Hermann Weyl

Erhard Scholz, Wuppertal
January 3rd, 2005

Hermann Weyl (b. November 9, 1885, at Elmshorn in northern Germany; d. December 8, 1955 at Zürich) studied at Göttingen under D. Hilbert, H. Minkowski, and F. Klein between 1904 and 1908 (dissertation), at a time when Göttingen mathematics was at its apogee. After his habilitation (1910) he taught three years at Göttingen. He became professor of mathematics at the ETH Zürich in 1913, where he lived and worked during his mathematically most productive years. In 1930 he hesitatingly accepted a call back to Göttingen as Hilbert's successor. His second Göttingen period remained a short episode, as Weyl would not adapt to the Nazi regime. He rather preferred to emigrate to the United States in 1933 where he was offered a position at the newly founded Institute for Advanced Studies. There he stayed until his retirement in 1951, after that sharing his time between Princeton and his beloved Zürich.

In his prolific work H. Weyl contributed to real and complex analyis, geometry and topology, Lie groups, number theory, to the foundations of mathematics, mathematical physics and philosophy. He was a tremendously inventive mathematician, an unconventional, deep thinker, and a fluid writer. To all of the fields mentioned he contributed at least one book (all in all 13 without translations during his lifetime) which, together with his other technical and conceptual innovations, often led to strong immediate response and were of lasting influence on the mathematical sciences of the 20th century. They surely will continue to do so well into the following one. In this note we can only indicate a highly selective profile of his work, centered around his most influential books which have become *classics* of mathematical literature.

H. Weyl started his research in the field of integral operators (Weyl 1908) and of differential equations with singular boundary conditions. At the time of his habilitation he anticipated among others, the defect theory of unbounded Hermitian operators in special cases and the asymptotic properties of eigenvalues of bounded self-adjoint operators (Weyl 1910, Weyl 1911). The analytical questions studied by him were often linked to physical problems (boundary value problems of elastic vibrations), sometimes to number theory or even to both as in a famous paper on equidistribution modulo 1 (Weyl 1916).

Weyl's fame started with his first book (Weyl 1913), which grew out of a lecture course at Göttingen in winter 1910/11 and built upon Klein's intuitive treatment of Riemannian function theory, Hilbert's justification of Dirichlet's principle, Hilbert's hints on the analysis situs of the plane in his work on the foundations of geometry, and Poincaré's construction of covering surfaces of algebraic curves. Weyl gave a new presentation of the foundation and of the main properties of Riemannian surfaces, which was highly influential for geometric function theory of the 20th century (although its first edition stood still on shaky grounds with respect to its basic concept of a real 2-dimensional manifold).

His second book (Weyl 1918*a*) was written after the First World War and under the impression of a need for a safe intellectual base in a time of cultural turnover. It opened Weyl's intervention into the foundations of mathematics, different from Hilbert's program of an axiomatic formalist fondation, and even in opposition to it. Here Weyl sketched how a reduced part of real analysis could be secured by constructions in a semi-formalized arithmetical language, respecting the restrictions of predicativity. Shortly after this publication, Weyl changed his mind and started to support Brouwer's more radical intuitionistic program. He attacked Hilbert's foundational views strongly and polemically in a famous programmatic article (Weyl 1921). In the late 1920s he developed more multifaceted views on the foundational questions. After the second World War he even returned to a weak preference of his arithmetical constructive approach of 1918.[1]

Overlapping with the work in the foundations of analysis and the concept of the continuum, Weyl started to analyze and to deepen the links between differential geometry and the newly founded general theory of relativity. His book *Raum - Zeit - Materie* (Space - Time - Matter), first published in 1918 and revised in five successive editions until 1923, was one of the first text books of the subject and among the most influential ones over decades to come. In its first part, the book gave an up to date introduction to Riemannian and Lorentzian geometry, from the third edition (1919) onwards extended by an introduction of a gauge geometric generalization of semi-Riemannian geometry, which Weyl had invented at the time of writing the first edition (Weyl 1918*d*). It served him as a starting point for a proposal of a geometrically unified field theory of gravitation and electromagnetism and an attempt to derive the basic matter structures from it (following and extending a research program of G. Mie and D. Hilbert) (Weyl 1918*c*). The book was only the "top of an iceberg" of other contributions to this active field of interchange between differential geometry and general relativity. Weyl pursued it in a broad conceptual and philosophical perspective. One of the outcomes of this approach was his *Analysis of the Problem of Space* (Weyl 1923) in which he sketched concepts which later would be turned into fibre bundles and the study of geometries characterized by gauge structures in them.[2]

About the middle of 1920s H. Weyl delved into what became his most in-

[1](Feferman 2000, van Dalen 2000, Hesseling 2003, Mancosu 1998, Scholz 2000, Schappacher 2003).

