

Cosmological Spacetimes Balanced by a Weyl Geometric Scale Covariant Scalar Field

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Abstract A Weyl geometric approach to cosmology is explored, with a scalar field ϕ of (scale) weight -1 as crucial ingredient besides classical matter. Its relation to Jordan-Brans-Dicke theory is analyzed; overlap and differences are discussed. The energy-stress tensor of the basic state of the scalar field consists of a vacuum-like term $\Lambda g_{\mu\nu}$ with Λ depending on the Weylian scale connection and, indirectly, on matter density. For a particularly simple class of Weyl geometric models (called *Einstein-Weyl universes*) the energy-stress tensor of the ϕ -field can keep space-time geometries in equilibrium. A short glance at observational data, in particular supernovae Ia (Riess et al. in *Astrophys. J.* 659:98ff, 2007), shows encouraging empirical properties of these models.

Keywords Cosmology · Cosmological models · Scalar field · Scale covariance · Weyl geometry · Mass generation · Pseudo conformal field theory · Einstein-Weyl universe

1 Introduction

For more than half a century, Friedman-Lemaitre (F-L) spacetimes have been serving as a successful paradigm for research in theoretical and observational cosmology. With the specification of the parameters inside this model class, $\Omega_m \approx 0.25$, $\Omega_\Lambda \approx 0.75$, new questions arise. Most striking among them are the questions of how to understand the ensuing “accelerated expansion” of the universe indicated by this paradigm after evaluating the observational data, and those concerning the strange behavior of “vacuum energy” [7]. The latter seems to dominate the dynamics of space-time and of matter in cosmically large regions, without itself being acted upon by

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the matter content of the universe. Such questions raise doubts with respect to the reality claim raised by the standard approach [14]. They make it worthwhile to study to what extent small modifications in the geometric and dynamical presuppositions lead to different answers to these questions, or even to a different overall picture of the questions themselves.

In this investigation we study which changes of perspective may occur if one introduces scale covariance in the sense of integrable Weyl geometry (IWG) into the consideration of cosmological physics and geometry. At first glance this may appear as a formal exercise (which it is to a certain degree), but the underlying intention is at least as physical as it is mathematical. The introduction of scale freedom into the basic equations of cosmology stands in agreement with a kindred Weyl geometric approach to scale covariant field theory [11, 12, 24] and overlaps partially with conformal studies of semi-Riemannian scalar-tensor theories [15, 19]. In distinction to the latter, Weylian geometry allows a mathematical overarching approach to cosmological redshift, without an ex-ante decision between the two causal hypotheses of its origin, space expansion or a field theoretic energy loss of photons over cosmic distances. Although in most papers the first alternative is considered as authoritative, it is quite interesting to see how in our frame the old hypothesis of a field theoretic reduction of photon energy, with respect to a family of cosmological observers, finds a striking mathematical characterization in Weyl geometry. Cosmological redshift may here be expressed by the scale connection of Weylian Robertson-Walker metrics, in a specific scale gauge (warp gauge).

The following paper gives a short introduction to basics of Weyl geometry and the applied conventions and notations (Sect. 2). After this preparation the scale invariant Lagrangian studied here is introduced (Sect. 3). Different to Weyl's fourth order Lagrangian for the metric, an adaptation of the standard Hilbert-Einstein action serves as the basis of our approach, coupled in such a way to a scale covariant scalar field ϕ of weight -1 that scale invariance of the whole term is achieved. This approach is taken over from W. Drechsler's and H. Tann's research in field theory, which explores an intriguing path towards deriving mass coefficients for the electroweak bosons by coupling to gravity and the scalar field. For gravity theory Weyl geometric scalar fields show similarities with conformal studies of Brans-Dicke type theories, but differ in their geometrical and scale invariance properties (Sect. 4). Section 5 of this paper gives a short outline and commentary of Drechsler/Tann's proposal of the Weyl geometric Higgs-like "mechanism".

In the next section the variational equations of the Lagrangian are presented. They lead to a scale co/invariant form of the Einstein equations, a Klein-Gordon equation for the scalar field and the Euler equation of ideal fluids (Sect. 6). Then we turn towards cosmological modeling in the frame of Weyl geometry. The isotropy and homogeneity conditions of Robertson-Walker metrics are adapted to this context and lead to scale covariant Robertson-Walker fluids. New interesting features arise in the Weyl geometric perspective, in particular with respect to the symbolic representation of cosmological redshift by a scale connection (*Hubble connection*) (Sect. 7). The most simple Weyl geometric models of cosmology (*Weyl universes*) are similar to the classical static geometries; but here they are endowed with a scale connection encoding cosmological redshift (Sect. 8).

Luckily, the geometry of Weyl universes is simple enough to allow an explicit calculation of the energy-stress tensor of the scalar field’s ground state (Sect. 9). Thus it can be investigated under which conditions the scalar field safeguards dynamical consistency (equilibrium) of Weyl universes. Those with positive space sectional curvature are called *Einstein-Weyl universes*. A consistency condition derived from the Klein-Gordon equation of the scalar field leads to specific coupling condition for Hubble redshift to sectional curvature of the spatial fibres and thus to the matter content of the universe.

The article is rounded off by a short look at data from observational cosmology (Sect. 10) and a discussion of the perspective for cosmology opened up by the Weyl geometric approach to gravity and of some of the open questions (Sect. 11).

2 Geometric Preliminaries and Notations

We work in a classical spacetime given by a differentiable manifold M of dimension $n = 4$, endowed with a *Weylian metric* [6, 17]. The latter may be given by an equivalence class $[(g, \varphi)]$ of pairs (g, φ) of a Lorentzian metric $g = (g_{\mu\nu})$ of signature $(-, +, +, +)$, called the *Riemannian component* of the metric, and a *scale connection* given by a differential 1-form $\varphi = (\varphi_\mu)$. Choosing a representative (g, φ) means to *gauge* the metric. A *scale gauge transformation* is achieved by rescaling the Riemannian component of the metric and an associated transformation of the scale connection

$$\tilde{g} = \Omega^2 g, \quad \tilde{\varphi} = \varphi - d \log \Omega, \tag{1}$$

where $\Omega > 0$ is a strictly positive real function on M .

Einstein’s famous argument against Weyl’s original version of scale gauge geometry (stability of atomic spectra) and—related to it—coherence with quantum physics [2] make it advisable, to say the least, to restrict the Weylian metric to one with *integrable scale* (“length”) *connection*, $d\varphi = 0$.

The integration of φ leads to a *scale* (or “length”) *transfer* function $\lambda(p_0, p_1)$ allowing to compare metrical quantities at different points of the manifold,

$$\lambda(p_0, p_1) = e^{\int_{u_0}^{u_1} \varphi(\gamma'(u))du}, \tag{2}$$

$\gamma(u)$ any differentiable path from a fixed reference point $p_0 = \gamma(u_0)$ to $p = \gamma(u_1)$. In simply connected regions the scale connection can be integrated away, $\tilde{g} = \lambda^2 g$, $\tilde{\varphi} = 0$, if $d\varphi = 0$. In this case the Weylian metric may be written in Riemannian form, but need not. By obvious reasons this gauge is called *Riemann gauge*. Thus one may work in *integrable Weyl geometry (IWG) without passing to Riemann gauge* by default.

There is a uniquely determined *Levi Civita connection* of the Weylian metric,

$$\Gamma_{\nu\lambda}^\mu = g^\mu_{\nu\lambda} + \delta_\nu^\mu \varphi_\lambda + \delta_\lambda^\mu \varphi_\nu - g_{\nu\lambda} \varphi^\mu. \tag{3}$$

Here $g^\mu_{\nu\lambda}$ denote the coefficients of the affine connection with respect to the Riemannian component g only. The Weyl geometric covariant derivative with respect to

$\Gamma_{\nu\lambda}^\mu$ will be denoted by ∇_μ ; the covariant derivative with respect to the Riemannian component of the metric only by ${}_g\nabla_\mu$. ∇_μ is an invariant operation for vector and tensor fields on M , which are themselves invariant under gauge transformations. The same can be said for *geodesics* γ_W of Weylian geometry, defined by ∇_μ , and for the *curvature tensor* $R = R^\alpha_{\beta\gamma\delta}$ and its contraction, the Ricci tensor *Ric*. All these are *invariant* under scale transformations.

For calculating geometric quantities (covariant derivatives, curvatures etc.) of a Weylian metric in the gauge (g, φ) one may start from the corresponding (Riemannian) ones, with respect to the Riemannian component g of the Weylian metric given by (g, φ) . Like for the affine connection we use the pre-subscript g to denote the latter, e.g., ${}_g\bar{R}$ for the scalar curvature of the Riemannian component. For $\dim M = n$ we know already from [42, p. 21]:

$$\bar{R} = {}_g\bar{R} - (n - 1)(n - 2)\varphi_\lambda\varphi^\lambda - 2(n - 1) {}_g\nabla_\lambda\varphi^\lambda, \tag{4}$$

$$\begin{aligned} (Ric)_{\mu\nu} = & {}_gRic_{\mu\nu} + (n - 2)(\varphi_\mu\varphi_\nu - {}_g\nabla_{(\mu}\varphi_{\nu)}) \\ & - g_{\mu\nu}((n - 2)\varphi_\lambda\varphi^\lambda + {}_g\nabla_\lambda\varphi^\lambda). \end{aligned} \tag{5}$$

For $n = 4$, in particular, that is

$$\bar{R} = {}_g\bar{R} - 6(\varphi_\lambda\varphi^\lambda + {}_g\nabla_\lambda\varphi^\lambda) \quad \text{etc.}$$

In order to make full use of the Weylian structure on M one often considers (real, complex etc.) functions f or (vector, tensor, spinor ...) fields F on M , which transform under gauge transformations like

$$f \mapsto \tilde{f} = \Omega^k f, \quad F \mapsto \tilde{F} = \Omega^l F.$$

k and l are the (scale or Weyl) *weights* of f respectively F . We write $w(f) := k$, $w(F) := l$ and speak of *Weyl functions* or *Weyl fields* on M . To be more precise mathematically, Weyl functions and Weyl fields are equivalence classes of ordinary (scale invariant) functions and fields. Obvious examples are: $w(g_{\mu\nu}) = 2$, $w(g^{\mu\nu}) = -2$ etc. As the curvature tensor $R = R^\alpha_{\beta\gamma\delta}$ of the Weylian metric and the Ricci curvature tensor *Ric* are scale invariant, scalar curvature $\bar{R} = g^{\alpha\beta} Ric_{\alpha\beta}$ is of weight $w(\bar{R}) = -2$.

