Corrigendum to "Hadamard operators on $\mathscr{D}'(\mathbb{R}^d)$" [Studia Math. 237 (2017), 137–152]

Dietmar Vogt

In Lemma 3.1 of [1] it is claimed that $\mathscr{E}'(\mathbb{R}^d) \subset \mathscr{O}'_H(\mathbb{R}^d)$. This is false as the following argument shows: For $T \in \mathscr{O}'_H(\mathbb{R}^d)$ and $\varphi \in \mathscr{D}(\mathbb{R}^d)$ we consider (assuming without restriction of generality $T = \theta^\beta t_\beta$, $t_\beta \in L_1(\mathbb{R}^d)$) the function $F(y) = T_x\varphi(xy) = (-1)^{|\beta|} \int t_\beta(x)\,(xy)^\beta \varphi^{(\beta)}(xy)dx$. It is easily seen that it is bounded. For $T = \delta^{(\alpha)} \in \mathscr{E}'(\mathbb{R}^d)$ we have $T_x\varphi(xy) = (-1)^{|\alpha|} y^\alpha \varphi^\alpha(0)$ which for $\alpha \neq 0$ and $\varphi^{(\alpha)}(0) \neq 0$ is unbounded.

Lemma 3.1 has to be replaced with the following correct version.

Lemma 3.1 *$\mathscr{E}'(\mathbb{R}^d) \cap L_\infty(\mathbb{R}^d) \subset \mathscr{O}'_C(\mathbb{R}^d) \cap \mathscr{O}'_H(\mathbb{R}^d)$.*

In consequence further results of this section have to be modified. The proofs remain with obvious modifications.

Lemma 3.2 *If $T - S \in \mathscr{E}'(\mathbb{R}^d) \cap L_\infty(\mathbb{R}^d)$ and $S \in \mathscr{O}'_H(\mathbb{R}^d)$ or $S \in \mathscr{O}'_C(\mathbb{R}^d)$ then $T \in \mathscr{O}'_H(\mathbb{R}^d)$ or $T \in \mathscr{O}'_C(\mathbb{R}^d)$, respectively.*

Proposition 3.3 *1. If T is bounded measurable in a neighborhood of 0 and $T \in \mathscr{O}'_C(\mathbb{R})$ then $T \in \mathscr{O}'_H(\mathbb{R})$.*
2. If $\operatorname{supp} T \subset W_\varepsilon$ for some $\varepsilon > 0$ and $T \in \mathscr{O}'_C(\mathbb{R}^d)$ then $T \in \mathscr{O}'_H(\mathbb{R}^d)$.

$\mathscr{O}'_H(\mathbb{R}) \cap L_\infty(\mathbb{R})$ is not contained in $\mathscr{O}'_C(\mathbb{R})$, as the following example shows.

Example 3.4 *If $T = e^{-ix}$, that is, $T\varphi = \int e^{-ix}\varphi(x)dx$, then*
1. $T \notin \mathscr{O}'_C(\mathbb{R})$, 2. $T \in \mathscr{O}'_H(\mathbb{R})$.

Example 3.5 *If $T = e^{i\pi x^2}$ then $T \in \mathscr{O}'_C(\mathbb{R}) \cap L_\infty(\mathbb{R}^d)$ and therefore $T \in \mathscr{O}'_H(\mathbb{R})$. The function $e^{i\pi x^2}$ is bounded, but its derivatives are not.*

References

[1] D. Vogt, Hadamard operators on $\mathscr{D}'(\mathbb{R}^d)$, *Studia Math.* **237** (2017), 137–152.

Bergische Universität Wuppertal, Dept. of Math.,
Gauß-Str. 20, D-42119 Wuppertal, Germany;
e-mail: dvogt@math.uni-wuppertal.de