[2](Vizgin 1994, Goenner 2004, Scholz 2001, Scholz 2004a, Corry 2004)

fluential contributions to pure mathematics, the study of the representations of semi-simple Lie groups. Combining E. Cartan's insights into the representation theory of Lie algebras (in later terminology) with methods developed by A. Hurwitz and I. Schur to study the representation of certain matrix groups (orthogonal, unitary, and special linear), invariant theoretic methods as used by E. Study and others, Weyl used his knowledge of the topology of manifolds and developed the core of the general theory of representations of Lie groups in a genuine blend of geometric, algebraic, and analytic methods (Weyl 1925/1926). Extended and refined, this work formed the core of his later book *The Classical Groups* written as a harvest of his work and his lecturing activities on this topic during his Princeton years (Weyl 1939).[3]

During his work on the representation theory of Lie groups Weyl actively followed the turn towards the "new" quantum mechanics and started to explore the possibilities opened up by the interplay of infinitesimal and integral group operations in quantum mechanics. In 1927/28 he gave a lecture at the ETH on the topic, which gave rise to his second book on mathematical physics *Group Theory and Quantum Mechanics* (Weyl 1928). Weyl emphasized the conceptual role of group methods in the symbolic representation of quantum structures, in particular the intriguing interplay of representations of the special linear group and permutations groups. In this interplay he also saw the mathematical clue to understanding the phenomenon of spin coupling, studied at the time by some of the young physicists turning towards quantum chemistry (F. London, W. Heitler e.a.). Recently it has attracted new interest in the different context of quantum computing. Moreover, Weyl explored the central role of the discrete symmetries (parity P, charge conjugation C, and time inversion T) in the first steps toward a quantized version of electrodynamics, thus anticipating structural elements which turned into important questions for physics three decades later. Not covered in the book, but published separately, was a second step for his gauge theory of the electromagnetic field. Quantum theorists had proposed to reconsider the gauge idea for the phase of wave functions rather than for scale gauge as in Weyl's original idea of 1918. Weyl took up the proposal and explored it after the rise of Dirac's spinor fields for the electron (as was done independently by V. Fock in a similar way) (Weyl 1929). He came to the conclusion that the new context demanded considering symmetry extension of the Lorentz group by unitary transformations in $U(1)$. That gave rise to a modified gauge theory of electromagnetism which was endorsed by leading theoretical physics (among them W. Pauli and E. Schrödinger, V. Fock) and served as a starting point for the next generation of physicists who molded the symbolic frame of gauge field theories in the 1950/60s.[4]

[3](Hawkins 2000, Borel 2001, Slodowy 1999)

[4](Coleman/Korté 2001, Mackey 1988, Mackey 1993, Yang 1986, O'Raifeartaigh/Straumann 2000, Scholz 2002, Scholz 2004b)

H. Weyl was not only a philosophically interested researcher in mathematics and physics. He gave active literary expressions to his philosophical reflections of scientific activity in many of his publications, among them five of his books. Most influential was his contribution to a philosophical handbook *Philosophy of Mathematics and Natural Science*, originally published in German in 1927, translated into English in 1949. It became a classic in the philosophy of science.

Many of Weyl's other contributions to mathematics, physics and the philosophy of science could not be mentioned in this overview. For broader and more detailed information see, among others (Newman 1957, Chevalley/Weil 1957, Dieudonné 1976, Coleman/Korté 2001, Frei/Stammbach 1992, Sigurdsson 1991, Deppert 1988, Chandrasekharan 1986, Wells 1988).

References

Borel, Armand. 2001. *Essays in the History of Lie Groups and Algebraic Groups.* Providence, RI, etc.: AMS and London Mathematical Society.

Chandrasekharan, K. (ed.). 1986. *Hermann Weyl: 1885 - 1985;* Centenary Lectures delivered by C.N. Yang, R. Penrose, and A. Borel at the Eidgenössische Technische Hochschule Zürich. Berlin etc.: Springer.

Chevalley, Claude; Weil, André. 1957. "Hermann Weyl (1885–1955)." *L'Enseignement* (2) 3:157–187.

Coleman, Robert; Korté, Herbert. 2001. Hermann Weyl: Mathematician, physicist, philosopher. In *(Scholz 2001).* pp. 161–388.

Corry, Leo. 2004. *David Hilbert and the Axiomatization of Physics (1898-1918).* From Grundlagen der Geometrie to Grundlagen der Physik. Dordrecht: Kluwer.