Formulas similar to (3) to (5) are derived for conformal transformations in semi-Riemannian gravity [15, Chap. 1.11, 19, Chap. 3]. But there the geometrical and physical meaning is slightly different. While in semi-Riemannian relativity these equations are used to calculate the affine connection and curvature quantities of an “original” metric g after a conformal mapping to a different one, $g_* = \Omega^2 g$, Weyl geometry considers conformal rescaling as a *gauge transformation in the original sense of the word*, expressing the change of measuring devices (or equivalently of dilatations). The aim her is to study scale *covariant* behavior of quantities and structures, with particular attentiveness to scale *invariant* aspects.

Note that the application of Weyl’s covariant derivative ∇ , associated to the Weyl geometric affine connection (3), to Weyl fields F of weight $w(F) \neq 0$ *does not* lead to a scale covariant quantity. This deficiency can be repaired by introducing a *scale*

covariant derivative D_μ of Weyl fields in addition to the scale invariant ∇_μ [10, 12, Appendix A]:

$$DF := \nabla F + w(F)\varphi \otimes F. \tag{6}$$

Thus, for example, a scale covariant vector field F^ν has the scale covariant derivative

$$D_\mu F^\nu := \partial_\mu F^\nu + \Gamma_{\mu\lambda}^\nu F^\lambda + w(F)\varphi_\mu F^\nu,$$

with the abbreviation $\partial_\mu := \frac{\partial}{\partial x^\mu}$ etc.

For the description of relativistic trajectories Dirac introduced *scale covariant* geodesics γ with weight of the tangent field $u := \gamma' \ w(u) = -1$, defined by

$$D_u u = 0 \tag{7}$$

with scale covariant derivation D , i.e.

$$\nabla_\mu \gamma'^\mu \gamma'^\nu - \varphi_\mu \gamma'^\mu \gamma'^\nu = 0 \quad \text{for } \nu = 0, \dots, n - 1.$$

They differ from Weyl’s scale invariant geodesics γ_W only by parametrization.

By construction $g(u, u)$ is scale invariant. In particular $g(u, u) = \pm 1$ for space-like or timelike geodesics; and *geodesic distance* $d_{(g,\varphi)}(p_0, p_1)$ between two points p_0, p_1 with respect to g is given by the parameter of the Diracian geodesics. Of course it depends on the scale gauge (g, φ) chosen and coincides with (semi-Riemannian) distance of *Weyl geometric geodesics* measured in the *Riemannian component* of (g, φ)

$$d_{(g,\varphi)}(p_0, p_1) = \int_{\tau_0}^{\tau_1} \sqrt{|g(\gamma', \gamma')|} d\tau = (\tau_1 - \tau_0). \tag{8}$$

Dirac’s scale covariant geodesics have the same scale weight as energy E and mass m , $w(E) = w(m) = -1$, which are postulated in order to keep scaling consistent with the Planck relation $E = h\nu$ and Einstein’s $E = mc^2$ (with true constants h and c). Thus mass or energy factors assigned to particles or field quanta can be described in a gauge independent manner in Dirac’s calculus: one just has to associate *constant mass factors* \hat{m} (more formally defined below) to the Diracian geodesics; the scale gauge dependence is implemented already in the latter.

For any nowhere vanishing Weyl function f on M with weight k there is a gauge (unique up to a constant), in which \tilde{f} is constant. It is given by (1) with

$$\Omega = f^{-\frac{1}{k}} \tag{9}$$

and will be called *f-gauge* of the Weylian metric. There are infinitely many gauges, some of which may be of mathematical importance in specific contexts. An *R*-gauge (in which scalar curvature is scaled to a constant) exists for manifolds with nowhere

vanishing scalar curvature. It ought to be called *Weyl gauge*, because Weyl assigned a particularly important role to it in his foundational thoughts about matter and geometry [43, 298f].

Similar to ordinary semi-Riemannian and conformal scalar tensor theories of gravity, one often considers a *nowhere vanishing scalar field* ϕ of weight $w(\phi) = -1$ in Weyl geometry. Originally the name-giving authors of Jordan-Brans-Dicke (J-B-D) theory hoped to find a “time varying” real scalar field as an empirically meaningful device, corresponding to the so-called *Jordan frame* of J-B-D theories [4]. But empirical evidence, gathered since the 1960s, and a theoretical reconsideration of the whole field since the 1980s have accumulated overwhelming arguments that the conformal picture of the theory with ϕ scaled to constant, the so-called *Einstein frame*, if any at all, ought to be considered as “physical”, i.e. of empirical content [16].

A similar view holds in the Weyl geometric approach. There the norm $|\phi|$ of the scalar field (now complex or even a Higgs-like isospinor complex two-component field) may be considered as setting the physical scale [11, 12]. That leads to an obvious method to extract the *scale invariant magnitude* \hat{X} of a scale covariant *local quantity* X of weight $w := w(X)$, given at point p . One just has to consider the proportion with the appropriately weighted power of $|\phi|$. In this sense the *observable magnitude* $\hat{X}(p)$ of a Weyl field X with $w = w(X)$ is given by

$$\hat{X}(p) := \frac{X(p)}{|\phi|^{-w}(p)} = X(p)|\phi|^w(p). \tag{10}$$

By definition \hat{X} is *scale invariant*.

For example the scale invariant length $\hat{l}(\xi)$ of a vector ξ at $p, \xi \in T_pM$, is $\hat{l}(\xi) = |\phi| |g(\xi, \xi)|^{\frac{1}{2}}$, independent of the scale gauge considered. Matter energy density in the sense of $\rho = T_{00}^{(mat)}$ (cf. (30)) has to be compared with observed quantities by $\hat{\rho} = \rho|\phi|^2$, etc. Geodesic distance in the sense of (8) is a non-local, scale dependent concept; but its observable $\hat{d}(p_0, p_1)$, i.e. the *scale invariant distance* between two points, is calculated by integrating local “observables” derived from the infinitesimal arc elements. For geodesics $ds = |g(\gamma', \gamma')|^{\frac{1}{2}} = 1$ and thus

$$\hat{d}(p_0, p_1) = \int_{\tau_0}^{\tau_1} |\phi| d\tau. \tag{11}$$

This is identical to geodesic distance in $|\phi|$ -gauge.

Choosing the scale gauge such that the norm of ϕ becomes constant may thus facilitate the calculation of scale invariant observables considerably. In $|\phi|$ -gauge \hat{X} is identical to X up to a (“global”) constant factor depending on measuring units. If lower $*$ denotes values in $|\phi|$ -gauge, we clearly have

$$\hat{X} \doteq const X_* \quad \text{with } const = |\phi_*|^{w(X)}. \tag{12}$$

The dotted \doteq indicates that the equality only holds in a specified gauge (respectively frame, if one considers the analogous situation in conformal scalar tensor theories, cf. Sect. 4). In this sense $|\phi|$ gauge is physically *preferred*. Observables \hat{X} are directly

read off from the quantities given in this gauge, $\hat{X} \sim X_*$. In particular for distances

$$\hat{d}(p_0, p_1) \doteq d_*(p_0, p_1),$$

up to a global constant. If scale invariant local quantities of Weyl geometric gravity (with a scalar field) are empirically meaningful, $|\phi|$ -gauge expresses directly the behavior of atomic clocks or other physically distinguished measuring devices. On the other hand, there may be mathematical or other reasons to calculate \tilde{X} in another scale gauge first.

The physical fruitfulness of Weylian geometry (in the scalar field approach) depends on the answer to the following question: Can measurements by atomic clocks characterized by scale invariant classical observables like above?—Those who stick to the default answer that this is not the case and Riemann gauge expresses observables directly will be led back to Einstein’s semi-Riemannian theory. If this were the only possibility, the generalization to IWG would be redundant. However, this is not at all the case when we consider a Weyl geometric version of scalar tensor gravity with the assumption that $|\phi|$ “sets the scale” (in the sense above).

3 Lagrangian

We start from scale invariant Lagrangians similar to those studied in conformal J-B-D type theories of gravity [15, 19] with a real scalar field ϕ . Tann [41] and Drechsler and Tann [12] have investigated the properties of a complex version of it in their field theoretic studies of a Weyl geometric unification of gravity with electromagnetism. Drechsler [11] even includes semi-classical fields of the standard model (fermionic and bosonic), extending ϕ to a Higgs-like isospinor spin 0 doublet. Here we deal exclusively with gravity and might specialize to a real scalar field, but we do not.

In order to indicate the symbolical interface to the extension of the Weyl geometric approach to electromagnetism and/or the standard model sector of elementary particle physics (EP), studied by Drechsler and Tann, we stick (formally) to a complex version of the scalar field ϕ , although for our purposes we are essentially concerned with $|\phi|$ only. The Lagrangian is

$$\mathcal{L} = \sqrt{|g|} (L^{(HE)} + L^{(\phi)} + \dots (L^{EP} \dots) + L^{(em)} + L^{(m)}), \tag{13}$$

where $|g| = |\det(g_{ij})|$. Standard model field theoretic Lagrangian terms, $L^{(EP)}$, are indicated in brackets (cf. Sect. 5). $\mathcal{L}^{(HE)}$ is the Hilbert-Einstein action in scale invariant form due to coupling of the scalar curvature to a complex scalar field ϕ , $w(\phi) = -1$. $\mathcal{L}^{(\phi)}$ is the scale invariant Lagrangian of the scalar field, $\mathcal{L}^{(m)}$ the Lagrangian of classical matter for an essentially phenomenological characterization of mean density matter. Here we consider the most simple form of a neutral fluid (even dust). A more sophisticated (general relativistic magnetohydrodynamical) $\mathcal{L}^{(m)}$ will be necessary for more refined studies, e.g. of structure formation arising from hot

intergalactic plasma of intergalactic jets etc.¹

$$\begin{aligned}
 L^{(HE)} &= \frac{1}{2}\xi(\phi^*\phi)^{\frac{n-2}{2}}\bar{R}, & w(\phi) &= -1, \\
 L^{(\phi)} &= -\left(\frac{1}{2}D^\mu\phi^*D_\mu\phi - V(\phi)\right), \\
 L^{(m)} &= \mu(1 + \epsilon), & w(\mu) &= -4, \\
 L^{(em)} &= \frac{[c]}{16\pi}F_{\mu\nu}F^{\mu\nu}
 \end{aligned}
 \tag{14}$$

$\xi = \frac{n-2}{4(n-1)}$ ($n =$ dimension of spacetime) is the known coupling constant establishing conformal invariance of the action $\mathcal{L}^{(HE)} + \mathcal{L}^{(\phi)}$, if covariant differentiation and scalar curvature refer to the *Riemannian component of the metric only* (${}_g\nabla_\mu$ and ${}_g\bar{R}$ in the notation above) [30, 41]. Here $n = 4$, $\xi = \frac{1}{6}$.

