Practice-related symbolic realism in H. Weyl's mature view of mathematical knowledge

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Introduction

Hermann Weyl went through diverse transformations of his views of mathematical knowledge. Most of them were described by himself in his look back in (Weyl 1954). In 1905, at the beginning of his university studies, he was thrown (by Hilbert's views on the foundations of geometry) from a youthful and naive Kantianism to a "positivism" in the sense of H. Poincaré and E. Mach. Five years later, he came under the influence of Husserl's phenomenology and turned away from positivism. At Zürich he got into close contact with F. Medicus who was an expert in the philosophy of post-Kantian German idealism and edited Fichte's works. After Weyl came back from service in the German army in 1916, his philosophical outlooks turned radically towards realism in the sense of German idealist philosophy, formed under the impression of his way of reading J.G. Fichte and, a little later, under the personal influence of L.E.J. Brouwer. In 1926 he had the chance to rework his philosophical outlook when he wrote his contribution *Philosophie der Mathematik und Naturwissenschaften* for the handbook of philosophy edited by M. Schröter and A. Bäumler (Weyl 1927*a*).[1] During this work Weyl got closer acquainted with Leibniz' philosophy, among others. He broadened and refined his philosophical views and started to reconsider his earlier exaggerated rejection of Hilbert's formalist views in the foundations of mathematics.

The book was written at the time of the pathbreaking invention of the new quantum mechanics by W. Heisenberg, M. Born. P. Jordan, and E. Schrödinger. It would have been too early to draw philosophical conclusions from these physical insights. In the years to come, Weyl not only contributed to the conceptual framework of quantum mechanics but also drew his own consequences for the understanding of nature and the way how mathematics

[1] Bäumler was the main editor, but M. Schröter was in charge of the section which Weyl contributed to. Later Bäumler became an ardent protagonist of Nazi ideology, while M. Schröter distantiated himself from Nazi philosophy and worked for a publishing house during the Nazi era.

could and can contribute to it. After the second great war Weyl continued to participate in the philosophical debate on mathematics and natural science. Among others, he refined and extended his book for an English edition (Weyl 1949*b*) and commented the questions from his peculiar perspective which was now broadened and presented in a moderate and sometimes even modest language, even where he still preferred drastic alternatives to generally accepted views (Weyl 1948, Weyl 1953). When we use the attribute *mature* views in this context, it has to be understood in the sense of post 1927/27. In this article we even refer mainly to Weyl's late contributions after 1948.

The meaning of *symbolic realism* will, hopefully, become clearer during this article. To give a short description in advance, remember that Weyl liked to consider science as a "symbolic construction" (with allusions to both, Leibniz and Poincaré) and mathematics as its symbol producing core. Criticizing Hilbert, he insisted that there were more than purely formal aspects in the reflection of mathematical knowledge. During his "maturation phase" in the 1920s he gave up his earlier strongly idealistic understanding of the "realism" of mathematical concepts and symbols (defended in the time between 1917 and 1921) and developed a more refined view of the quasi-realistic character of mathematical knowledge through its link to broader scientific practices, in particular of physics and mathematized technologies, and the existence of a semantical "input" derived from it.

It may be worthwhile to notice that Weyl's later "symbolical realism" contained an approach to Hilbert's "formalism" which differed from logical positivism and the later analytical philosophy of science. Because of the striking difference between the *view* of mathematics given in Hilbert's foundational contributions and the latter's actual practice as a mathematician,[2] Weyl's mature form of symbolic realism was probably even closer to Hilbert's own understanding than the picture of Hilbert's formalism present in large parts of the analytical philosophy literature. Moreover, it was no longer built on strongly idealist ontologies of mathematical and physical reality, as had been the case for Weyl's earlier *Sturm und Drang* realism. It was rather cautiously based on cooperation with other, empirically bound sciences, tending towards coherence with cultural practices, reflected by him as existential experience of the individual in a world of irritating insecurities.

Thus this article will present an argument which may be structured in three theses:

(1) H. Weyl lived through a detachment from classical metaphysics in parts of European intellectual culture during the early 20th century, and he suffered from it. In addition, and linked to it, he battled with the aporetic problems of self-interpretation of the mathematical sciences

[2]See (**?**)

arising from the foundational debate in mathematics, from special and general relativity, and from quantum mechanics (section 1).

