Multiple Precision Interval Arithmetic:
Comparing some Packages

Passo Fundo/Brazil, August 2004
Overview

1. Types of packages
2. Languages and Libraries
3. Packages for dedicated Scientific Computing Software Environments
Some Packages

Types of Packages

• Libraries for Programming Languages
  – C/C++: C-XSC, MPFI, GMP-XSC

• Packages for Scientific Computing Environments
  – Matlab: Intlab
  – Maple: intpakX
Objectives

Libraries:
- Extend mathematical capabilities of the language
- Efficiency
- Keep features of the language

Scientific Computing Packages:
- Support special scientific computing features (symbolical computations, matrix computations)
- Convenience
- Make use of graphics
MPFI

MPFI:
- Interval extension of the MPFR library
- based on GNU Multiple precision library

MPFR: Extension of GMP with correct rounding (suitable as a base for interval computation)

Authors of MPFI: Nathalie Revol, Fabrice Rouillier, ENS Lyon, France
MPFI Overview

- C library compliant to IEEE 754 floating point arithmetic
- Arbitrary precision variables (↔ arbitrary number of mantissa digits)
- Real arithmetic
- Correct rounding (taken from MPFR)
MPFI Features

- All computations done via C functions
- Full set of basic operations and standard functions available
- Set operations and some special functions also available
- String I/O
Library: MPFI

Examples of functions

- Initialization and assignment:
  ```c
  int mpfi_init_set (mpfi_t rop, mpfi_t op)
  ```
  Initializes `rop` and sets its value from `op`, outward rounded so that `op` belongs to `rop`. The precision of `rop` will be taken from the active default precision, as set by `mpfr_set_default_prec`. The returned value indicates whether none, one or both endpoints are exact.

- Setting the precision:
  ```c
  int mpfi_set_prec (mpfi_t x, mp_prec_t prec)
  ```
  Resets the precision of `x` to be exactly `prec` bits. The previous value stored in `x` is lost. It is equivalent to a call to `mpfi_clear(x)` followed by a call to `mpfi_init2(x, prec)`, but more efficient.
Examples of functions

- Basic operations:
  - `int mpfi_add (mpfi_t rop, mpfi_t op1, mpfi_t op2)`
  - `int mpfi_add_d (mpfi_t rop, mpfi_t op1, double op2)`

Carries out the appropriate addition.

→ Separate versions for different parameter types

**MPFI: Interval features added to multiple precision library**
GMP-XSC

- Also based on GMP
- Package designed to serve special purposes in integral computation
- Designed as a future add-on for the XSC languages
- Faster than MPI, but no full set of operations

Author: Knut Petras, TU Braunschweig, Germany
XSC languages

• Pascal and C/C++ Libraries, C-XSC extending “standard“ C/C++ floating point arithmetic
• Fixed multiple precision called staggered precision (using \( n \) floating point variables)
• Real and complex arithmetic, vector and matrix computations
• Correct rounding

Authors: W. Krämer et al., Universities of Wuppertal and Karlsruhe, Germany
C-XSC Features

- Full set of basic operations and standard functions
- Special functions and string I/O available
- Overloaded C/C++ operators → Denotation of expressions does not need to be changed
- Exact scalar product via dotprecision accumulator

C-XSC: Multiple precision added to interval library
C-XSC Example

stagprec=6;  // set staggered precision
str1 >> RndNext >> x1;  // string input and rounding
l_imatrix L_IA1(dim,dim),L_IA2(dim,dim),L_IR(dim,dim);
   // construct staggered variables
L_IR=L_IA1*L_IA2;  // Matrix multiplication
   with staggered data
Intlab

- Interval package for Matlab (all Matlab Code)
- Fast real/complex interval computations based on BLAS routines
- Rudimentary multiple precision arithmetic (according to the author)

Author: S. Rump, TU Hamburg-Harburg
Intlab Features

Intlab Multiple precision arithmetic:

- „long“ data type
- „Automatic“ choice of operators (≈ Overloading)
- Reduced set of operations: basic operations (+, -, *, /), exponential and power function
- No vectors/matrices possible
Packages

intpakX

• Interval package for Maple (all Maple code)
• Arbitrary precision: Built-in Maple feature (precision can be set to an arbitrary value)
• Real arithmetic and complex disc arithmetic
• Correct rounding
• Some built-in applications

Authors: W. Krämer, I. Geulig, M. Grimmer, Universities of Wuppertal and Karlsruhe, Germany
Packages: intpakX

**intpakX Features**

- Full set of basic operations and standard functions for real arithmetic
- Reduced set of operations for complex disc arithmetic
- Special (but no overloaded) operators → Denotation of expressions similar to pseudocode
- Graphical output of the included applications (see detailed intpakX description)
## Comparison - Timing

### Matrix multiplications:

<table>
<thead>
<tr>
<th>Data Type/Matrix Size</th>
<th>15 Digits</th>
<th>90 Digits</th>
<th>540 Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maple float (20×20)</td>
<td>0.86</td>
<td>1.85</td>
<td>6.86</td>
</tr>
<tr>
<td>intpakX interval (20×20)</td>
<td>20.16</td>
<td>23.38</td>
<td>63.59</td>
</tr>
</tbody>
</table>

### C-XSC:

<table>
<thead>
<tr>
<th>Size</th>
<th>imatrix (2 reals)</th>
<th>l_imatrix (2 reals)</th>
<th>l_imatrix (6 reals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20x20</td>
<td>0.07</td>
<td>0.15</td>
<td>0.68</td>
</tr>
<tr>
<td>200x200</td>
<td>63.70</td>
<td>132.38</td>
<td>663.07</td>
</tr>
</tbody>
</table>

### GMP-XSC:

<table>
<thead>
<tr>
<th>Size</th>
<th>15</th>
<th>30</th>
<th>90</th>
<th>540 Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>20x20</td>
<td>0.07</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>200x200</td>
<td>79.10</td>
<td>81.60</td>
<td>86.20</td>
<td>121.28</td>
</tr>
</tbody>
</table>
Comparison - Timing

Maple:

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple float (90 Digits)</th>
<th>intpakX int. (90 Digits)</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin(x)</td>
<td>4.63</td>
<td>19.42</td>
<td>4.1</td>
</tr>
<tr>
<td>exp(x)</td>
<td>2.60</td>
<td>4.20</td>
<td>1.6</td>
</tr>
</tbody>
</table>

C-XSC:

<table>
<thead>
<tr>
<th>Function</th>
<th>interval</th>
<th>l_interval (2 reals)</th>
<th>l_interval (6 reals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin(x)</td>
<td>0.0014</td>
<td>17.61</td>
<td>57.20</td>
</tr>
<tr>
<td>exp(x)</td>
<td>0.0012</td>
<td>17.74</td>
<td>78.78</td>
</tr>
</tbody>
</table>

GMP-XSC:

<table>
<thead>
<tr>
<th>Function</th>
<th>15</th>
<th>30</th>
<th>90 Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin(x)</td>
<td>0.22</td>
<td>0.30</td>
<td>0.74</td>
</tr>
<tr>
<td>exp(x)</td>
<td>0.16</td>
<td>0.23</td>
<td>0.52</td>
</tr>
</tbody>
</table>
References/Contact

**MPFI:**
http://perso.ens-lyon.fr/nathalie.revol/software.html

**GMP-XSC:**
http://www.tu-bs.de/~petras/software.html

**XSC languages:**
http://www.xsc.de

**intpakX:**
http://www.math.uni-wuppertal.de/wrswt/software/intpakX/
or from Waterloo Maple on

**Intlab:**