The potential term in $\mathcal{L}^{(\phi)}$ is formal placeholder for a quadratic mass like and a biquadratic self interaction term

$$V(\phi) = \lambda_2(\phi^*\phi) + \lambda_4(\phi^*\phi)^2, \quad w(\lambda_2) = -2, \quad w(\lambda_4) = 0, \tag{15}$$

with scale invariant coupling constant λ_4 like in [11] and scale covariant quadratic coefficient λ_2 . Formally, V looks like the Lagrange term of a scale covariant cosmological “constant”. We shall see, however (33), that the energy stress tensor of ϕ contains other, more important contributions.

Our *matter Lagrangian* consists of a fluid term with energy density μ and internal energy ratio ϵ similar to the one in [22, 69f], with functions μ, ϵ on spacetime of weight $w(\mu) = -4$, $w(\epsilon) = 0$. $L^{(m)}$ is related to *timelike unit vectorfields* $X = (X^\mu)$ of weight $w(X) = -1$ representing the flow and constrained by the condition that during variation of the flow lines its energy density flow

$$j := \mu(1 + \epsilon)X$$

satisfies the local energy conservation of the matter current

$$\operatorname{div} j = D_\mu j^\mu = 0. \tag{16}$$

For abbreviation we set

$$\rho := \mu(1 + \epsilon). \tag{17}$$

As an alternative, one might try to model classical matter by a second scalar field Φ with the same scale weight as ϕ [19, Chap. 3.3]. The coupling to the scalar field ϕ [19, (3.60)ff] could be transformed into a Weyl geometric kinetic term $g^{\mu\nu}D_\mu\Phi D_\nu\Phi$ with scale covariant derivative $D_\mu\Phi = (\partial_\mu - \varphi_\mu)\Phi$. But for cosmological applications the (observational) restriction of negligible pressure, would lead to an artificial

¹For a first heuristic discussion of structure formation compatible with Einstein-Weyl models, cf. [18, Chap. 6].

coupling between matter and the scalar field ϕ . We therefore choose here the approach adapted from [22].

In terms of extension of gauge groups, the Weyl geometric approach to gravity works in the frame of the scale extended Lorentz or Poincaré group, sometimes called the (metrical) Weyl group

$$W \cong \mathbb{R}^4 \ltimes SO(1, 3) \times \mathbb{R}^+.$$

For inclusion of standard model (EP) fields it has to be extended by internal symmetries $SU_3 \times SU_2 \times U(1)_Y$.

To bring the constants in agreement with Einstein’s theory, the constant in $|\phi|$ -gauge has to be chosen such that the coefficient of the Hilbert Einstein action becomes

$$\frac{1}{2}\xi|\phi_*|^2 \doteq \frac{[c^4]}{16\pi G}. \tag{18}$$

For $n = 4$ that means

$$|\phi_*|^2 \doteq 6\frac{c^4}{8\pi G}. \tag{19}$$

In other words

$$|\phi_*|\sqrt{\hbar c} \doteq \frac{1}{2}\sqrt{\frac{3}{\pi}}E_{Pl} \approx \frac{1}{2}E_{Pl}, \quad |\phi_*|^{-1}\sqrt{\hbar c} \doteq 2\sqrt{\frac{\pi}{3}}l_{Pl} \approx 2l_{Pl}, \tag{20}$$

with E_{Pl}, l_{Pl} Planck energy, respectively Planck length.

Some authors conjecture [23, 24, 27, 40] that a condensation, close to the Planck scale, of an underlying non-trivial scale bosonic field φ with $d\varphi \neq 0$ may give a deeper physical reason for the assumption that ϕ “sets the scale” in the sense of (10) and (18). If this were true, the scalar field ϕ , and with it the integrable scale connection φ taken into consideration here, would probably characterize a macroscopic state function of some kind of scale boson condensate. This is an interesting thought, but at present a reality claim for this conjecture would be premature.

4 $|\phi|$ -Gauge and Scalar Tensor Theories of Gravity

In a formal sense, our Lagrangian may be considered as belonging to the wider family of scalar-tensor theories of gravity. The scale invariant Hilbert-Einstein action is analogous to the one of Jordan-Brans-Dicke theory. One should keep in mind, however, that the conceptual frame and the (model) dynamics are different. In J-B-D theories rescaling of the metric $g_* = \Omega^2 g$ expresses a conformal mapping in which the affine connection and curvature quantities derived from g are “pulled back” to the new frame and expressed in terms of g_* . Two conformal pictures, usually called “frames” (g or g_*), represent possible different physical models [16]. Thus the question arises which of the pictures (frames) may be “physical”, if any. Faraoni et al. [16] give strong arguments in favor of the conformal picture in which the factor $|\phi|^2$ in the Hilbert-Einstein action is normalized to a constant, the so-called *Einstein frame*.

Some authors have started to look for a bridge between scalar tensor theories and Weyl geometry [38, 39].

In the Weyl geometric approach all scale gauges are, in principle, equivalent. Weyl geometry is a scale invariant structure; local physical quantities X (locally defined “lengths”, energy densities, pressure, ...) are scale covariant (transform according to their gauge weight), but have scale invariant observable quantities \hat{X} , cf. (10). In this sense, Weyl geometry is a *gauge theory* like any other (needless to remind that it has given the name to the whole family). On the other hand, scale invariant quantities \hat{X} can be read off directly in $|\phi|$ -gauge up to a constant factor. In this respect and different to other gauge theories, $|\phi|$ -gauge provides us with a *preferred scale*. This corresponds well to the established knowledge that atomic clocks etc. define a physical scale, a fact which cannot be neglected in any reasonable theory of gravity, cf. [32, 33].

In integrable Weyl geometry, which we consider here exclusively (cf. Sect. 2), the scale connection φ is “pure gauge”, i.e. has curvature zero, and can be integrated away. So there are two distinguished gauges, one in which the scale connection is gauged away and one in which the norm of the scalar field is trivialized, i.e. made constant. The complex, or isospin, phase of ϕ plays its part only if electromagnetic fields or weak interaction is considered (is “switched on”), cf. [11, 12]; here we abstract from its dynamical role:

$$\begin{aligned}
 (\tilde{g}, \tilde{\varphi}), \quad \tilde{\phi}, \quad \text{with } \tilde{\varphi} = 0 \quad & \text{(Riemann gauge),} \\
 (g_*, \varphi_*), \quad \phi_*, \quad \text{with } |\phi_*| = \text{const} \quad & \text{(|}\phi\text{|-gauge).}
 \end{aligned}$$

Formally the Einstein frame of J-B-D theories corresponds to $|\phi|$ -gauge, Jordan frame (more precisely one of its choices) to Riemann gauge.

Scale connection φ_* (of $|\phi|$ -gauge) and scalar field $|\tilde{\phi}|$ (in Riemann gauge) determine each other. Structurally speaking they are different aspects of the same entity (in *integrable* Weyl geometry). The scalar field in Riemann gauge, more precisely its norm, can be written as

$$|\tilde{\phi}| = |\tilde{\phi}(p_0)|e^{-\sigma} \quad \text{with } \sigma(p) = \int_{p_0}^p \varphi_*(\gamma'), \tag{21}$$

γ connecting path between p_0, p , and φ_* the scale connection in $|\phi|$ -gauge. It is just the inverse of Weyl’s “length” (scale) transfer function (2) in $|\phi|$ -gauge up to a constant, $|\tilde{\phi}| \sim \lambda(p_0, p)^{-1}$. The other way round, the scale connection φ_* in $|\phi|$ -gauge can be derived from the scalar field $\tilde{\phi}$ in Riemann gauge,

$$\varphi_* = d\sigma = -d \log |\tilde{\phi}| = -\frac{d|\tilde{\phi}|}{|\tilde{\phi}|}, \quad \text{i.e. } \varphi_\mu = \partial_\mu \sigma. \tag{22}$$

The dynamics of φ (in $|\phi|$ -gauge) is governed by the Lagrangian of the ϕ -field in (14), $\mathcal{L}^{(\phi)} = \sqrt{|g|}(-\frac{1}{2}D^\mu \phi^* D_\mu \phi + V(\phi))$. In IWG φ cannot have a scale curvature term “of its own” ($d\varphi$ vanishes). This does *not* mean that the Weyl geometric extension of classical gravity is dynamically trivial. In Riemann gauge its non-triviality is obvious. At first glance it may appear trivial in $|\phi|$ -gauge, because $|\phi| = \text{const}$.

A second glance shows, however, that it is not, due to the scale connection terms of the covariant derivative. The dynamics of the scalar field in Riemann gauge is now expressed by a Lagrangian term in the scale connection $\varphi = d\sigma$, i.e., in the derivatives of σ .

It may be useful to compare the $|\phi|$ -gauge Lagrangian

$$\mathcal{L}^{(EH,\phi)} \doteq \sqrt{|g_*|} \left(\frac{1}{2} \xi \overline{R_*} - \frac{1}{2} D^\mu \phi_*^* D_\mu \phi_* + V(\phi_*) \right) \tag{23}$$

with the corresponding expression of semi-Riemannian scalar tensor theory. If non-gravitational (em or ew) interactions are abstracted from, $|\phi|$ can be considered as an essentially real field

$$\phi = |\phi|.$$

The corresponding expression in Einstein frame [19, Chap. 3.2]

$$\sqrt{|g_*|} \left(\frac{1}{2} \xi_{g_*} \overline{R_*} - \frac{1}{2} (1 + 6\xi) g_*^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V(\phi_*) \right) \tag{24}$$

is variationally equivalent to (23). It just has shifted the $6\varphi_*^\mu \varphi_{*\mu}$ term of (4), plugged into (23), to the kinetic term in $\partial_\mu \sigma = \varphi_{*\mu}$. The last term, $6_g \nabla_\lambda \varphi^\lambda$, in (4) is a gradient and has no consequence for the variational equations.²

Nobody would consider semi-Riemannian scalar tensor theories in Einstein frame dynamically trivial. This comparison may thus help to understand that even a scale connection with $d\varphi = 0$ can play a dynamical role in Weyl geometric gravity. Below we shall study a simple example, where φ even assumes *constant values* in large cosmological “average”, respectively idealization. We have to keep in mind that even then the Weylian scale connection φ indicates a dynamical element of spacetime. “Statics” is nothing but a dynamical constellation in equilibrium.

5 Extension to the Field Theoretic Sector

Field theoretic contributions to the Lagrangian (electroweak, Yukawa, fermionic), adapted from conformal field theory, are studied in [11, 24] and other works:

$$\begin{aligned} L^{(ew)} &= \alpha_1 (W_{\mu\nu} W^{\mu\nu} + B_{\mu\nu} B^{\mu\nu}) \\ L^{(Y)} &= \alpha_2 ((\bar{\psi}_L \tilde{\phi}) \Psi_R + (\bar{\psi}_R \tilde{\phi}) \Psi_L) \\ L^{(\Psi)} &= \frac{i}{2} (\bar{\Psi}_L \gamma^\mu \tilde{D}_\mu \Psi_L - \bar{\Psi}_L \tilde{D}_\mu \gamma^\mu \Psi_L) \\ &\quad + \frac{i}{2} (\bar{\Psi}_R \gamma^\mu \tilde{D}_\mu \Psi_R - \bar{\Psi}_R \tilde{D}_\mu \gamma^\mu \Psi_R) \end{aligned}$$

²Fujii and Maeda’s σ contains a factor $\sqrt{1+6\xi}$, compared with our’s and a different sign convention for V [19, (3.28), (3.30)].

