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Naturwissenschaftliche Fakultät I
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Prof. Dr. K. Klamroth
Barbara Pfeiffer

Integer and Nonlinear Optimization
Exercise 11

Problem 1
Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x_1, x_2) = \frac{x_1 + x_2}{3 + x_1^2 + x_2^2 + x_1 x_2}$$

(a) Find all stationary points of f, i.e. all points $\overline{\underline{x}}$ with $\nabla f(\overline{\underline{x}}) = \underline{0}$.

(b) Determine whether these are (strict) local minima (maxima) or neither.

Problem 2
Prove Theorems 10.2 and 10.3 of the lecture.

Problem 3
Show that a function $f : \mathbb{R}^n \to \mathbb{R}$ is quasiconvex if and only if the level-sets

$$L_{\leq}(y) := \{\underline{x} \in \mathbb{R}^n : f(\underline{x}) \leq y\}$$

are convex for all $y \in \mathbb{R}$.
Give an (analytical) example of a function which is quasiconvex but not convex.

Problem 4
Definition: A function $\theta : \mathbb{R} \to \mathbb{R}$ is called *strictly unimodal* over the interval $[a, b]$ if there exists a $\bar{\lambda} \in [a, b]$ that minimizes θ over $[a, b]$ and for $\lambda_1, \lambda_2 \in [a, b]$ such that $\theta(\lambda_1) \neq \theta(\bar{\lambda})$, $\theta(\lambda_2) \neq \theta(\bar{\lambda})$, and $\lambda_1 < \lambda_2$, we have

$$\begin{array}{lcl} \lambda_2 \leq \bar{\lambda} & \Rightarrow & \theta(\lambda_1) > \theta(\lambda_2) \\ \lambda_1 \geq \bar{\lambda} & \Rightarrow & \theta(\lambda_1) < \theta(\lambda_2). \end{array}$$

Show that if θ is strictly unimodal and continuous over $[a, b]$, then θ is strictly quasiconvex over $[a, b]$. Conversely show that if θ is strictly quasiconvex over $[a, b]$ and has a minimum in this interval, then it is strictly unimodal over $[a, b]$.