(2) Our protagonist searched for a solution to the problems of self-interpretation of mathematics which arose from the detachment from classical metaphysics in a kind of *symbolic realism.* In his later years, Weyl even reflected the problems of the mathematical sciences in the medium of existential philosophy which, in his view, gave an adequate expression to the crisis of metaphysics in high modernity (sections 2 and 3).

(3) From our own perspective, it is worthwhile to adopt the core of Weyl's symbolic realism and to integrate it into a broader cultural philosophy of practice (section 4).

1. Detachment from classical metaphysics

Looking back in the late 1940s to what had happened in the first third of the 20th century in and with the mathematical sciences, Weyl could be much clearer in several respects than at the time when he wrote his book on the philosophy of mathematics and the natural sciences, which happened to be the case exactly during the transition towards the new quantum mechanics. Now, he could present the turn from the early and the classical modern mechanistic conception of nature to a "purely symbolical" one of high modernity with even greater perspicuity than in the midst of it. He characterized the traditional "mechanistic construction of the world (mechanistische Weltkonstruktion)" (Weyl 1948, 295) by two complementary ingredients,

— a *spatio-temporal science of geometry and motion*, which was understood by important early protagonists of this view (among them Kepler, Descartes and Newton) as reflecting God's spirit,

— and an *atomism* in the explanation of matter, derived from Demokritos through Gassendi and Galilei to Huygens and Newton.

Thus the traditional mechanistic world construction integrated, in a kind of "consensus" as Weyl expressed it (Weyl 1948, 295), ontological *idealism* with respect to space and time and *materialism* with respect to matter structures. The traditional world view was built upon a balance between these two components and allowed different specifications.

Weyl contrasted this classical view of the relationship between mathematics and nature with the modern one:

> . . . in place of a real spatio-temporal-material being we are only left with a *construction in pure symbols.* (Weyl 1948, 295, emphasis here as in the sequel in original)[3]

[3]" . . . anstatt eines realen räumlich-zeitlich-materiellen Seins behalten wir nur eine *Konstruktion in reinen Symbolen* übrig."

He immediately made sure, however, that one should not understand this symbolism in a formalist sense. Even the "pure symbols" had origin and meaning.

> I want to turn towards mathematics to enquire for meaning and origin of the symbols, and there we will detect *man*, inasmuch as he is a creative mind, as the masterbuilder of the world of symbols. (ibid.)[4]

Moreover, the creative power of the producing mind is not left to arbitrariness. Although Weyl agreed that the traditional ontological bound of mathematics had been cut, he still supposed a a binding law to exist. Without attempting here to analyze where it came from, he insisted on a necessary restriction of creativity:

> Only in committing the liberty of mind to lawfulness, the mind is able to comprehend the constraints of the world reproducing them and of its own being in the world. (ibid.)[5]

Already here, we find a clear expression that the detachment from the bounds of classical metaphysics did not at all lead Weyl to admit arbitrariness of symbolic construction. It remains to find out, where he saw the "lawfulness" to arise from. Let us first see, however, how Weyl described the modern condition for the mathematical sciences in foundations of mathematics, relativity and quantum mechanics.

It is worthwhile to notice that, more than two decades after having written (Weyl 1927*a*) and after a cautious and limited reapprochement to Hilbert's foundational positions, he continued to be nearly as sharp in his philosophical rejection of transfinite set theory as he had been then. He drew a direct line between classical metaphysics which had "written a cipher referring to transcendent reality by posing God as absolute being ..." and repeated his old verdict against the transfinite as an actual infinity in the sense of mainstream modern mathematics:

> Mathematics too has executed a jump to the absolute in, I would say, naive objectivity, without being aware of the dangers ....(Weyl 1948, 327f.).[6]

[4]"Ich werde mich dann zur Mathematik wenden, um Auskunft über Sinn und Ursprung der Symbole zu erhalten, und wir werden da den *Menschen.* sofern er schöpferischer Geist ist, als den Baumeister der Symbolwelt entdecken."