Ψ denotes left and right handed spinor fields of spin $S(\Psi) = \frac{1}{2}$; (γ^μ) is a field of Dirac matrices depending on scale gauge. Weyl weights are $w(\Psi) = -\frac{3}{2}$, $w(\gamma^\mu) = -1$. $\tilde{\phi}$ is the scalar field (spin 0) $w(\tilde{\phi}) = -1$, extended to an isospin $\frac{1}{2}$ bundle, i.e., locally with values in \mathbb{C}^2 . \tilde{D}_μ denotes the covariant derivative lifted to the spinor bundle, respectively the isospinor bundle, taking the electroweak connection with $W_{\mu\nu}$ (values in $SU(2)$) and $B_{\mu\nu}$ (values in $u(1)_Y$) into account ($w(W_{\mu\nu}) = w(B_{\mu\nu}) = 0$) [11]. After substitution of D_μ by \tilde{D}_μ in $L^{(\phi)}$, the total Lagrangian becomes

$$L = \frac{[c^4]}{16\pi N} (L^{(HE)} + L^{(\phi)} + L^{(ew)} + L^{(Y)} + L^{(\Psi)} + \dots) + \dots + L^{(m)} + [L^{(em)}]. \tag{25}$$

The electromagnetic action $L^{(em)}$ arises after symmetry reduction, induced by fixing the gauge of electroweak symmetry imposing the condition $\tilde{\phi}_o = (0, |\phi|)$. In $|\phi|$ -gauge it is normed to a constant. In this context Drechsler sets it to the ew energy scale, $\sqrt{2}|\phi|[\hbar c]^{-\frac{1}{2}} \doteq v \approx 246$ GeV, i.e., scaled down to laboratory units by a “global” factor 10^{-17} with respect to (18).

The infinitesimal operations of the ew group then lead to a “non-linear realization” in the stabilizer $U(1)_{em}$ of $\tilde{\phi}_o$ and contribute to the covariant derivatives and the energy momentum tensor of the ϕ field. In this way the energy-momentum tensor of ϕ indicates the *acquisition of mass* of the electroweak bosons, even without assuming a “Mexican hat” type potential and without any need of a speculative symmetry break in the early universe.

Drechsler’s study shows that *mass may be acquired by coupling the ew-bosons to gravity* through the intermediation of the ϕ -field. This is a conceptually convincing alternative to the usual Higgs mechanism. In similar approaches, Hung Cheng [24] and Pawłowski and Raczka [29] have arrived at similar expressions by deriving mass terms perturbatively on the tree level from the same scale invariant Lagrangian without the Mexican hat potential. This is a remarkable agreement. With the Large Hadron Collider (LHC) coming close to starting its operation, such considerations deserve more attention by theoretical high energy physicists.

In their investigation Drechsler and Tann consider a mass term of the scalar field as a *scale symmetry* breaking device, by substituting $-M_0^2|\phi|^2$ in $V(\phi)$ for $-\lambda_2|\phi|^2$, where $w(M_0) = 0$, $M_0 \neq 0$ [11, 12, 41]. This choice, although possible, is not compulsory for the analysis of a Higgs-like mechanism which couples ew-bosons to gravity [20], neither does it seem advisable. The similarity of this approach to the one of [29] which relies on unbroken conformal scale covariance and a conformally weakened gravitational action, indicates that this type of coupling does not depend on the scale breaking condition $w(M_0) = 0$ for a scalar field mass. Therefore it seems preferable to assume $M_0 = 0$ (or if $M_0 \neq 0$, gauge weight $w(M_0) = -1$ and $M_0 \equiv \lambda_2$), in order to keep closer to the Weyl geometric setting.

6 Variational Equations

Variation with respect to ϕ^* leads to a Klein-Gordon equation for the scalar field [12, (2.13)], which couples to scalar curvature,

$$D^\mu D_\mu \phi + \left(\xi \bar{R} + \frac{2}{\phi} \frac{\partial V}{\partial \phi^*} \right) \phi = 0. \tag{26}$$

The factor

$$\xi \bar{R} + \frac{2}{\phi} \frac{\partial V}{\partial \phi^*} =: \tilde{m}_\phi^2 = \frac{m_\phi^2 c^4}{\hbar^2} \tag{27}$$

functions as a mass-like factor of the ϕ -field. The contributions of the quadratic and biquadratic terms of V to (27) are intrinsic to the ϕ -field. If they vanish, m_ϕ is derived exclusively from the mass-energy content of spacetime via scalar curvature \bar{R} and the Einstein equation.

Variation of flow lines with adjustment of ρ such that the mass energy current is conserved, i.e. respects the constraint (16), leads to an Euler equation for the acceleration of the flow $\dot{X}^\mu := D_\lambda X^\mu X^\lambda$,

$$(\rho + p_m) D_\lambda X^\mu X^\lambda = -\partial_\lambda p_m (g^{\lambda\mu} + X^\lambda X^\mu), \tag{28}$$

where $p_m = \mu^2 \frac{d\epsilon}{d\mu}$ is the pressure of the fluid and $\rho = \mu(1 + \epsilon)$ as above (17), cf. [22, p. 96] for the semi-Riemannian case.

Variation with respect to $\delta g^{\mu\nu}$ gives the scale covariant Einstein equation

$$Ric - \frac{\bar{R}}{2} g = (\xi |\phi|^2)^{-1} (T^{(m)} + T^{(\phi)} + [\dots + T^{(Z)} \dots]) \tag{29}$$

with a classical matter tensor compatible with (28)

$$T_{\mu\nu}^{(m)} = -2 \frac{1}{\sqrt{|g|}} \frac{\delta \mathcal{L}^m}{\delta g^{\mu\nu}} = (\rho + p_m) X_\mu X_\nu + p_m g_{\mu\nu}, \tag{30}$$

and field tensors

$$T^{(Z)} := -2 \frac{1}{\sqrt{|g|}} \frac{\delta \mathcal{L}^{(Z)}}{\delta g^{\mu\nu}}, \quad Z \text{ for } \phi, ew, \Psi, Y,$$

of scale weight $w(T) = -2$. They are calculated in [11]; here we only need $T^{(\phi)}$.

\tilde{G} defined by generalization of (18)

$$\frac{8\pi \tilde{G}}{[c^4]} := \xi^{-1} |\phi|^{-2}$$

may be considered as a *scale covariant version of the gravitational “constant”*. Its weight corresponds to what one expects from considering the physical dimension of G ,

$$[[G]] = [[L^3 M^{-1} T^{-2}]] = [[LM^{-1}]] = 2 = w(\tilde{G}).$$

L, M, T denote length, mass, time quantities respectively, $[[\dots]]$ the corresponding metrological (“phenomenological”) scale weights. $w(\tilde{G})$ correctly cancels the weight of the doubly covariant energy-momentum tensor with $w(T_{\mu\nu}) = -2$. If one wants, one may even find *some* of the intentions of the original J-B-D theory (e.g., “time dependence” of the gravitational constant) reflected in the behavior of the Weyl geometric \tilde{G} in Riemannian gauge. In $|\phi|$ -gauge the gravitational coupling is constant; that corresponds to the Einstein frame picture of conformal J-B-D theory and underpins the importance of this gauge as a *good candidate* for proportionality to measurements according to atomic clocks, without further reductions like (10).

The r.h.s. of (29) will be abbreviated by

$$\Theta := (\xi|\phi|^2)^{-1}(T^{(m)} + T^{(\phi)} + [\dots + T^{(Z)} \dots]) \tag{31}$$

and its constituents by $\Theta^{(m)} := (\xi|\phi|^2)^{-1}T^{(m)}$ etc.

The energy-stress tensor of the Weyl geometric ϕ -field has been calculated in [41, (372)] and [12, (3.17)]. It is consistent with the (non-variationally motivated) proposal of Callan et al. [5] to consider an “improved” energy tensor:

$$T^{(\phi)} = D_{(\mu}\phi^* D_{\nu)}\phi - \xi D_{(\mu}D_{\nu)}(\phi^*\phi) - g_{\mu\nu} \left(\frac{1}{2} D_\lambda \phi^* D^\lambda \phi - \xi D^\lambda D_\lambda (\phi^*\phi) - V((\phi^*, \phi)) \right). \tag{32}$$

Crucial for Drechsler/Tann’s calculation is the observation that the coupling of \bar{R} with $|\phi|^2$ leads to additional, in general non-vanishing, terms for the variational derivation $\delta g^{\mu\nu}$ of the Hilbert-Einstein Lagrangian. These terms (those with factor ξ in the formula above) agree with the additional terms of the “improved” energy tensor of Callan-Coleman-Jackiw [41, 98–100].

This modification has to be taken into account also in scalar-tensor theories more broadly. Although it is being used in some of the present literature, e.g. [37], it has apparently found no broad attention. In [15, (7.29)] the “improved” form of Callan energy-momentum tensor is discussed as one of several different alternatives for an “effective” energy-momentum tensor. Faraoni sees here the source of the problem of non-uniqueness of the “physically correct” energy momentum tensor of a scalar field.

Tann’s and Drechsler’s derivation shows, however, that there is a clear and unique variational answer (32) to the question. It also indicates that the *truncated form of the energy tensor* (without the ξ -terms) *is in general incorrect* for theories with a quadratic coupling of the scalar field to the Hilbert-Einstein action, independent of the wider geometrical frame (conformal semi-Riemannian or Weyl geometric). We shall see that already for simple examples this may have important dynamical consequences (Sect. 9).

We even may conjecture that (32) opens a path towards a solution of the long-standing problem of *localization of gravitational energy*, mentioned in this context by other authors, cf. [15, 157]. As ϕ is an integral part of the gravitational structure, one may guess that (32) itself may represent the energy stress tensor of the gravitational field. At least, it is a well-defined energy tensor and is closely related to the gravitational structure. Moreover it is uniquely defined by the variational principle.

