ANOTHER LOOK AT THE PIONEER ANOMALY

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ABSTRACT. The unexpected frequency shift observed by the Pioneer team, usually described as an anomalous acceleration, is discussed from the perspective of two alternative assumptions on the origin of cosmological redshift (Hubble effect). One is the standard assumption of an “expanding space”, the other is the assumption of an energy loss of photons without expanding space sections (“downscaling photons”). We find that the Pioneer frequency shift is an anomaly from the point of view of the expanding space hypothesis only. If one assumes a downscaling photon hypothesis for the Hubble effect, the anomaly dissolves and can be identified with a direct manifestation of the Hubble law inside the solar system.

1. Introduction

Many attempts have been made to understand the so-called anomalous acceleration $a_P$ of the Pioneer spacecrafts 10 and 11 from a cosmological point of view, (Rosales/Sanchez 1999, Rosales 2004, Masreliez 2005, Nottale 2006, Carrera/Giulini 2006, Fahr/Siewert 2006a, Fahr/Siewert 2006b) and others. Different obstacles obliterated these attempts. An explanation of $a_P$ as a dynamical effect of the general relativistic modification of low velocity orbits by cosmological terms in Robertson-Walker models is ruled out, because the cosmological corrections are proportional to $-H_0 v$, with $v \sim 10\text{ km} s^{-1}$ the spacecraft velocity. They are a factor $\frac{v}{c}$, i.e. 4 orders of magnitude, too low (Scholz 2005, equus. (20), (63)). M. Lachièze-Rey arrives at even smaller values in his recent approximation (Lachièze-Rey 2007).

Cosmological corrections of distance measurements, or a phase shift of the photons (Berry phase) have been proposed by (Rosales/Sanchez 1999, Rosales 2004). These may lead to effects which look like a (fictitious) acceleration. But according to (Carrera/Giulini 2006) the derivation is erroneous and the effect is even smaller, carrying a factor on the order of magnitude $(\frac{v}{c})^3$. Other approaches are shortly discussed in the basic study (Anderson e. a. 2002, section XI C). None of these has led to conclusive results, but the motivation for a search in this direction persists.

Since the beginning of the study of the Pioneer anomaly a surprising numerical coincidence between the Hubble constant $H_0$ and the anomalous Pioneer acceleration $a_P$ divided by $c$, the velocity of light has been observed.

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Date: 22. 03. 2007.
The best fit for $a_P$ to the experimental data prior to a bias estimate is

$$a_{P,\text{exp}} = (7.84 \pm 1.33) \times 10^{-8} \text{ cm s}^{-2}$$

(Anderson et al. 2002, equ. (23)), here supplemented by the estimation of the error intervals of (table 2, ibid.). This leads to

$$|a_{P,\text{exp}} c^{-1}| \approx (2.6 \pm 0.44) \times 10^{-18} \text{ s}^{-1},$$

which compares well with the value for the Hubble constant

$$H_0 \approx (2.3 \pm 0.23) \times 10^{-18} \text{ s}^{-1}$$

usually given in the form $H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \pm 10\%$. Even the usually quoted “bias corrected” value $a_P = (8.74 \pm 1.33) \times 10^{-8} \text{ cm s}^{-2}$, with $|a_P c^{-1}| \approx (2.92 \pm 0.44) \times 10^{-18} \text{ s}^{-1}$, leads to overlapping 1σ intervals. But the agreement seems to hold only for the absolute value; its sign poses a question of its own. The anomalous acceleration is directed towards the sun and is opposite to the spacecraft’s velocity. If one takes orientation into account, one gets $a_P < 0$, while obviously $H_0 > 0$.

In the following note we show that a field theoretic cause of cosmological redshift could easily explain the Pioneer frequency shift. At first glance this may seem to be a pointless enterprise. Special (ad-hoc) versions of the hypothesis of a field theoretic reduction of photon energy during their course through space-time have been discussed in the 20th century under the nickname “tired light”. They have received strong criticism and seem to be empirically refuted. At present, most physicists consider also the general hypothesis with great suspicion (understandably) or even devalidated (wrongly).

