset$

(3) $A = \begin{pmatrix} A' \\ I \end{pmatrix}$, where I is an $n \times n$ identity matrix.

Problem 4
Consider the Knapsack set

$$S = \{\underline{x} \in \{0,1\}^6 : 12x_1 + 9x_2 + 7x_3 + 5x_4 + 5x_5 + 3x_6 \leq 14\}$$

Set $x_1 = x_2 = x_4 = 0$, and consider the cover inequality
$x_3 + x_5 + x_6 \leq 2$ that is valid for $S' = S \cap \{\underline{x} \in \{0,1\}^6 : x_1 = x_2 = x_4 = 0\}$.
"Lift" this inequality to obtain a valid inequality $\alpha_1 x_1 + \alpha_2 x_2 + \alpha_4 x_4 + x_3 + x_5 + x_6 \leq 2$ for S with $\alpha_1, \alpha_2, \alpha_4 \geq 0$ that is as strong as possible.