$\Theta^{(\phi)} = \xi^{-1}|\phi|^{-2}T^{(\phi)}$ decomposes (additively) into a *vacuum-like* term proportional to the Riemannian component of the metric

$$\Theta_{\mu\nu}^{(\Lambda)} = -\Lambda g_{\mu\nu}, \quad \text{with} \tag{33}$$

$$\Lambda := (\xi|\phi|^2)^{-1} \left(\frac{1}{2}D_\lambda\phi^*D^\lambda\phi - \xi D^\lambda D_\lambda(\phi^*\phi) - V((\phi^*, \phi)) \right),$$

and a *matter-like* residual term

$$\Theta_{\mu\nu}^{(\phi^{res})} = (\xi|\phi|^2)^{-1} (D_{(\mu}\phi^*D_{\nu)}\phi - \xi D_{(\mu}D_{\nu)}(\phi^*\phi)). \tag{34}$$

Clearly Λ is no constant but a scale covariant quantity (of weight -2). By the scalar field equation (26) it depends on scalar curvature of spacetime and matter density. Its weight is $w(\Lambda) = -4$.

If $T^{(\phi^{res})}$ reduces to its $(0, 0)$ component, it acquires the form of a *dark matter* term. There are indications that the scalar field contributions close to galactic mass concentrations may be helpful for understanding dark matter [26, 27].

Another energy-momentum tensor of a long range field (after *ew* symmetry reduction) is the e.m. energy stress tensor. As usual it is

$$T_{\mu\nu}^{(em)} = \frac{[c]}{4\pi} \left(F_{\mu\lambda}F_\nu^\lambda - \frac{1}{4}g_{\mu\nu}F_{\kappa\lambda}F^{\kappa\lambda} \right). \tag{35}$$

In our context $T^{(em)}$ is negligible. So is the internal energy of the fluid. For the purpose of a first idealized approximation of cosmic geometry in the following sections we work with $\epsilon = 0$, i.e. with dust matter,

$$p_m = 0, \quad \rho = \mu. \tag{36}$$

For the sake of abbreviation we also use the matter density parameter

$$\tilde{\rho} = \xi^{-1}|\phi|^{-2}\rho. \tag{37}$$

Remember that we have not included a dynamical term proportional to $f^{\mu\nu}f_{\mu\nu}$ into the Lagrangian, $f := d\varphi$ curvature of the Weylian scale connection. So we exclude, for the time being, considerations which might become crucial close to the Planck scale, presumably supplemented by scale invariant higher order terms in the curvature [40], mentioned at the end of Sect. 3.

7 Cosmological Modeling

Any semi-Riemannian manifold can be considered in the extended framework of integrable Weyl geometry. For cosmological studies, Robertson-Walker manifolds are particularly important. They are spacetimes of type

$$M \approx \mathbb{R} \times M_\kappa^{(3)}$$

with $M_\kappa^{(3)}$ a Riemannian 3-space of constant sectional curvature κ , usually (but not necessarily) simply connected. If in spherical coordinates (r, Θ, Φ)

$$d\sigma_\kappa^2 = \frac{dr^2}{1 - \kappa r^2} + r^2(d\Theta^2 + \sin^2 \Theta d\Phi^2) \tag{38}$$

denotes the metric on the spacelike fibre $M_\kappa^{(3)}$, the Weylian metric $[(g, \varphi)]$ on M is specified by its Riemann gauge $(\tilde{g}, 0)$ like in standard cosmology:

$$\tilde{g}: ds^2 = -d\tau^2 + a(\tau)^2 d\sigma_\kappa^2 \tag{39}$$

$\tau = x_0$ is a local or global coordinate (cosmological time parameter of the semi-Riemannian gauge) in \mathbb{R} , the first factor of M . We shall speak of *Robertson-Walker-Weyl (R-W-W) manifolds*.

In the semi-Riemannian perspective $a(\tau)$, the *warp function* of $(M, [(g, \varphi)])$, is usually interpreted as an expansion of space sections. The Weyl geometric perspective shows that this need not be so. For example, there is a gauge (g_w, φ_w) in which the “expansion is scaled away”:

$$g_w = \Omega_w^2 g \quad \text{with } \Omega_w := \frac{1}{a}. \tag{40}$$

With

$$t := \int^{\tau} \frac{du}{a(u)} = h^{-1}(\tau) \quad \text{and its inverse function } h(t) = \tau \tag{41}$$

we get a gauge

$$\begin{aligned} g_w(x) &:= -dt^2 + d\sigma_\kappa^2, \\ \varphi_w(x) &= d \log(a \circ h) = (a' \circ h) dt = a'(h(t)) dt, \end{aligned} \tag{42}$$

in which the Riemannian component of the metric looks static. According to (9), it may be called *warp gauge* of the Robertson-Walker manifold, because the warp function is scaled to a constant. The other way round, the warp function is nothing but the integrated scale transfer of warp gauge (2)

$$a(p) = a(p_0) e^{\int_0^1 \varphi_w(\gamma')}. \tag{43}$$

The geodesic path structure is invariant under scale transformations of IWG. In agreement with Diracian geodesics of weight -1 the observer field $X^i = \partial/\partial x_i$ has also to be given the weight $w(X) = -1$. Then the energy $e(p)$ of a photon along a null-geodesic γ , observed at p by an observer of the family X is given by

$$e(p) = g_p(\gamma'(p), X(p)).$$

It is of weight $w(e) = 2 - 1 - 1 = 0$ and thus scale invariant. Therefore *redshift* (cosmological or gravitational) of a photon emitted at p_0 and observed at p_1 with

respect to observers of the family X ,

$$z + 1 = \frac{e(p_0)}{e(p_1)} = \frac{g_{p_0}(\gamma'(p_0), X(p_0))}{g_{p_1}(\gamma'(p_1), X(p_1))}, \tag{44}$$

is also *scale invariant* (γ null-geodesic connecting p_0, p_1).

We see that cosmological redshift is not necessarily characterized by a warp function $a(x_0)$; in warp gauge it is expressed by the scale connection φ_w and can be read off directly from Weyl’s “length” transfer because of $z + 1 = \frac{a(p)}{a(p_0)}$ and (43):

$$z + 1 = e^{\int_0^1 \varphi(\gamma') d\tau} = \lambda(p_0, p_1), \quad \gamma \text{ connecting path.} \tag{45}$$

We therefore call φ_w the *Hubble connection* of the R-W-W model. It is timelike, $\varphi_w = H(t)dt$, with $H(t) = a'(h(t))$.

If Hubble redshift is not due to space expansion but to a field theoretic energy loss of photon energy with respect to the observer family, the warp gauge picture will be more appropriate to express physical geometry than Riemann gauge. In this case, the Hubble connection should not be understood as an independent property of cosmic spacetime, but rather depends on the mean mass-energy density in the universe. Different authors starting from [45] to the present have tried to find a higher order gravitational effect which establishes such a relation. A convincing answer has not yet been found. If however the Hubble connection is “physical”, *Mach’s principle* suggests that it should be due to the mean distribution of cosmic masses. As simplest possibility, we may conjecture that a linear relation between H^2 and mass density might hold in large means *in warp gauge*,

$$H^2 = \eta_1 \tilde{\rho} + \eta_0, \quad \eta_1 > 0 \quad (H^2 \text{ conjecture}). \tag{46}$$

In the models studied below such a coupling of H^2 to mass density is a consequence of the scalar field equation and the Einstein equation (59).

As geodesic distance (8) is no local observable and not scale invariant, the question arises which of the gauges, Riemann or warp gauge (or any other one), expresses the measurement by atomic clocks. In the context of Weyl geometric scalar field theory the question can be reformulated: Does $|\phi|$ -gauge coincide with one of these gauges and if so, with which? Ontologically speaking, the two gauges, Riemann or warp, correspond to *two different hypotheses* on the cause of cosmological redshift: *space expansion* (Riemann gauge) or *a field theoretic energy loss of photons* (warp gauge). Weyl geometry allows to translate between the two hypotheses and provides a theoretical framework for a systematic comparison.

8 Weyl Universes

In order to get a feeling for the new perspectives opened up by Weyl geometric scalar fields, we investigate the simplest examples of R-W-W cosmologies with redshift. In warp gauge their Weylian metric is given by a constant Riemannian component

$ds^2 = -dt^2 + d\sigma_\kappa^2$ (where $\kappa \in \mathbb{R}$ denotes the sectional curvature of the spatial fibres $M_\kappa^{(3)}$) and a constant scale connection with only a time component:

$$ds^2 = -dt^2 + \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\phi^2 \right), \tag{47}$$

$$\varphi = (H, 0, 0, 0), \quad H > 0 \quad \text{constant.}$$

H is called Hubble constant (literally). We encounter here a Weyl geometric generalization of the *classical static models* of cosmology, but now *including redshift* (45):

$$z + 1 = e^{H(t_2 - t_1)}.$$

These models will be called *Weyl universes*. Different to the classical static cosmologies this type of static geometry can be upheld, under certain conditions, in a natural way by the dynamical effects of the scalar field ϕ , respectively the Weylian scale connection φ (Sect. 9).

The integration of the scale connection leads to an exponential length transfer function $\lambda(t) = e^{Ht}$. Transition to the R-W metric presupposes a change of the cosmological time parameter $\tau = H^{-1}e^{Ht}$; then

$$a(\tau) = H\tau. \tag{48}$$

Thus this class deals with a Weyl geometric version of *linearly warped* (“expanding”) R-W cosmologies.

Up to (Weyl geometric) isomorphism, Weyl universes are characterized by *one* metrical parameter (*module*) only,

$$\zeta := \frac{\kappa}{H^2}. \tag{49}$$

In warp gauge the components of the affine connection with respect to spherical coordinates (47) are

$$\begin{aligned} \Gamma_{00}^0 &\doteq H, & \Gamma_{0\alpha}^\alpha &\doteq H \quad (\alpha = 1, 2, 3), \\ \Gamma_{11}^0 &\doteq H(1 - \kappa r^2)^{-1}, & \Gamma_{22}^0 &\doteq Hr^2, & \Gamma_{33}^0 &\doteq Hr^2 \sin^2 \vartheta, \\ \Gamma_{\alpha\beta}^\gamma &\doteq {}_g\Gamma_{\alpha\beta}^\gamma \quad (\alpha, \beta, \gamma = 1, 2, 3). \end{aligned} \tag{50}$$

Ricci and scalar curvature are

$$\begin{aligned} Ric &\doteq 2(\kappa + H^2)d\sigma_\kappa^2, \\ \bar{R} &\doteq 6(\kappa + H^2). \end{aligned} \tag{51}$$

Similar to those of the classical static models, Weyl universes have constant entries of the energy momentum tensor but contain quadratic Hubble terms H^2 in addition to spacelike sectional curvature terms:

$$\Theta_{00} \doteq 3(\kappa + H^2), \quad \Theta_{\alpha\alpha} \doteq -(\kappa + H^2)(d\sigma_\kappa^2)_{\alpha\alpha} \quad (\alpha = 1, 2, 3). \tag{52}$$

That corresponds to a total energy density ρ and pressure p

$$\tilde{\rho} \doteq 8\pi G\rho \doteq 3(\kappa + H^2), \quad \tilde{p} \doteq 8\pi Gp \doteq -(\kappa + H^2).$$

For $\kappa > 0$ we obtain *Einstein-Weyl models* similar to the classical Einstein universe.