One has to note, that an analysis of Robertson-Walker manifolds in the framework of integrable Weyl geometry and scale covariant gravity leads to alternative metrical gauges to the Riemannian one and to interesting cosmological models, also from the empirical point of view (Scholz 2004, Scholz 2007). We therefore have reasons to explore hypothetically the consequences of a *downscaling photon hypothesis* for the evaluation of the Pioneer data. In contrast to the older “tired light” approaches, this downscaling is expressed by a non-vanishing scale connection of Weyl geometry (in the gauge distinguished by observational instruments and procedures); it is thus part of the geometrical structure itself. Physically it signifies a (hypothetical) “higher order” gravitational effect on photons, and cannot be interpreted as a scattering phenomenon superimposed on the inertial structure of null geodesics, which forces photons to deviate from the latter. If this structural approach (“formal” from the physical point of view) leads to a better understanding of physics in different phenomenal ranges, the question of how the Weyl geometric downscaling can be understood physically (in the end, quantum field theoretically) has to be posed anew. Already now, we may like to have another look at the older tired light mechanisms from the point of view, whether they may agree with Weyl geometric scale transfer, or whether they don’t. It is not the object of the following paper to analyze this question. Short remarks on this question can be found in section 5 below.
Here we show that the anomalous frequency shift of the Pioneer experiment behaves quantitatively as if it had the same origin as cosmological redshift, correct sign included, if one only broadens the theoretical perspective as indicated. For this special question we only need very elementary mathematics, as we deal with cosmologically tiny regions which can be treated in the linearized (“infinitesimal”) regime. The following analysis of the frequency shift data can therefore be read without further knowledge of scale covariant gravity or Weyl geometry.

The next section presents a condensed view of the data and their evaluation. Section 3 and 4 discuss how the Pioneer frequency shift is easily understood, if one assumes the hypothesis of downscaling photon energy. It should be kept in mind that IWG provides a broader geometrical background to the analysis of the localized question dealt with here. Section 5 compares the result we have found with the problems to understand the Pioneer effect in the framework of expanding space cosmologies, and section 6 draws a short conclusion.

In order to adapt our language to the possibility that Hubble’s observation may have an expression already on the solar system level, we here prefer the more general terminology of Hubble effect in place of the terminology of “cosmological redshift” with its narrower connotation.

2. Pioneer data

The Pioneer probes 10 and 11 were navigated on the basis of Doppler measurements which used integration times between 60 and 1000 s. No ranging data were taken; position information was inferred from diurnal variation of the Doppler shift resulting from earth rotation. These variations allowed to determine the celestial coordinates (declination and right ascension) for each of the probes in certain intervals of the flight time. The spatial coordinates of the spacecraft could be determined by using best fit PPN orbit determination methods. Although the team concedes that the radio Doppler observable was not the optimal method for the purpose of a 3-dimensional orbit reconstruction (Turyshev e. a. 2005, 3) (separate signal run time ranging data would have been preferable), the radiometric tracking data sufficed to indicate unanimously a model anomaly.\footnote{Two different well established and often tested evaluation programs were used, the orbit determination program, ODP, of Jet Propulsion Laboratory, JPL, and another one, CHASMP, of the Aerospace Corporation (Anderson e. a. 2002, 8f.). Both led to the same result.}

For Pioneer 10 the data used in the present main evaluation (another one is in preparation) consisted of 20 055 Doppler data points. They were taken between 3 January 1987 and 22 July 1998, while the space probe moved in an interval between 40 \( AU \) and 70.5 \( AU \) heliocentric distance. Pioneer 11 evaluation used 11 616 Doppler data points (between 5 January 1987 and 1 October 1990, 22.37 \( AU \) to 31.7 \( AU \)). In the literature on the Pioneer effect the terminology “Doppler data” is often understood literally. Here we use it purely conventionally for the total frequency shift \( \Delta \nu \) of a two way tracking signal \( \nu \rightarrow \nu' \rightarrow \nu'' \) from the ground station to the probe and back, with \( \Delta \nu = \nu - \nu'' \) in the sign convention of the Pioneer team (Anderson e. a. 2002, 8f.).
a. 2002, footnote 38). In addition, the Pioneer mission recorded telemetric data on the state of the spacecraft and for results of scientific measurements. These are not of much relevance for our analysis.

The main evaluation period for Pioneer 10 was partitioned into three intervals, I: 3 Jan 1987 to 17 July 1990, II: 17 July 1990 to 12 July 1992, III: 12 July 1992 to 22 July 1998. Orbit data and the unexpected frequency shift (respectively acceleration $a_P$) were fitted separately for each interval.

