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Naturwissenschaftliche Fakultät I
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Integer and Nonlinear Optimization
Exercise 10

Problem 1
Determine which of the following functions are convex and which are not. (You may use derivatives.)

(a) $f(x_1, x_2) = x_1^2 + 2x_1x_2 - 10x_1 + 5x_2$

(b) $f(x_1, x_2, x_3) = -x_1^2 - 3x_2^2 - 2x_3^2 + 4x_1x_2 + 2x_1x_3 + 4x_2x_3$

(c) $f(x_1, x_2) = e^{x_1} + a \log x_2$, where $a < 0$.

Problem 2
Prove Lemma 8.6 of the lecture.

Problem 3
Prove Lemma 8.8 of the lecture.

Problem 4
Show that the following inequalities are satisfied:

(a) $\prod_{j=1}^n x_j^{t_j} \le \sum_{j=1}^n t_j x_j$

(b) $\left(\sum_{j=1}^n t_j x_j\right)\left(\sum_{j=1}^n \frac{t_j}{x_j}\right) \ge 1$

where $t_j > 0$, $\sum_{j=1}^n t_j = 1$ and $x_j \in \mathbb{R} \setminus \{0\}$ $\forall j = 1, \ldots, n$, and $\sum_{j=1}^n t_j x_j \neq 0$. (Hint: Use the inequality $f(\sum_{j=1}^n t_j x_j) \le \sum_{j=1}^n t_j f(x_j)$ for convex functions f.)

Problem 5
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a real-valued function. The *epigraph* of f is defined as

$$\operatorname{epi}(f) := \left\{(\underline{x}, y) \in \mathbb{R}^{n+1} \,:\, f(\underline{x}) \le y\right\}.$$

The *strict epigraph* of f is defined analogously, with $\le$ replaced by $<$.
Show that f is convex if and only if the epigraph of f is convex if and only if the strict epigraph of f is convex.