[5]"Nur indem die Freiheit des Geistes sich selber bindet an das Gesetz, begreift der Geist nachkonstruierend die Gebundenheit der Welt und seines eigenen Daseins in der Welt."

[6]"[Die Metaphysik] schreibt eine auf das Transzendente verweisende Chiffre, wenn sie Gott als absolutes Sein setzt, ... Auch die Mathematik hat, ohne sich der Gefahr bewußt zu sein, in naiver Sachlichkeit, möchte ich sagen, den Sprung zum Absoluten vollzogen."

We see that even the late Weyl warned against undisclosed metaphysical remnants in modern mathematics nearly as strongly as he had done thirty years before as a young philosophical and mathematical radical. He criticized that the acceptance of classical first order predicate logic, with its unrestricted logical use of the existential quantifier and the principle of the *tertium non datur*, already signifies a kind of metaphyical apellation to an "infinite all-ness (unendliche Allheit)" which "is not from this world".

> [He who does so] is already standing on the other shore: the number system, an open domain of possibilities which can only be conceived in the process of becoming, has turned for him into an embodiment of absolute existence. (Weyl 1948, 328)[7]

Although an understanding of transfinite set theory as a symbolical logical possibility *without* any ontological committment and thus in strong contrast to G. Cantor's philosophical interpretation of it, had already been expressed by Felix Hausdorff about 40 years earlier in a move to rid mathematics from classical metaphyics,[8] our protagonist did not perceive of such a possibility of thought. He continued his polemics by the warning that on such a "logical transcendentalism resides the power of classical mathematics" (ibid., 329). He himself had been obliged to acknowledge this power in his path-breaking work on the representation of Lie groups and in other parts of his mathematical work.[9] Notwithstanding these experiences, he repeated old phrases, although now openly attributed to his former ally L.E.J. Brouwer: If mathematics attempts to master the infinite by finite tools, it achieves so only by a "fraud" in its logical-transcendent form,

> ... — by a gigantic, although highly successful fraud, comparable to paper money in the economic realm. (Weyl 1948, 330)[10]

A similar paper money metaphor had been used by Weyl in his famous radical article on the *crisis* in the foundations of mathematics (Weyl 1921, 156f.).

[7] "Wer die an die unendliche Allheit appellierende Alternative ... als sinnvoll hinnimmt, steht bereits am jenseitigen Ufer: das Zahlsystem ist ihm aus einem offenen, nur im Werden zu erfassenden Bereich von Möglichkeiten zu einem Inbegriff absoluter Existenz geworden, das 'nicht von dieser Welt ist'."

[8] (Stegmaier 2002, Scholz 1996)

[9] (Hawkins 2000, Coleman/Korté 2001)

[10] "Ist es das Ziel der Mathematik, das Unendliche durch endliche Mittel zu meistern, so erreichte sie das in ihrer logisch-transzendenten Form, wenn wir Brouwer glauben wollen, nur durch einen Betrug — durch einen gigantischen, freilich höchst erfolgreichen Betrug; vergleichbar dem Papiergeld auf ökonomischen Gebiet."

2. In search of a "post-classical" metaphysics

H. Weyl's position in the foundations of mathematics has often been described and discussed,[11] therefore I continue directly with his characterization of the role of modern mathematics in physical knowledge. The metaphysical implications are here more directly visible.

Weyl would not accept a formal hypothetical role for mathematical theory in the natural (or other) sciences. He even started the discussion of this question by a remark which emphasized a metaphysical aspect, although different from the classical one characterized above (in the sense of reference to the "mind of God" or comparable transcendent referents).

> One cannot deny that a theoretical desire is living in us, which is simply incomprehensible from a purely phenomenological point of view. Its productive urge is symbollically conforming the transcendent and is driven by the metaphysical belief in the reality of the external world (...). (Weyl 1948, 333)[12]

For Weyl this was the crucial point. In his view, mathematics did more than to offer mere tools for the formation of mathematical models of processes or structures, in a purely pragmatic sense. A good mathematical theory of nature was the result of such a "productive urge" and expressed, if well done, an aspect of transcendent reality in "symbolical form". In his view, the modern criticism of traditional metaphysics would never be able to achieve a complete purge of *all* metaphysical elements in the knowledge construction of the mathematical sciences. Any such knowledge at least requires a *productive force (Schaffensdrang)* driven by a *metaphysical belief* in some *transcendent* world core, without which no meaningful communicative scientific practice would be possible.