Deppert, W. e.a. (ed.s.). 1988. *Exact Sciences and Their Philosophical Foundations.* Frankfurt/Main: Peter Lang. Weyl, Kiel Kongress 1985.

Dieudonné, Jean. 1976. Weyl, Hermann. In *Dictionary of Scientific Biography*, ed. C.G. Gillispie. Vol. 14 pp. 281–285.

Feferman, Solomon. 2000. The significance of Weyl's Das Kontinuum. In *(Hendricks 2000).* pp. 179–194.

Frei, Günther; Stammbach, Urs. 1992. *Hermann Weyl und die Mathematik an der ETH Zürich, 1923 – 1930.* Basel etc.: Birkhäuser.

Goenner, Hubert. 2004. "On the history of unified field theories." *Living Reviews in Relativity* . [http://relativity.livingreviews.org/Articles/lrr-2004-2, visited June 2, 2004].

Hawkins, Thomas. 2000. *Emergence of the Theory of Lie Groups. An Essay in the History of Mathematics 1869–1926.* Berlin etc.: Springer.

Hendricks, Vincent F.; Pedersen, Stigandur; Jørgensen Klaus F., ed. 2000. *Proof Theory: History and Philosophical Significance.* Dordrecht etc.: Kluwer.

Hesseling, Dennis. 2003. *Gnomes in the Fog. The Reception of Brouwer's Intuitionism in the 1920s.* Basel: Birkhäuser.

Mackey, George. 1988. "Hermann Weyl and the application of group theory to quantum mechanics.". In (Deppert 1988, 131–160).

Mackey, George. 1993. "The mathematical papers.". In Wigner, Eugene. *Collected Works.* vol. A I, Berlin etc.: Springer, 241–290.

Mancosu, Paolo. 1998. *From Brouwer to Hilbert.* The Debate on the Foundations of Mathematics in the 1920s. Oxford etc.: University Press.

Newman, M.H.A. 1957. "Hermann Weyl, 1885–1955." *Biographical Memoirs of Fellows of the Royal Society London* 3:305–328.

O'Raifeartaigh, Lochlainn. 1997. *The Dawning of Gauge Theory.* Princeton: University Press.

O'Raifeartaigh, Lochlainn; Straumann, Norbert. 2000. "Gauge theory: Historical origins and some modern developments." *Reviews of Modern Physics* 72:1–23.

Schappacher, Norbert. 2003. "Politisches in der Mathematik: Versuch einer Spurensicherung." *Mathematische Semesterberichte* 50:1–27.

Scholz, Erhard. 2000. Hermann Weyl on the concept of continuum. In *Proof Theory: History and Philosophical Significance*, ed. Vincent Hendricks; S.A. Pedersen; K. Frovin. Dordrecht: Kluwer pp. 195–220.

Scholz, Erhard (ed.). 2001. *Hermann Weyl's* Raum - Zeit - Materie *and a General Introduction to His Scientific Work.* Basel etc.: Birkhäuser.

Scholz, Erhard. 2002. "Local spinor structures in V. Fock's and H. Weyl's work on the Dirac equation (1929).". Preprint Wuppertal, to appear in D. Flament, C. Houzel. (eds.) *Géométrie au XXième siècle.* Paris [http://arxiv.org/physics/0409158].

Scholz, Erhard. 2004*a*. "Hermann Weyl's analysis of the "problem of space" and the origin of gauge structures." *Science in Context* 17:165–197.

Scholz, Erhard. 2004*b*. "The introduction of groups into quantum theory." *Preprint Wuppertal*, submitted to *Historia Mathematica* .

Sigurdsson, Skúli. 1991. "Hermann Weyl, Mathematics and Physics, 1900 – 1927." Cambridge, Mass.: PhD Dissertation, Harvard University.

Slodowy, Peter. 1999. "The early development of the representation theory of semisimple Lie groups: A. Hurwitz, I. Schur, H. Weyl." *Jahresbericht der Deutschen Mathematiker-Vereinigung* 101:97–115.

van Dalen, Dirk. 2000. *Brouwer and Weyl on proof theory and philosophy of mathematics.* pp. 117–152.

Vizgin, Vladimir. 1994. *Unified Field Theories in the First Third of the 20th Century.* Translated from the Russian by J. B. Barbour. Basel etc.: Birkhäuser.

Wells, Raymond O. (ed.). 1988. *The Mathematical Heritage of Hermann Weyl:* Proceedings of the Symposium on the Mathematical Heritage of Hermann Weyl, held at Duke University, Durham, North Carolina, May 12 - 16, 1987. Providence, R.I.: American Mathematical Society.

Weyl, Hermann. 1908. "Singuläre Integralgleichungen mit besonderer Berücksichtigung des Fourierschen Integraltheorems." Göttingen: Dissertation. In (Weyl 1968, I, 1-87) [1].