The next question will be, whether the equilibrium condition between energy density and negative pressure necessary for upholding such a geometry may be secured by the scalar field.

9 Energy Momentum of the Scalar Field and Dynamical Consistency

Here and in the following sections we work in warp gauge, i.e., spacelike fibres $M_\kappa^{(3)}$ are gauged to constant (time-independent) sectional curvature, if not stated otherwise. Therefore the following equations are in general no longer scale invariant.

In warp gauge the Beltrami-d'Alembert operator of Weyl universes is given by

$$\square\phi = D^\lambda D_\lambda\phi \doteq -(\partial_0^2 - 3H\partial_0 + 2H^2)\phi + \tilde{\nabla}^\alpha \tilde{\nabla}_\alpha\phi, \tag{53}$$

where $\lambda = 0, 1, 2, 3$, $\alpha = 1, 2, 3$, and $\tilde{\nabla}^\alpha$ denotes the covariant derivatives of the Riemannian component of the metric *along spatial fibres*.

Separation of variables

$$\phi(t, \tilde{x}) = e^{i\omega t} f(\tilde{x}), \quad \tilde{x} = (x_1, x_2, x_3), \omega \in \mathbb{R},$$

with an eigensolution f of the Beltrami-Laplace operator and $w(f) = -2$,

$$\tilde{\nabla}^\alpha \tilde{\nabla}_\alpha f = \lambda f, \quad \lambda \in \mathbb{R},$$

leads to

$$\square\phi \doteq (\omega^2 + 3H^2\omega i + \lambda - 2H^2)\phi.$$

Reality of the mass like factor of the K-G equation implies $\omega = 0$. Moreover, for Einstein-Weyl universes, $\kappa > 0$, f is a spherical harmonic on the 3-sphere S^3 with eigenvalue λ . The only spherical harmonic with constant norm is $f \equiv const$, $\lambda = 0$, and thus $\phi = Re(\phi) = const$ is a ground state solution (after separation of variables) of the scalar field equation in warp gauge. We conclude that *warp gauge of Weyl universes coincides with $|\phi|$ -gauge* and

$$\square\phi \doteq -2H^2\phi.$$

As we work in this section in $|\phi|$ gauge anyhow, the denotation ϕ_* used where different gauges are compared is here simplified to ϕ .

Using (51) we see that the K-G equation (26) is satisfied, iff

$$H^2 \doteq \kappa + \frac{2}{\phi} \frac{\partial V}{\partial \phi^*}. \tag{54}$$

The contributions to the energy momentum tensor (32) in $|\phi|$ -gauge (identical to warp gauge) are given by:

$$D_0\phi \doteq -H\phi, \quad D_\alpha\phi \doteq 0 \quad (\alpha = 1, 2, 3)$$

and therefore

$$D_\lambda\phi^*D^\lambda\phi \doteq -H^2|\phi|^2. \tag{55}$$

For constant functions f of weight $w(f) = -2$, like $|\phi|^2$, the scale covariant gradient *does not vanish* because of the weight correction of (6),

$$D_0f \doteq (\partial_0 + w(f)H)f \doteq -2Hf, \quad D^0f \doteq 2Hf,$$

while all other components vanish, $D_\alpha f \doteq 0$. For the next covariant derivative the only non-vanishing component of the affine connection is $\Gamma_{00}^0 \doteq H$. That leads to

$$D_0D_0f \doteq (\partial_0 - \Gamma_{00}^0 + w(D_0f)H)D_0f \doteq 6H^2f.$$

Thus $\frac{1}{2}D_0\phi^*D^0\phi - \frac{1}{6}D^0D_0|\phi|^2 \doteq \frac{H^2}{2}|\phi|^2$. From (33) we find

$$\Lambda \doteq 3H^2 - 6\frac{V}{|\phi|^2}. \tag{56}$$

The truncated version of (32) would lead to a different value and thus to a completely different dynamics of the whole system.

$T^{(\phi_{res})}$ vanishes and

$$\Theta^{(\phi)} = -\Lambda g. \tag{57}$$

Formally $\Theta^{(\phi)}$ looks like the “vacuum tensor” of the received approach. Note, however, that here the coefficient Λ is no universal constant but couples to the mass content of the universe via H^2 , the relation (54) and the energy component of the Einstein equation (29). The (0, 0)-component of the latter is

$$\begin{aligned} 3(\kappa + H^2) &\doteq \tilde{\rho} + \Lambda \doteq \tilde{\rho} + 3H^2 - 6\frac{V}{|\phi|^2}, \\ 3\kappa &\doteq \tilde{\rho} - 6\frac{V}{|\phi|^2}, \end{aligned} \tag{58}$$

where the convention (37) for $\tilde{\rho}$ has been applied. Equation (54) and the observation that in the V considered here $\frac{1}{\phi}\frac{\partial V}{\partial\phi^*} - \frac{V}{|\phi|^2} = \lambda_4|\phi|^2$ imply

$$H^2 \doteq \frac{\tilde{\rho}}{3} + 2\lambda_4|\phi|^2, \quad \Lambda \doteq \tilde{\rho} - 6\lambda_2. \tag{59}$$

Thus the H^2 -conjecture (46) turns out to be true for the case of Weyl universes. In agreement with Mach’s “principle” H^2 and Λ depend on the mass content of the universe. This agrees with the basic principle of physics that a causally important

structure of spacetime and matter dynamics should not be independent of the matter content of the universe. The basic principle is satisfied for the energy tensor of the scalar field of Weyl universes and for Λ in general. According to [14] this is a *desideratum for any realistic cosmological model*.

Now it is clear which conditions have to be satisfied by ϕ and $V(\phi)$, if Weyl universes are to be kept in equilibrium by the scalar field. In this case the total amount of mass-energy density and the negative pressure of the vacuum-like term of the scalar field tensor have to counterbalance each other (*dynamical consistency* of Weyl universes). The general balance condition for total energy density ρ and pressure p of fluids is [13]

$$\rho + 3p = 0.$$

Because of $\tilde{p} = -\Lambda$ it becomes in Einstein universes:

$$\tilde{\rho} + \Lambda \doteq \Theta_{00} \doteq 3\Lambda \quad \longleftrightarrow \quad \tilde{\rho} \doteq 2\Lambda. \tag{60}$$

Altogether K-G equation (54), Einstein equation (58) and the consistency condition for Weyl universes (60), including (56), give an easily surveyable set of conditions (the Euler equation (28) is trivially satisfied)

$$\begin{aligned} \kappa + 2\lambda_2 + 4\lambda_4|\phi|^2 &\doteq H^2, \\ 3\kappa - \tilde{\rho} + 6\lambda_2 + 6\lambda_4|\phi|^2 &\doteq 0, \\ \tilde{\rho} + 12\lambda_2 + 12\lambda_4|\phi|^2 &\doteq 6H^2. \end{aligned} \tag{61}$$

To get a first impression what this means in terms of energy densities for low values of ζ we list some examples including comparison with $\rho_{crit} = \frac{3H^2[c^4]}{8\pi G} \doteq 3H^2\xi|\phi_*|^2$.

Examples A moderate curvature module 1 arises for

$$\begin{aligned} \tilde{\rho} &\doteq 4H^2, & \Lambda &\doteq 2H^2, & \kappa &\doteq H^2, & \lambda_2 &\doteq \frac{H^2}{3}, & \lambda_4|\phi|^2 &\doteq -\frac{H^2}{6}, \\ \Omega_m &= \frac{4}{3}, & \Omega_\Lambda &= \frac{2}{3}, & \zeta &= 1. \end{aligned} \tag{62}$$

For $\lambda_4 = 0$, on the other hand, we get

$$\Omega_m = 1, \quad \Omega_\Lambda = \frac{1}{2}, \quad \zeta = \frac{1}{2}, \quad \lambda_2 \doteq \frac{H^2}{4}. \tag{63}$$

It cannot come as a surprise that we find relatively high mass energy densities, as we are working with positive space curvature. They increase with higher curvature values, e.g., $\Omega_m = 2, \Omega_\Lambda = 1$ for $\zeta = 2$ etc.

Considering present mass density estimations, this might appear as a reason to discard these models. But there are other reasons to shed a second glance at them, at least as “toy” (i.e. methodological) constellations. In the light of the conjecture (Sect. 6) that $T^{(\phi)}$ may be considered as energy tensor of the gravitational structure (extended by ϕ), these simple models demonstrate the *possibility* of a cosmic

geometry balanced by the gravitational stress energy tensor itself (cf. in a different context [14]). This may be important for attacking the problem of stability. We do not do this here; but turn to a second glance at the empirical properties of these models. This also illuminates the more general question how Weyl geometric models behave under empirical scrutiny.

10 First Comparison with Data

First of all, it is clear that the precision of the empirical tests of GRT inside the solar system lie far away from cosmological corrections in any approach. In Weyl geometry the cosmological corrections to weak field low velocity orbits amount to an additional coordinate acceleration $\ddot{x} = -H\dot{x} =: a_H$ [35, Appendix II]. For typical low velocities of planets or satellites $\sim 10 \text{ km s}^{-1}$, this is 9 orders of magnitude below solar gravitational acceleration at distance 10 AU (astronomical units) from the sun, and 4 orders of magnitude below the anomalous acceleration a_P of the Pioneer spacecrafts determined in the late 1990s [1]. Present solar system tests of GRT work at an error margin corresponding to acceleration sensitivity several orders of magnitude larger [44].

Thus, for the time being, the Weyl geometric cosmological corrections cannot be checked empirically by their *dynamical* effects on the level of solar system in terms of parametrized postnewtonian gravity (PPN). On the other hand the Hubble connection leads to an additional redshift $\Delta v \approx Hc^{-1}v\Delta t$ over time intervals Δt for space probes of the Pioneer type with nearly radial velocity v . This corresponds to the absolute value of the anomalous Pioneer acceleration, but is of wrong sign, if compared with the interpretation of the Pioneer team. Follow up experiments will be able to clarify the situation [9].