In brackets following the remark just quoted, Weyl added that this conformation of the transcendent is driven not only by the belief in the external world, but simultaneously (gleichartig) by beliefs in the "reality of the own self, of the foreign thou, and of God". Weyl did not endow his "God" with any peculiar feature that preformed the a priori forms of scientific knowledge. His "God" seemed not far away from Spinoza's; we would not change the argument much, if we substituted other historical names for it, which are legend, like the "tao" (Laotse), the "transcendent chaos" (F. Hausdorff),

[11] See (Mancosu 1998, 65–85) and the following English translation of Weyl's paper; more details in (Coleman/Korté 2001, Feferman 2000, van Dalen 1984, van Dalen 2000, Hesseling 2003, Majer 1988).

[12] "Es ist nicht zu leugnen, daß in uns ein vom bloß phänomenalen Standpunkt schlechterdings unverständliches theoretisches Bedürfnis lebendig ist, dessen auf symbolische Gestaltung des Transzendenten gerichteter Schaffensdrang Befriedigung verlangt und das getrieben wird von dem metaphysischen Glauben an die Realität der Außenwelt (neben den sich gleichartig der Glaube an die Realität des eigenen Ich, des fremden Du und Gottes stellt)."

or even less transcendent sounding ones like "nature/matter" in the original sense of K. Marx.[13] All of these refer to "some transcendent core of the world" beyond the individual self (eigenes Ich) and a communicative other (fremdes Du) without giving rise to a claim of being able to preform scientific knowledge of the world.

This very general remark on the constitutive role of mathematics for the drive towards a representation of transcendent reality was made more explicit by a discussion of its different appearance in relativity theory and in quantum mechanics. Already in his book, Weyl had extensively discussed the role of mathematical symbolism in relativity theory (Weyl 1927*a*). Now in 1948, he briefly resumed how the *principle of relativity* served as the main tool to assure the best possible symbolic representation of some "transcendent reality" in the mathematical theory of nature. Natural laws have to be invariant not only with respect to all transformations between posssible inertial observer systems, as in special relativity, but even with respect to all admissible, i.e. continuous, tranformations between nets of subdivisions of the time-spatial continuum of event localizations. This was Weyl's form to express in even more fundamental terms what also Einstein aimed at in his search for general covariance of natural laws, in the sense of G. Ricci Curbastro and T. Levi-Civita. Phrased in more recent language, Weyl underpinned the *covariance* principle by postulating *invariance* of the mathematical description of the physical world with respect to morphisms of the groupoid of constructively defined topological transformations of the space-time continuum.

Only by this move the "projection of the given", as Weyl called it, on the background of the a priori possible constructed by the mathematical mind acquires a well-determined form. He thus substituted the logico-symbolically possible in the place of a cognitive *a priori*, by which the latter lost its formerly assumed (or even "demonstrated") necessary form. After the modern revolution the a priori has, according to Weyl, to be considered as a *product of the creativity of the free mind.* Of course, this arose new problems, and Weyl thought to compensate for this loss by the principle of invariance.

> [I]t is quite evident from the liberty of mind, that in its constructions some *arbitrariness* is necessarily inbuilt; this can, however, afterwards be compensated for by the principle of invariance. (ibid, 336)[14]

[13] We definitely have to distinguish between K. Marx's natural philosophy and 20th century versions of "dialectical materialism" with its strong input of classical or, even worse, pre-classical scholastic metaphysics in which the determative power of "God" reappeared in the guise of "matter" attributes. This has already been shown by Schmidt (1962), long before the downfall of the institutional strongholds of "dialectical materialism" as an official philosophy/ideology in parts of the world of our past.

[14] "[E]s ist einigermaßen aus der Freiheit des Geistes verständlich, daß in seine Konstruktion unvermeidlich *Willkür* eingeht, daß aber diese nachträglich durch ein Prinzip der Invarianz unschädlich gemacht werden kann."