As no ranging data had been taken, the model velocity $v_{mod}$ calculated by the orbit determination programs contains the integrated knowledge about the space probe’s kinematics in each of the time intervals. In this sense, we may consider $v_{mod}$ as kind of higher order observational data (“higher order”, because it was not directly measured but relied on a sophisticated evaluation of the raw data). The measured total absolute and relative frequency shifts are

$$\Delta \nu := \nu - \nu'' \text{ and } z := \frac{\Delta \nu}{\nu''}.$$  

If one takes the “Doppler” origin of $\Delta \nu$ literally, the probe’s apparent velocity $\tilde{v}$ seems to be given by

$$\tilde{v} = \frac{\Delta \nu}{c} = \frac{z}{\nu''}.$$  

The Pioneer anomaly consists in a systematic difference between this apparent velocity $\tilde{v}$ and $v_{mod}$

$$\Delta v := \tilde{v} - v_{mod} > 0.$$  

$\Delta v$ turned out to be proportional to the distance $d$ of the spaceprobe, respectively the one way signal running time $t$ between it and the ground station

$$\Delta v \approx C \cdot 2t.$$  

This suggested an interpretation of the constant as an acceleration $a_P$ with $|a_P| = C$ of equ. (6), where the factor 2 expresses the two way tracking procedure. It turned out that $v_{mod} < \tilde{v}$. If positive orientation is defined by the flight direction, the assumed acceleration assumes a negative sign,

$$a_P := -C.$$  

Once it rose above the numerical dirt effects induced by measurement errors $a_P$ turned out to be approximately constant over the whole period of 12 years of data collection, across the different time intervals and even for the different space probes (Pioneer 10 and 11). Thus it seems a very natural point to look for a common external origin of the effect, among them cosmological as one of the possibilities.

One has to keep in mind that the experimental observable of the anomaly was an unmodelled frequency shift (which the Pioneer team calls “Doppler”), rather than an acceleration. According to the research program of the overall mission, the observed phenomenon was translated into terms of an apparent acceleration (Anderson e. a. 2002, 39). Moreover, the team added a warning against any rash attempt for a cosmological explanation. The observed anomalous frequency seems to be a “slight blueshift on top of a larger red
shift” (Anderson e. a. 2002, 17), i.e., of irritating sign for a straight forward cosmological correction.

Figure 1. Pioneer anomaly $|a_P|$ in $10^{-13}$ kms$^{-2}$, (Anderson e.a. 2002, fig. 7)

3. A SLIGHT GENERALIZATION FOR THE DESCRIPTION OF COSMOLOGICAL REDSHIFT

At the time cosmological redshift was detected (in the late 1920s and the 1930s) it was well known that it may have different explanations by expanding space sections of cosmological spacetime or by an energy loss of photons due to higher order gravitational effects (later called “tired light” hypothesis). At certain stages of the development of a science it seems advisable to reconsider hypotheses from a new perspective, which have been neglected during a phase of in track research. In our case, both explanations have a common mathematical description if one uses integrable Weyl geometry extended by Dirac’s scale covariant differentiation (Dirac 1973). Integrable Weyl geometry gives a framework in which both hypotheses can be modelled and transformed into another by a change of scale gauge. The expanding space hypothesis corresponds to the (generally used) Einstein-Riemann gauge, the downscaling photon hypothesis to another one which has been called warp gauge in (Scholz 2007). In the latter the whole warping of the space fibres (the apparent expansion) appears “scaled away” (from the classical, semi-Riemannian viewpoint).

We are here dealing with cosmological effects on a (cosmologically) very small scale level, which can be treated in the linearized regime. This is characterized by a cosmological frequency shift $z_H$ of photons $\nu \rightarrow \nu'$ travelling over a time $t$,

$$z_H = H_0 t, \quad \frac{\nu'}{\nu} = 1 + z_H.$$
We need not go into any detail of the equivalent descriptions in Weyl geometry for our analysis. Here we only have to allow for the logical possibility that equ. (8), the Hubble law, may hold without an underlying space expansion, as a possible alternative to the generally accepted expanding space hypothesis.

We have seen that for the analysis of the Pioneer anomaly neither dynamical effects of the cosmological modification to the equations of motion, nor corrections to the metrical evaluation of the empirical data for distance and time measurement need to be taken into account. Both effects are at least 4 orders of magnitude lower (section 1). Thus our task is simply to analyze which consequences are to be expected, if one takes the Hubble shift $z_H$ of equ. (8) into account for determining the velocities of low speed trajectories, and to compare it with the result which one expects in an expanding space approach.

