Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Integer and Nonlinear Optimization
Exercise 5

Problem 1
Prove the following result:

Lemma A: Let $P = \{\underline{x} \in \mathbb{R}^n : A\underline{x} \leq \underline{b}\} \neq \emptyset$.
Then $\underline{x}$ is an extreme point of P if and only if $\underline{x}$ is a 0-dimensional face o P.

Problem 2
Find all extreme points of the polyhedron

$$\begin{array}{rcr} x_1 + x_2 & \geq & 1 \\ x_1 + 2x_3 & \geq & 2 \\ -x_2 + x_3 & \geq & -4 \\ \underline{x} & \in & \mathbb{R}^3 \end{array}$$

(Hint: Use Lemma A from Problem 1.)

Problem 3
Prove the following result:

Lemma B: Let $P = \{\underline{x} \in \mathbb{R}^n : A\underline{x} \leq \underline{b}\} \neq \emptyset$ and $P^0 = \{\underline{r} \in \mathbb{R}^n : A\underline{r} \leq \underline{0}\}$. Then $\underline{r} \in P^0 \setminus \{\underline{0}\}$ is an extreme ray of P if and only if $\{\lambda\underline{r} : \lambda \in \mathbb{R}_+\}$ is a 1-dimensional face of P^0.

Problem 4
Use Lemma B from Problem 3 to find all extreme rays of the polyhedron

$$\begin{array}{rcr} x_1 + x_2 & \geq & 1 \\ x_1 + 2x_3 & \geq & 2 \\ -x_2 + x_3 & \geq & -4 \\ \underline{x} & \in & \mathbb{R}^3 \end{array}$$

Problem 5
Consider the following polyhedron:

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & \leq & 1 \\ -x_1 & - & x_2 & - & x_3 & \leq & -1 \\ x_1 & & & + & x_3 & \leq & 1 \\ -x_1 & & & & & \leq & 0 \\ & & -x_2 & & & \leq & 0 \\ & & & & x_3 & \leq & 2 \\ x_1 & + & x_2 & + & 2x_3 & \leq & 2 \\ & & & & \underline{x} & \in & \mathbb{R}^3 \end{array}$$

(a) Show that $(\underline{\pi}^1, \pi_0^1) = ((-1, -1, 1), 1)$ and $(\underline{\pi}^2, \pi_0^2) = ((2, -7, 2), 2)$ are valid inequalities for P and determine the dimension of the faces F_1 and F_2 represented by $(\underline{\pi}^1, \pi_0^1)$ and $(\underline{\pi}^2, \pi_0^2)$.

(b) Show that $F_3 = \{\underline{x} \in P : x_1 + x_3 = 1\}$ is a facet of P.

(c) Find a minimal description of P, i.e. so that none of the inequalities describing P is redundant.