We see how, in Weyl's analysis, the principle of invariance took over the position of the former metaphysical binding law for the construction of empirical knowledge, which had been anchored by philosophical minds of early modernity in the "spirit of God" and was substituted in Kant's criticism by his a priori categories and forms of intuition. In his way, Weyl alluded to a kind of *relative a priori*. This seemingly contradictory term relates, on the one hand side, to the *a priori* function of the symbolical knowledge of mathematics with respect to the empirical one. On the other hand, it reflects the shift away from necessary structures derivable once and for all and indicates the dependence on a historically achieved and changing stage of knowledge to which it is *relative*. Such a relative a priori constituted a bridge, so to speak, between the productivity of the "free mind" and the "urge" to give account of some "transcendent" core of reality which appears to the knowlege acquiring subject through the empirical phenomena. So far, Weyl essentially repeated in condensed form what he had written already in his book during the 1920s.

The epistemological consequences of quantum physics for the role of mathematics in the enterprise to understand nature were stronger than those of relativity. Weyl's philosophical views were strongly influenced by the rise of the new quantum mechanics; he revisited his expectation of mathematical theory in natural sciences and his characterization of the concept of matter.[15] He subscribed to an enlightened Copenhagen-Göttingen interpretation of quantum mechanics, as far as its interior scientific semantics was concerned. In this sense he supported Born's probabilistic interpretation of the Schrödinger "wave" function from the outset against Schrödinger's, Einstein's and others' attempts to keep to a classical field theoretical interpretation of it. He was not satisfied, however, with the philosophical self-interpretation of the new quantum mechanics or its interpretations by contemporary philosophers.

Weyl accepted the Heisenberg-Bohr insight of the essential role of complementarity in the sense of pairing of conjugate observables in the mathematical description of quantum systems. Very early (already in autumn 1925) he convinced himself that Heisenberg's commutation relation

$$QP - PQ = [Q, P] = i\hbar$$

for any pair q and p of conjugate variables represented by symmetric operators Q and P, like a linear space coordinate and its corresponding momentum, was *the* constitutive insight of the new theory and probably a clue for an elaboration of a mathematical theory of quantum reality. Already in late 1925 Weyl had started to think about a derivation of Heisenberg's relation more fundamentally as the property of a normal form of projective representations of abelian groups. His (still preliminary) thoughts were finally published in (Weyl 1927*b*). They formed the starting point for the

[15](Scholz 2004*a*)

Stone/von Neumann representation theorem in quantum mechanics. Moreover, they contained an idea for a quantization procedure relying on properties of Fourier transformations on abelian groups, which in the 1970s was taken up in and generalized to "Weyl quantization".[16]

Weyl was not glad however, to say the least, with the more general "philosophy of complementarity" which had been proposed by Niels Bohr in the 1920/30s and continued to be propagated by him after the war. No doubt, there was a certain plausibility in describing oppositional features of life and culture in terms of "complementarity". Among them Weyl counted as well chosen examples the pairs freedom of will — natural causality, living an experience — cognition or reflection of it, moreover in ethics, justness versus love. Nevertheless he warned:

> May it not be that the idea of complementarity, which in quantum physics corresponds to a state of affairs with an exact mathematical expression, could be misused in a similar way as is the case for the idea of relativity? (...) I want no more than to pose a question. (Weyl 1948, 338f.)[17]

Slightly streamlining, we may describe the Copenhagen interpretation of quantum mechanics as a package made up of a positivistic-formalist interpretation of the mathematics of quantum mechanics, with an open minded philosophy of complementarity in the interpretation of the empirical phenomena of quantum physics.[18] Both together formed something like a "hard core" of the interpretation. To this some of its protagonists, in particular N. Bohr, added a complementarity outlook on broader, cultural philosophy in the sense of *Lebensphilosophie*. Others, like W. Heisenberg, stipulated, in their more personal reflections a kind of platonist ontology underlying quantum reality, while he demanded adherence to a positivist outlook for the crowd of working physicists. Weyl modestly, but clearly rejected the *Lebensphilosophie* part of the parcel, although he had great admiration for Bohr as a philosopher of science, inasmuch he was rethinking basic concepts of physics in the light of the new empirical evidence of quantum physics. He also did not subscribe to the formalist ("positivist") interpretation of the role of mathematics in the construction of the theory, which was defended by M. Born, W. Heisenberg and J. von Neumann, fitting to the mainstream Göttingen spirit of the time, and was accepted by N. Bohr as part of the compromise with W. Heisernberg and W. Pauli in the discussions on the interpretation of quantum mechanics.[19]