4. DOWNSCALING PHOTON HYPOTHESIS

If we assume that cosmological redshift is a vacuum or field theoretic effect (without space expansion), it should be present on all scales, although not always observable because of limited measurement precision. In this case, the observed absolute frequency shift $\Delta \nu$ of a two way tracking signal $\nu \mapsto \nu' \mapsto \nu''$ like above has a relative value

$$z = \frac{\Delta \nu}{\nu''}$$

composed of a pure Doppler term

$$z_D = 2 \frac{v}{c}$$

and a Hubble term

$$z_H = 2H_0 t.$$

Here $v$ denotes the velocity of the spacecraft with respect to the observer system, $c$ the velocity of light, and $t$ is the one way running time of the tracking signal (factor 2 because of measuring a two way signal). Notice that under our hypothesis $z_H$ is not linked to a space kinematical velocity component. Up to higher (second) order quantities the whole frequency shift is

$$z = z_D + z_H.$$

If in the data evaluation the total shift $z$ is considered as Doppler, the velocity of the spacecraft is systematically overestimated. High precision orbit data will therefore indicate an observed trajectory which falls back against what one expects from $\tilde{v}$ (equ. (9)). This looks like an unmodelled “fall” of the spacecraft towards the ground station(s).

If the measurement precision is high enough, like in the case of Pioneer 10 and 11, and the dynamical model is reliable, one has

$$z_{\text{mod}} := \frac{v_{\text{mod}}}{c} \approx z_D$$
(inside the marges of measurement errors). Thus it is clear that an “unmodel-
elled” redshift correction $\Delta z$ arises

$$\Delta z := z - z_{\text{mod}} \approx z_H .$$

As $\Delta z$ has to be subtracted to bridge the gap between the observed red-
shift $z$ and the model redshift $z_{\text{mod}}$, an “anomalous” blueward frequency
correction has to be applied.

In (Anderson e. a. 2002, equ. (15)) the Pioneer team has expressed the
frequency shift by an unexpected acceleration $a_P = -|a_P| < 0$ (compare our
equ. (7))

$$\nu_{\text{obs}} - \nu_{\text{mod}} = -\frac{2}{c}|a_P|t .$$

Because

$$\frac{\nu_{\text{obs}} - \nu_{\text{mod}}}{\nu} = 1 + \frac{1}{1 + z_{\text{obs}}} - \frac{1}{1 + z_{\text{mod}}} = z_{\text{mod}} - z_{\text{obs}} + o(z) ,$$

this comes down, up to higher order terms in $z$ (i.e., far inside the bounds
of the observational error), to

$$\Delta z \approx \frac{2}{c}|a_P| t = -2 \frac{a_P}{c} t .$$

The “coincidence”

$$|a_P| c = - \frac{a_P}{c} \approx H_0$$

is immediate. It looses any surprise, if it is considered from the point
of view of the downscaling photon hypothesis. In this framework the Pioneer
frequency shift is nothing but the Hubble effect.

It is easy to understand, why the “anomalous acceleration” started to be
observable only at the distance of about 10 AU. The frequency stability
of the hydrogen maser in the Pioneer crafts was at the order of magnitude
$10^{-15}$ for the integration times in question (Anderson e. a. 2002, 7). In order
that $\Delta z = 2H_0 t$ enters the next order of magnitude, the signal running time
$t$ has to be such that

$$2H_0 t \sim 10^{-14} \iff t \sim 5 \cdot 10^3 s .$$

That corresponds to $d = ct \sim 10$ AU, shortly beyond the Saturn orbit. At
smaller distances the Hubble effect vanishes below the threshold of observ-
ability given by the maser stability.

The experimental precision data $a_{p_{\text{exp}}}$ quoted in our introduction (equ.
(2)), taken together with (3), underpin equation (15) observationally:

$$(2.6 \pm 0.44) \times 10^{-18} s^{-1} \approx (2.3 \pm 0.23) \times 10^{-18} s^{-1}$$

Let us repeat why, under the hypothesis of a downscaling photon origin of
the Hubble effect, no sign problem arises for the Pioneer frequency shift. It
is true that the Hubble effect $z_H$ shifts toward the red and adds to the pure
Doppler shift $z_D$. But the (fitted) correction term $\Delta z$ has to be subtracted

\footnote{Readers who know J. Masreliez’ theory of scale expanding cosmos (SEC) may notice
that Weyl geometry allows to give a mathematical foundation to the SEC approach.
Masreliez’ tired light explanation has nothing to do, however, with the analysis given
here. He argues (dubiously) with two different time scales and derives a Pioneer like effect
of wrong sign (Masreliez 2005, equ. 6.3).}
from the observed total redshift \( z \), if one wants to isolate the pure Doppler shift from the corpus of the empirical data:

\[
z_D = z - z_H
\]

Only then we arrive at velocity data which are consistent with the model calculations. Thus the correction terms appears as a “blue shift on top of a red shift”.

