Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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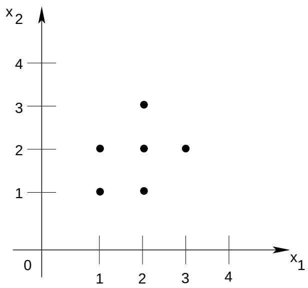
Integer and Nonlinear Optimization
Exercise 2

Please submit your answers on Thursday, October 30th, 2003, in the class ($14^{15} - 15^{45}$).

Problem 1
Consider the set S of points $x \in \mathbb{Z}^2_+$ as depicted in the drawing.

a) Show that S is equal to the set of integer points satisfying

$$\begin{aligned} x_1 + x_2 &\geq \tfrac{3}{2} \\ x_1 + x_2 &\leq \tfrac{11}{2} \\ \tfrac{2}{5}x_1 + 4x_2 &\geq 2 \\ -\tfrac{16}{9}x_1 + x_2 &\geq -4 \\ -x_1 + x_2 &\leq \tfrac{3}{2}. \end{aligned}$$



Why does the LP with these constraints and objective funcion $\min \underline{c}\,\underline{x}$ not have an integer solution for every choice of $\underline{c}$?

b) Give a tight linear description $\tilde{A}\,\underline{x} \leq \tilde{\underline{b}}$ of the set S, i.e. such that the feasible region is the convex hull of S. (The convex hull is the smallest convex set containing S.)

Problem 2
A delivery company must make deliveries to 10 customers whose respective demands are given by d_j, $j = 1, \ldots, 10$. The company has 4 trucks available with capacities L_k and daily operating costs c_k, $k = 1, \ldots, 4$. A single truck cannot deliver to more than 5 customers, and customer pairs $\{1, 7\}, \{2, 6\}$ and $\{2, 9\}$ cannot be visited by the same truck.
Find a mixed-integer programming model for this problem.

Problem 3
Consider the following two ideas for solving the knapsack problem: The objects are put into the knapsack

a) in decreasing order of their value (most valuable first), regardless of the weight;

b) in increasing order of their weight (lightest first), regardless of their value.

Stop if no more objects can be put into the knapsack due to weight limitation. Does any of these methods solve the problem? Justify your answer.

See back side!

Problem 4

Show that for the continuous knapsack problem

$$\begin{array}{lll} \max & \sum_{i=1}^{n} c_i x_i & \\ \text{s.t.} & \sum_{i=1}^{n} a_i x_i \leq b & \\ & 0 \leq x_i \leq 1 & \forall i = 1, \ldots, n \end{array}$$

the following strategy provides an optimal solution:

1. Order the elements such that $\frac{c_1}{a_1} \geq \ldots \geq \frac{c_n}{a_n}$.
2. Choose as many objects as possible according to this order and fill the gap by a fractional amount of the next element.

Does this algorithm without the fractional last item also solve the integer knapsack problem?