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[16] (Mackey 1988, Mackey 1993)

[17] "Aber besteht nicht doch die Gefahr, daß hier mit der Idee der Komplementarität, die innerhalb der Quantenphysik einem mathematisch genau zu präzisierenden Tatbestand entspricht, ein ähnlicher Mißbrauch getrieben wird wie mit der Idee der Relativität? ... Ich werfe nicht mehr als eine Frage auf."

[18] (Chevalley 1995, Chevalley 1993)

[19] (Beller 1999, Hendry 1984)

The positivistic-formalist agreement on the role of mathematical theory served well to cover up underlying differences in the metaphysical outlooks of the Copenhagen-Göttingen contributors to quantum mechanics. In fact, it was quite useful as a substitute for a working philosophy which allowed to avoid to spell out subtle differences of a metaphysical kind. From a historian's point of view, such differences appear to be indissolubly linked to social, personal and intellectual experiences of the individual contributing scientists. Differences of this kind are a natural ingredient and flavour of cultural life. Weyl, for his side, was not content with any of these views; he rather looked for other allies, closer to his own philosophical experiences. In *this respect* he now sided with Oskar Becker who in the late 1920s had been a kind of spokesman of Husserl's phenomenology among mathematical scientists, although in that time he had differences with him on a proper understanding of geometry after the advent of relativity theory.[20]

In an article also written at the end of the 1920s, called *Das Symbolische in der Mathematik*, Becker had characterized the first step by Hilbert, von Neumann and Northeim toward an axiomatic characterization of quantum mechanics (von Neumann e.a. 1928) as a kind of return to the ancient magical origins of the mathematical sciences. Weyl quoted Becker literally:

> In a way one jumps into the 'interpretation' of nature with the complete, ontologically incomprehensible 'mathematical apparatus'; the apparatus works a like a magic key which opens up the field of physical problems — but it does so in the sense of a symbolical representation only, not in the sense of an interpretation really 'discovering' the phenomena (Becker 1927/28)[21]

Becker continued, and Weyl quoted him, apparently with a recondite smile:

> The basic direction of such a symbolical approach comes from time immaterial, archaic, even 'pre-historic': the most modern 'exact' science returns again to magic from which it originally descended. (ibid)[22]

Apparently Weyl sided with Becker's move to break the hermeticity of the formalist approach to quantum mechanics and the clear characterization the formal system as a symbolical representation of external ("transcendent")

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[20](Mancosu/Ryckman 2002)

[21]"Man springt also gewissermaßen mit dem vollständigen, ontologisch unverständlichen 'mathematischen Apparat' in die 'Deutung' der Natur hinein; der Apparat ist wie ein magischer Schlüssel, der das physikalische Problem erschließt, — aber nur erschließt im Sinne einer symbolischen Repräsentation, nicht im Sinne einer die Phänomene wirklich 'entdeckenden' Interpretation." Quoted in (Weyl 1953, 535).

[22]"Die Grundrichtung dieses symbolischen Weges ist uralt, arachaisch, ja geradezu 'prähistorisch': die modernste 'exakte' Wissenschaft wird wieder zur Magie, aus der sie ursprünglich abstammt." (ibid.)

reality. On the other hand, he indicated that one might not necessarily be glad about such a return to the "archaic origins" of science. To express this uneasiness, he quoted E.T. Bell who had criticized the reappearance of numerology in the works of A. Eddington and others. Weyl did not disclose, in this text, where he positioned himself in this respect. It is clear, however, that Becker's joy about the resurrection of archaic metaphysics could not be shared without reservation by those who wanted to uphold an enlightening role for science.