5. COMPARISON WITH EXPANDING SPACE HYPOTHESIS AND DISCUSSION

Let us resume. Assuming the downscaling photon hypothesis, the Pioneer shift becomes an obvious and natural consequence of the Hubble effect by extremely simple calculations. Basically, a simple proportionality is all one needs to consider. Of course, one has also to distinguish + and −. The physically additive effect \( z_H \) has to be subtracted from the total signal for the correction. More subtle mathematics comes into the play only for a deeper understanding of the framework geometry (Weyl geometry) and field theory (scale covariant gravity).

From a physical point of view, the result is more interesting. If our assumption (field theoretic origin of cosmological redshift) is right, the Pioneer effect turns out to be a new experimental manifestation of the Hubble law, well known at a different distance scale level. The expanding space hypothesis leads to no satisfying cosmological explanation of the frequency shift. If the downscaling photon assumption hits the point, it can even be proved that a model anomaly arises necessarily in the expanding space interpretation.

The comparison of both perspectives is strongly facilitated by relating them to the frame of a slightly generalized background theory of relativistic cosmology. From the point of view of Weyl-Dirac geometry the warp function \( f(t) \) of Robertson-Walker cosmologies may be understood as a gauge factor arising from integrating a scale (length) connection. The latter is expressed by a differential one-form,

\[
\varphi = \sum_{i=0}^{3} \varphi_i dx_i ,
\]

and models the cosmological redshift partially or completely, depending on the physical assumptions for the origin of the Hubble effect. In our case, the downscaling photon assumption is modelled by the classical relativistic (or PPN) equation of the orbit dynamic, on which an additional Weylian length (scale) connection \( \varphi \) with

\[
\varphi_0 = H_0 , \quad \varphi_\alpha = 0 \quad \text{for} \quad \alpha = 1, 2, 3
\]

is superimposed. That gives negligible dynamical modifications, while the scale connection describes an energy loss for photons

\[
\Delta E = - H_0 \Delta t
\]

over a running time \( \Delta t \) (energy is of scale weight −1). \( \Delta E \) is identical with the loss one would find in a Robertson-Walker geometry arising from integrating the scale connection ((Scholz 2005). The Weylian scale connection
\( \varphi = H_0 dt \) describes how metrical data at one point are transferred to another one (here along null geodesics). Weyl geometry therefore predicts an apparent clock drift by a factor \( H_0 t \) in the linearized regime (Anderson et al. 2002, equ. (16)). It is a result of the transfer process only and does not indicate a different “rate of clock ticking” in any meaningful sense.

This observation leads back to the question, whether the scale connectivity explanation of the Pioneer effect (17) contradicts established empirical knowledge which invalidates the older variants of “tired light” assumptions of the 20th century, from F. Zwicky (1929) to J.-P. Vigier (1990). Surely Weyl geometric downscaling of photon energy shares a common motif with them. But the specific mechanisms which have been considered as possible causes for the energy loss, like Zwicky’s gravitational “drag” of light, assumed ad-hoc in a first, so to speak heroic, attempt (Zwicky 1929), specific kinds of photon-photon interaction (Freundlich 1954, Pecker et al. 1972), or a non-zero rest-mass of the photon (Vigier 1990), presupposed a “naive”, or even an unspecified relativistic background geometry. They were not part of the gravitational structure itself. Thus most (all?) of the arguments for an empirical invalidation of tired light theories by astronomical observations, are irrelevant for the Weyl geometric downscaling hypothesis, or even support the latter. Most clearly, the observation of time stretch for supernovae light curves (Goldhaber et al. 2001), fits beautifully to a Weyl geometric description of signal transfer. In integrable Weyl geometry, any time interval information transmitted by photon signals acquires the inverse factor \( (1 + z)^{-1} \) of the corresponding redshift \( z \) as scale transfer (energy \( E \) and time \( T \) are of complementary gauge weights in Weyl geometry, \( \left[ E \right] = -1, \left[ T \right] = 1 \)). Other devalidations of the older tired light theories argue against naive or even “guessed” formula for the distance in the underlying geometry, e.g., (Lubin/Sandage 2001, 20). The Tolman brightness characteristic in Weyl universes (the most simple models of the Weyl geometric approach) behaves like in a comparable expanding space model (and differs from the present standard model only very little up to \( z \approx 1.2 \) (Scholz 2007, 26)).