At present a first test of the model with data from observational cosmology is possible by confronting it with the high precision supernovae data available now for about a decade [31], recently updated [34]. In the Weyl geometric approach the damping of the energy flux of cosmological sources is due to four independent contributions: In addition to damping by redshift $\sim (1+z)$ (energy transfer of single photons with respect to X), the internal time dilation due to scale transfer of time intervals (2) reduces the flux by another factor $\sim (1+z)$ (reduction of number of photons per time). Moreover the area increase $A(z)$ of light spheres at redshift z in the respective geometry (here in spherical geometry) and an extinction exponent ϵ have to be taken into account. As distance $d \sim (1+z)$, the absorption contributes another factor $\sim (1+z)^\epsilon$. The energy flux $F(z)$ is thus given by

$$F(z) \sim (z+1)^{-(2+\epsilon)} A(z)^{-1}.$$

For the module $\zeta = \kappa H^{-2}$ the area of spheres is

$$A(z) = \frac{4\pi}{\kappa} \sin^2(\sqrt{\zeta} \ln(1+z)).$$

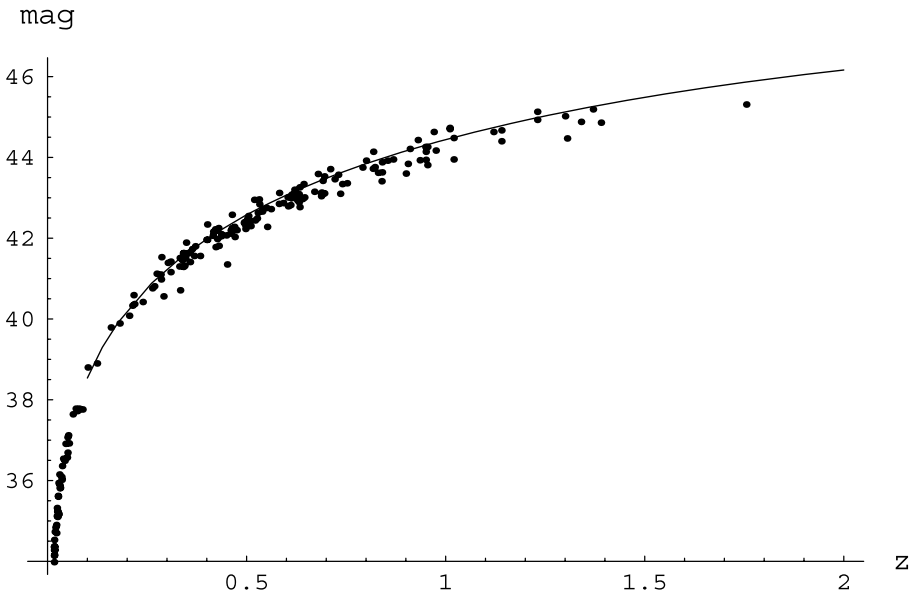


Fig. 1 Magnitudes of 191 supernovae Ia (mag) $z \leq 1.755$ [34], and prediction in Weyl geometric model $\zeta = 1$

Then the logarithmic relative magnitudes m of sources with absolute magnitude M become (cf. [35])

$$m(z, \zeta, \epsilon, M) = 5 \log_{10} \left[(1+z)^{\frac{2+\epsilon}{2}} \zeta^{-\frac{1}{2}} \sin(\log(1+z)\zeta^{\frac{1}{2}}) \right] + C_M \quad (64)$$

where the constant C_M is related to the absolute magnitude of the source by

$$C_M = M - 5 \log_{10}(H_1 10^{-5}).$$

A fit of the redshift-magnitude characteristic of Einstein-Weyl universes with the set of 191 SNIa data in [34] leads to best values $\epsilon_0 \approx 1$ and $\zeta_0 \approx 2.5$ with confidence intervals $1.46 \leq \zeta \leq 3.6$ and $0.65 \leq \epsilon \leq 1.3$.³ The root mean square error is $\sigma_{\text{Weyl}}(\zeta) \leq 0.22$ and increases very slowly with change of ζ . In the whole confidence interval it is below the mean square error of the data $\sigma_{\text{dat}} \approx 0.24$ (given by Riess et al.) and below the error of the standard model fit $\sigma_{\text{SMC}} \approx 0.23$. For $\epsilon \approx 0.65$ (lower bound of its confidence interval) the root mean square error of the Weyl model predictions for $\zeta = 0.5$, compared with the data, is still $\sigma_{\text{Weyl}}(\zeta) \approx 0.231$ comparable to the quality of the SMC and $< \sigma_{\text{dat}}$. It surpasses the latter only for $\zeta \leq 0.2$. According to this criterion our examples (62), (63) survive the test of the supernovae data as well as the standard approach (Fig. 1).

³In [35, (38)] a more aprioristic deduction of the energy damping has been used, fixing ϵ to 1. The empirically minded approach given here follows a proposal of E. Fischer (personal communication).

At the moment supernovae data do not allow to discriminate between the Weyl geometric approach and the Friedman-Lemaitre one of the SMC. That may change, once precise supernovae data are available in the redshift interval $2 < z < 4$.

Of course many more data sets have to be evaluated, before a judgment on the empirical reliability of the Weyl geometric approach can be given. The cosmic microwave background, e.g., appears in our framework as a thermalized background equilibrium state of the quantized Maxwell field. A corresponding mathematical proof of a perfect Planck spectrum of a high entropy state of the Maxwell field in the Einstein universe has been given by I.E. Segal [36]. Anisotropies seem to correlate with inhomogeneities of nearby mass distributions in the observable cosmological sky by the Sunyaev-Zeldovic (SZ) effect [28]. For more distant clusters that is completely different: The almost lack of SZ effects for larger distances has been characterized as “paradoxical” by leading astronomers [3]. It seems to indicate that the origin of the microwave background does not lie beyond these clusters. Empirically the assumption of a *deep redshift origin* of the anisotropies is therefore no longer as safe as originally assumed, perhaps even doubtful.

Some empirical evidence, like quasar data, goes similarly well in hand with the Weyl geometric cosmological approach as with the SMC, other worse. Most importantly, present estimations of mass density lie far below the values indicated by our models. But the last word on mass density values may not have been spoken yet. The determination of the present values for Ω_m (≈ 0.25) is strongly dependent on the standard approach of cosmology. We should not be faulted by what philosophers call the experimenters “regress” (testing theories by evaluatoric means which presuppose already parts of the theory to be tested) and keep our eyes open for future developments [18, 25].

On the other hand, other data are better reconcilable with the Weyl models than with the SMC. In particular the lack of a positive correlation of the metallicity of galaxies and quasars with cosmological redshift z seems no good token for a universe in global and longtime evolution. Moreover, the observation of high redshift X-ray quasars with very high metallicity (BAL quasar APM 08279+5255 with $z \approx 3.91$ and Fe/O ratio of about 3) appears discomfoting from the expanding space perspective. Present understanding of metallicity breeding indicates that a time interval of about 3 Gyr is needed to produce this abundance ratio, while the age of SMC at $z \approx 3.91$ is about $t \approx 1.7$ Gyr, just above half the age needed [21].

Many more data sets have to be investigated carefully comparing different points of view afforded by differing theoretical frames. It is too early to claim anything like secure judgment on this issue.

11 Conclusion and Discussion of Open Questions

The extension of the Weyl geometric approach from field theory to cosmology leads to a formally satisfying weak generalization of the Einstein equation by making all its constituents scale covariant, (29). The corresponding Lagrangian (14) uses a minimal modification of the classical Lagrangians. It is inspired by a corresponding scale covariant approach to semi-classical field theory of W. Drechsler and H. Tann and

is similar to the one used in J-B-D type scalar-tensor theories. Weyl geometric gravity theory has features similar to those of conformal J-B-D theories (Sect. 4); but it builds upon different geometrical concepts. The scale connection φ , the specific new geometrical structure of Weyl geometry, shows remarkable physical properties. Integrated it describes transfer properties of metric dependent quantities (2), and its dynamics is basically that of the scalar field. Both can be transformed into another (21), (22). Local observables can be formulated scale gauge invariantly (10), but have a direct representation in a preferred gauge (12).

A difference to large parts of the work in J-B-D scalar-tensor theory arises from the actual consequences drawn from coupling the scalar field to the Riemann-Einstein action for variation with respect to $\delta g^{\mu\nu}$. Tann and Drechsler have shown that a correct evaluation of $\frac{\delta(l\phi)^2\bar{R}}{\delta g^{\mu\nu}}$ leads to the same additional terms in the energy tensor of the scalar field (32) as postulated by Callan/Coleman/Jackiw by different (quantum physical) considerations. This argument has apparently found not much attention in the literature on J-B-D theories, although it should have done so. As long as this is not the case, the dynamics of scalar fields in J-B-D theories and in Weyl geometry seems to be different.

The scale covariant perspective sheds new light on the class of Robertson-Walker solutions of the Einstein equation. Weyl geometry suggests to consider non-expanding versions of homogeneous and isotropic cosmological geometries, in which the redshift is encoded by a Weylian scale connection with only a time component $\varphi = H dt$, the Hubble connection (42). Thus the question arises, whether the warp function of Robertson Walker models does describe a real expansion, as usually assumed, or whether it is no more than a mathematical feature of the Riemannian component of a scale gauge without immediate physical significance.

For a first approach to this question we have investigated special solutions of the coupled system of a scale covariant Euler type fluid equation (28), in the simplest case dust, the scalar field equation (26), and a scale invariant version of the Einstein equation (29). This leads to the intriguingly simple geometrical structure of *Weyl universes* (47) and gives a first impression of the new features which can arise in Weyl geometric gravity. The Riemannian component of the metric of these models coincides with that of the classical static solutions of cosmology; but in addition we have a time-homogeneous Hubble connection. The scalar field's energy-stress tensor can be evaluated explicitly, (56) and (57). It shows good physical properties, *if it is considered in the untruncated form* of (32). Formally it looks like the vacuum tensor, $\Theta = -\Lambda g$, of the standard approach; physically, however, it is different. For the case of Weyl universes a close link between the coefficient H of the Hubble connection and mean cosmic mass energy density can be established (59). This can be considered as a kind of implementation of Mach's principle.

In several aspects Weyl geometric models behave differently from what is known and expected for classical F-L models of cosmology. In this sense they may be useful to challenge some deeply entrenched convictions of the received view:

- (1) Cosmological redshift need not necessarily be due to an "expansion" of spacetime, corresponding to a realistic interpretation of the warp function of Robertson-Walker models. It may just as well be expressed by the scale

- connection (*Hubble connection*) corresponding to a Weyl geometric scalar field (45), (22).
- (2) Scalar fields can develop a dynamical contribution (32) which may stabilize the geometry even to the extreme case of a “static” Weyl geometry (although linearly expanding in the Riemann picture) (60). The arising problems of stability and of a possible tuning of the parameters λ_2, λ_4 of V have been left open here. More sophisticated examples will have to be investigated; some of them may show oscillatory behavior.
 - (3) This approach leads to cosmological models beyond the standard approach which are, in any case, methodologically interesting (62), (63). Perhaps they even are of wider empirical interest.
 - (4) Our case study of Weyl universes shows that in particular the consequences of Tann’s and Drechsler’s calculation of the variational consequences of coupling the scalar field to the Einstein-Hilbert action have to be taken seriously already on the semi-classical level.