Weyl, Hermann. 1910. "Über gewöhnliche Differentialgleichungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen." *Mathematische Annalen* 68:220–298. In (Weyl 1968, I, 248-297) [8].

Weyl, Hermann. 1911. "Über die asymptotische Verteilung der Eigenwerte." *Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen; Mathematisch-physikalische Klasse* pp. 110–117. In (Weyl 1968, I, 368–375) [13].

Weyl, Hermann. 1913. *Die Idee der Riemannschen Fläche.* Leipzig: Teubner. ^{2}1923, ^{4}1955, 51997. English (Weyl 1913/1964).

Weyl, Hermann. 1913/1964. *The Concept of a Riemann Surface.* Reading: Mass.: Addison-Wesley. English translation of (Weyl 1913, 3rd ed.) by G. MacLane.

Weyl, Hermann. 1916. "Über die Gleichverteilung von Zahlen mod. Eins." *Mathematische Annalen* 77:313–352. In (Weyl 1968, I, 563–599) [23].

Weyl, Hermann. 1918*a*. *Das Kontinuum. Kritische Untersuchungen über die Grundlagen der Analysis.* Leipzig: Veit. ²1932 Berlin: de Gruyter. English (Weyl 1918/1987).

Weyl, Hermann. 1918/1987. *The Continuum.* English translation by S. Pollard and T Bole. Kirksville, Missouri: Thomas Jefferson University Press. Corrected republication New York Dover 1994.

Weyl, Hermann. 1918*b*. *Raum, - Zeit - Materie.* Berlin etc.: Springer. Weitere Auflagen: ²1919, ³1919, ⁴1921, ⁵1923, ⁶1970, ⁷1988, ⁸1993.

Weyl, Hermann. 1918*c*. "Gravitation und Elektrizität." *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* pp. 465–480. GA II, 29–42, [31]. In (Weyl 1968, II, 29–42) [31], English in (O'Raifeartaigh 1997, 24–37).

Weyl, Hermann. 1918*d*. "Reine Infinitesimalgeometrie." *Mathematische Zeitschrift* 2:384–411. In (Weyl 1968, II, 1–28) [30].

Weyl, Hermann. 1921. "Über die neue Grundlagenkrise der Mathematik." *Mathematische Zeitschrift* 10:39–79, GA II, 143–180, [41]. English in (Mancosu 1998, 86–121).

Weyl, Hermann. 1922. *Space, Time, Matter.* Translated from the 4th German edition by H. Brose. London: Methuen.

Weyl, Hermann. 1923. *Mathematische Analyse des Raumproblems.* Vorlesungen gehalten in Barcelona und Madrid. Berlin etc.: Springer. Nachdruck Darmstadt: Wissenschaftliche Buchgesellschaft 1963.

Weyl, Hermann. 1925/1926. "Theorie der Darstellung kontinuierlicher halbeinfacher Gruppen durch lineare Transformationen. I, II,III und Nachtrag." *Mathematische Zeitschrift* 23, 24:*23* 271–309, *24* 328—-395, 789–791. In (Weyl 1968, II, 543–646) [68].

Weyl, Hermann. 1927. *Philosophie der Mathematik und Naturwissenschaft,* Handbuch der Philosophie, Abt. 2A. . München: Oldenbourg. Weitere Auflagen ²1949, ³1966. English with comments and appendices (Weyl 1949).

Weyl, Hermann. 1928. *Gruppentheorie und Quantenmechanik.* Leipzig: Hirzel. ²1931, English 1931.

Weyl, Hermann. 1929. "Elektron und Gravitation." *Zeitschrift für Physik* 56:330–352. In (Weyl 1968, III, 245–267) [85]. English in (O'Raifeartaigh 1997, 121–144).

Weyl, Hermann. 1931. *The Theory of Groups and Quantum Mechanics* Translated from the 2nd Geman edition by H.P. Robertson. New York: Dutten.

Weyl, Hermann. 1939. *The Classical Groups, Their Invariants and Representations.* 2nd ed. 1946. Princeton: University Press. ²1946.

Weyl, Hermann. 1949. *Philosophy of Mathematics. and Natural Science.* 2nd ed. 1950. Princeton: University Press. ²1950.

Weyl, Hermann. 1955. *Selecta.* Basel: Birkhäuser.

Weyl, Hermann. 1968. *Gesammelte Abhandlungen, 4 vols.* Ed. K. Chandrasekharan. Berlin etc.: Springer.

Yang, Chen Ning. 1986. "Hermann Weyl's contribution to physics.". In (Chandrasekharan 1986, 7–21).