In the integrable Weyl geometry version of general relativity the explanations of cosmological redshift by expanding space sections or by a field theoretic reduction of photon energy become mathematically interchangeable by a transformation of the scale gauge. This poses the new physical question, which of the gauges corresponds to material measurements. Cosmological observations indicating which of the two gauges expresses physical (material) measurements are difficult to evaluate and are still inconclusive.

The Pioneer effect, on the other hand, has the qualities of a picture-book experimentum crucis which allows to decide empirically between the two hypotheses. Pioneer type observations are able to supply two data sets, frequency shift and distance data. In an optimal experiment there would be independently measured data on signal running time, here they are reconstructed by orbit model calculations. A comparable duplication of empirical information seems to be impossible for cosmological observations proper. Of course, the Pioneer mission was not designed for that purpose. The realization that there was an unexpected frequency shift was the outcome of careful and painstaking work of more than a decade data preparation and
exploratory data analysis of an irritating effect. That it has the qualities of both types of experimental enterprises, should make it interesting for philosophers and historians of science, besides the more direct disciplinary repercussions we may expect in physics and cosmology.

Of course, we need to have a broader and more precisely checked experimental basis, before we can be sure to replace the expanding space hypothesis by a downscaling photon explanation of the Hubble effect. Follow up experiments of the Pioneer type could decide clearly between a Hubble effect explanation and other explanatory schemes discussed in the literature. Although the design of future Pioneer like experiments concentrates on the direction and quantity of a true acceleration (Nieto/Turyshchev 2004, Nieto e. a. 2005, Izzo/Rathke 2005), the planned missions will also be able to discriminate sharply between the two gauge perspectives and the corresponding physical hypotheses. It is planned that two (or even three) data sets are raised independently, most important in our context ranging data \( t(\tau) \) and frequency shift \( z(\tau) \) (\( \tau \) orbit parameter expressed in ephemeris time and \( t \) the signal running time as above). Then the time derivative of ranging distances \( v := \frac{c dt}{d\tau} \) and the apparent velocity \( \tilde{v} = cz \) derived from the total frequency shift \( z \) can be used for a direct comparison of experimental data. From our point of view, we expect agreement between \( v \) and \( v_{\text{mod}} \) derived from orbit modelling like in the Pioneer experiment, and \( \frac{\tilde{v} - v}{c} = H_0 t \). Increasing precision may even give new empirical information on the Hubble constant.

6. Conclusion

In our view, the Pioneer effect should not be understood as a real acceleration. It essentially consists of a frequency gap, \( \Delta z \), between a measured total redshift \( z \) and the one, \( z_{\text{mod}} \), derived from orbit calculations. \( \Delta z \) turned out to be proportional to the running time \( t \) of the signal

\[
\Delta z = \text{const} \cdot t .
\]

For the research context of the Pioneer group it was more than natural to express the discrepancy by an acceleration like term, \( \text{const} = \frac{|a|}{c} \). But as it was known since the first ‘anomalous’ observations that

\[
\text{const} \approx H_0 ,
\]

a strictly empiricist interpretation of the Pioneer frequency gap as just another expression for the Hubble law in different experimental disguise would be a natural next step. Considering the fact that the redshift derived from orbit data, \( z_{\text{mod}} \), contains the whole knowledge about a “pure” Doppler shift, as far as the latter can be inferred from orbit observation, one might consider these as higher order empirical data. Disregarding any possible cognitive tension with the expanding space hypothesis and, if necessary, in open opposition to the latter, the data may be taken as an empirical indication for the validity of the Hubble law, equ. (8), inside the solar system.

That this has not been done already years ago, adds another case to a striking observation of Felix Hausdorff, “Formalism is the true empiricism . . . ” (Hausdorff RuZ), [because it helps to go beyond the limits of culturally inherited conceptual limits, E.S.]. Originally this remark was formulated
in the context of non-Euclidean geometry, but it is of wider import. It seems that a formalist analysis of cosmological geometry by integrable Weyl geometry had to precede the realization that the Pioneer effect may be read as a direct solar system expression of the Hubble effect, and probably has to.

In any case, the real surprise of the Pioneer measurements seems to be the demonstration that the Hubble effect may already be observable in high precision experiments in the outer solar system. In this regard we expect that the Pioneer experiment has considerable further consequences for our theoretical understanding of the foundations of cosmology and, indirectly, perhaps also for gravitational physics.

References
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