The Einstein-Weyl models should be studied more broadly from the point of view of observational cosmology. Here we had to concentrate on one aspect only, the supernovae data. They are well fitted by this models and clearly favour positive curvature values in the Weyl universe class, $0.2 \leq \zeta < 3.6$. This indicates higher mass density values than accepted at present. Our main example (62) has values $\Omega_m = \frac{4}{3}$, $\Omega_\Lambda = \frac{2}{3}$, $\zeta = 1$. Many will consider this already as an indicator of lacking empirical adequacy. But we have reasons to be more cautious in this respect; we should wait for further clarification of this issue (Sect. 10).

Moreover, there may arise motivations from another side to improve the approach to cosmology presented here, if the Weyl geometric approach turns out fruitful in the field theoretic sector. The scale covariant scalar field prepares the path towards a different approach to the usual Higgs mechanism for understanding the mass acquirement of ew bosons [11]. An analysis of the consequences of the Weyl geometric “pseudo-Higgs” ϕ -field without boson for the perturbative calculations of the (modified) standard model of elementary particle physics is a desideratum. For a comparison with upcoming experimental results at the LHC it may even become indispensable. Should the empirical evidence fail to support the present expectation of a massive Higgs boson, and even exclude it in the whole energy interval which at the moment is still theoretically and experimentally consistent with the present standard model EP (120–800 GeV) after several years of data collection, the scale covariant scalar field would be a good candidate for providing a new conceptual bridge between elementary particle physics and cosmology. In this case the missing link between gravity and the quantum had to be sought for in a direction explored in first aspects from the point of view of in quantum gravity, e.g., by [40]. If the “pseudo-Higgs” explanation of ew boson mass would be supported by a negative experimental results for the massive Higgs boson, we even had to face the possibility that effects of gravity may be observable in high energy physics (and in fact have been observed already) at laboratory energy scales, much lower than usually expected.

Such strong perspectives will very likely be turned down or corroborated in the coming few years by experiment. Even if they should be invalidated, our cosmological considerations will not have been in vain. The Weyl geometric models show a

route how the anomalous behavior of cosmic vacuum energy may be dissolved in a weak extension of classical GRT, without sacrificing the empirical phenomena or even the overall link to experiment. If this achievement stabilizes and can be extended to the problem of dark matter, the cosmological implications alone would be worth the trouble to work out more details of the Weyl geometric approach.

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References

1. Anderson, J.D., Laing, P.A., Lau, E., Liu, A., Nieto, M., Turyshev, S.: Indication, from Pioneer 10/11, Galileo, and Ulysses data, of an apparent anomalous, weak, long range acceleration. *Phys. Rev. Lett.* **81**, 2858 (1998). [arXiv:gr-qc/9808081](https://arxiv.org/abs/gr-qc/9808081)
2. Audretsch, J., Gähler, F., Straumann, N.: Wave fields in Weyl spaces and conditions for the existence of a preferred pseudo-Riemannian structure. *Commun. Math. Phys.* **95**, 41–51 (1984)
3. Bielby, R.M., Shanks, T.: Anomalous SZ contribution to 3 year WMAP data. *Mon. Not. R. Astron. Soc.* (2007). [arXiv:astro-ph/0703470](https://arxiv.org/abs/astro-ph/0703470)
4. Brans, C.: The roots of scalar-tensor theories: an approximate history (2004). [arXiv:gr-qc/0506063](https://arxiv.org/abs/gr-qc/0506063)
5. Callan, C., Coleman, S., Jackiw, R.: A new improved energy-momentum tensor. *Ann. Phys.* **59**, 42–73 (1970)
6. Canuto, V., Adams, P.J., Hsieh, S.-H., Tsiang, E.: Scale covariant theory of gravitation and astrophysical application. *Phys. Rev. D* **16**, 1643–1663 (1977)
7. Carroll, S.: The cosmological constant. *Living Rev. Relativ.* **4**, 1–77 (2001). Also in [arXiv:astro-ph/0004075v2](https://arxiv.org/abs/astro-ph/0004075v2), visited 10 November 2003
8. Cartier, P.: La géométrie infinitésimale pure et le boson de Higgs. Talk given at the conference *Géométrie au vingtième siècle, 1930–2000*, Paris 2001. <http://semioweb.msh-paris.fr/mathopales/geoconf2000/videos.asp>
9. Christophe, B., Andersen, P.H., Anderson, J.D.: *Odyssee: A solar system mission* (2007). [arXiv:0711.2007v2](https://arxiv.org/abs/0711.2007v2) [gr-qc]
10. Dirac, P.A.M.: Long range forces and broken symmetries. *Proc. R. Soc. Lond. A* **333**, 403–418 (1973)
11. Drechsler, W.: Mass generation by Weyl-symmetry breaking. *Found. Phys.* **29**, 1327–1369 (1999)
12. Drechsler, W., Tann, H.: Broken Weyl invariance and the origin of mass. *Found. Phys.* **29**(7), 1023–1064 (1999). [arXiv:gr-qc/98020v1](https://arxiv.org/abs/gr-qc/98020v1)
13. Ellis, G.: 83 years of general relativity and cosmology: progress and problems. *Class. Quantum Gravity* **16**, A43–A75 (1999)
14. Fahr, H.-J., Heyl, M.: Cosmic vacuum energy decay and creation of cosmic matter. *Die Naturwissenschaften* **94**, 709–724 (2007)
15. Faraoni, V.: *Cosmology in Scalar-Tensor Gravity*. Kluwer, Dordrecht (2004)
16. Faraoni, V., Gunzig, E., Nardone, P.: Transformations in classical gravitational theories and in cosmology. *Fundam. Cosm. Phys.* **20**, 121ff (1998). [arXiv:gr-qc/9811047](https://arxiv.org/abs/gr-qc/9811047)
17. Folland, G.B.: Weyl manifolds. *J. Differ. Geom.* **4**, 145–153 (1970)
18. Fischer, E.: An equilibrium balance of the universe. Preprint [arXiv:0708.3577](https://arxiv.org/abs/0708.3577) (2007)

19. Fujii, Y., Maeda, K.-C.: *The Scalar-Tensor Theory of Gravitation*. Cambridge University Press, Cambridge (2003)
20. Fulton, T., Rohrlich, F., Witten, L.: Conformal invariance in physics. *Rev. Mod. Phys.* **34**, 442–457 (1962)
21. Hasinger, G., Komossa, S.: The X-ray evolving universe: (ionized) absorption and dust, from nearby Seyfert galaxies to high redshift quasars. Report MPE Garching (2007). [arXiv:astro-ph/0207321](https://arxiv.org/abs/astro-ph/0207321)
22. Hawking, S., Ellis, G.: *The Large Scale Structure of Space-Time*. Cambridge University Press, Cambridge (1973)
23. Hehl, F.W., McCrea, D., Mielke, E., Ne'eman, Y.: Progress in metric-affine gauge theories of gravity with local scale invariance. *Found. Phys.* **19**, 1075–1100 (1989)
24. Hung, C.: Possible existence of Weyl's vector meson. *Phys. Rev. Lett.* **61**, 2182–2184 (1988)
25. Lieu, R.: Λ CDM cosmology: how much suppression of credible evidence, and does the model really lead its competitors, using all evidence? Preprint [arXiv:0705.2462](https://arxiv.org/abs/0705.2462) (2007) [astro-ph]
26. Mannheim, P.: Alternatives to dark matter and dark energy (2005). [arXiv:astro-ph/0505266](https://arxiv.org/abs/astro-ph/0505266)
27. Mielke, E., Fuchs, B., Schunck, F.: Dark matter halos as Bose-Einstein condensates. In: *Proceedings Tenth Marcel Grossmann Meeting, Rio de Janeiro, 2003*, pp. 39–58. World Scientific, Singapore (2006)
28. Myers, A.D., Shanks, T., Outram, P.J., Frith, W.J., Wolfendale, A.W.: Evidence for an extended SZ effect in WMAP data. *Mon. Not. R. Astron. Soc.* (2004). [arXiv:astro-ph/0306180v2](https://arxiv.org/abs/astro-ph/0306180v2)
29. Pawłowski, M., Rączka, R.: A Higgs-free model for fundamental interactions. Part I: Formulation of the model. Preprint ILAS/EP-3-1995 (1995)
30. Penrose, R.: Zero rest mass fields including gravitation: asymptotic behavior. *Proc. R. Soc. Lond. A* **284**, 159–203 (1965)
31. Perlmutter, S., Aldering, G., Goldhaber, G., et al.: Measurement of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.* **517**, 565–586 (1999)
32. Quiros, I.: The Weyl anomaly and the nature of the background geometry. Preprint [arXiv:gr-qc/0011056](https://arxiv.org/abs/gr-qc/0011056) (2000)
33. Quiros, I.: Transformation of units and world geometry. Preprint [arXiv:gr-qc/0004014v3](https://arxiv.org/abs/gr-qc/0004014v3) (2008)
34. Riess, G.A., Strolger, L.-G., Casertano, S., et al.: New Hubble Space Telescope discoveries of type Ia supernovae at $z \geq 1$: Narrowing constraints on the early behavior of dark energy. *Astrophys. J.* **659**, 98ff (2007). [arXiv:astro-ph/0611572](https://arxiv.org/abs/astro-ph/0611572)
35. Scholz, E.: On the geometry of cosmological model building. Preprint [arXiv:gr-qc/0511113](https://arxiv.org/abs/gr-qc/0511113) (2005)
36. Segal, I.E.: Radiation in the Einstein universe and the cosmic background. *Phys. Rev. D* **28**, 2393–2401 (1983)
37. Shojai, F., Golshani, M.: On the geometrization of Bohmian mechanics: A new approach to quantum gravity. *Int. J. Mod. Phys. A* **13**, 677–693 (1998)
38. Shojai, A.: Quantum, gravity and geometry. *Int. J. Mod. Phys. A* **15**, 1757–1771 (2000)
39. Shojai, A., Shojai, F.: Weyl geometry and quantum gravity. Preprint [arXiv:gr-qc/0306099](https://arxiv.org/abs/gr-qc/0306099) (2003)
40. Smolin, L.: Towards a theory of spacetime structure at very short distances. *Nucl. Phys. B* **160**, 253–268 (1979)
41. Tann, H.: Einbettung der Quantentheorie eines Skalarfeldes in eine Weyl Geometrie—Weyl Symmetrie und ihre Brechung. Utz, München (1998)
42. Weyl, H.: Reine Infinitesimalgeometrie. *Math. Z.* **2**, 384–411 (1918). In GA II, 1–28
43. Weyl, H.: *Raum-Zeit-Materie*, 5. Auflage. Springer, Berlin (1923)
44. Will, C.: The confrontation between general relativity and experiment. *Living Reviews in Relativity* **4**, 1–97 (2001)
45. Zwicky, F.: On the possibilities of a gravitational drag of light. *Phys. Rev.* **33**, 1623f (1929)