If we take other texts into account, it becomes clearer that Weyl liked Becker's allusion to an *ahnendes Erkennen* (apprehensive cognition) of the transcendent, here in the sense of the quantum reality of a "material agency" (Weyl 1953, Weyl 1924). In his long phase of involvement in mathematical physics he had strongly experienced, on the other side, that an intuitive apprehension as indicated in Becker's text was highly precarious. In a commentary at the turn to the 1930s on the status of unified field theories and quantum phyiscs he had made it clear that the mathematical methods of quantum mechanics only acquired strength and true meaning through their specific connections with the empirical practices of quantum mechanics, in particular those of spectroscopy (Weyl 1931). Thus the connections to the empirical practices had to be taken into consideration, not only for an account of an empiricist philosophy of science, but also for any metaphysical reflection of quantum physics, which would draw proper consequences of the crisis of classical metaphysics.

3. Weyl's symbolic realism

In a text resulting from two lectures given at the *Eranos* meeting 1948, an interdisciplinary meeting at Ascona, Weyl gave a beautiful review of the modern mathematical sciences as an expression of the the symbol producing activity in modern culture (Weyl 1948). In these lectures Weyl compared, among others, the way how relativity theory had transformed classical mechanics with the consequences of the rise of quantum mechanics. Relativity had managed to transform classical mechanical knowledge by a kind of *Aufhebung* in the Hegelian sense, i.e., it revoked and lifted the former notions all at once (Weyl 1948, 339). The classical concepts of space, time, position, momentum, energy etc. had been deeply transformed, but could be related back to the classical ones, once the new stage of knowledge had been established. The classical perspective of a physical reality lying behind the empirical phenomena could now be expressed by fields on Minkowski space or, in general relativity, on a Lorentzian manifold. Classical mechanics was then preserved in a well defined transition of the theoretical structure in the sense of limiting processes.[23]

[23] Velocity $v \rightarrow 0$ in special relativity; in general relativity more involved, but conceptually completely clarified (in years of work by Einstein) by a double limiting process, first a

Such an analysis had been used by F. Gonseth in his dialectical epistemology to which Weyl referred in his lecture (ibid.). Weyl accepted that such a "dialectization" of knowledge in the sense of Gonseth made sense for the transition from classical to relativistic mechanics.

> In the new relativistic picture the original concepts are 'lifted' in Hegel's double meaning of the word. That may be correct in a historical sense; but it would be nothing but a 'historical' dialectic. This is because it is possible to give a completely clear and intuitive description of relativistic space and time, which allows to specify the meaning of the concepts of velocity etc. in the new frame without any reference to the absolute standpoint. (Weyl 1948, 3339f.)[24]

In quantum physics things turned out to be more complicated and open ended, much closer to what Weyl demanded from a dialectical relationship which was more than a "historical" one. In the characterization of the epistemic constellation of quantum mechanics Weyl followed Bohr's analysis of the necessary distinction of a the quantum physical process itself and the description of the measuring process by classical experimental language.

> The case may be different in quantum theory. Here one has to distinguish sharply between the hidden physical process which can only be represented by the symbolism of quantum physics, although it may be referred to by such words as electron, proton, quantum of action etc., and the actual observation and measurement. According to Bohr, we have to talk about the latter in the intuitively comprehensible language of classical physics; or ought we better say: in the language of everyday life? (Weyl 1948, 340)[25]

Weyl had thought about the question whether it was only due to the early stage of development of quantum physics that one had to rely on these

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weak field approximation of Newtonian gravitation, then $v \to 0$. Cf. (Renn/Sauer 1999).

[24]"In dem neuen relativistischen Bilde sind die ursprünglichen Begriffe, in Hegels Doppelsinn des Wortes, 'aufgehoben'. Das mag historisch zutreffend sein; aber es wäre eben doch nur eine 'historische' Dialektik. Denn von der Relativitätstheorie von Raum und Zeit läßt sich eine vollkommen klare und anschauliche Darstellung geben, die, ohne Anleihen bei dem absoluten Standpunkt zu machen, genau bezeichnen kann, welche Bedeutung die Begriffe Geschwindigkeit usw. in dem neuen Rahmen haben."

[25]"Die Sache liegt vielleicht anders in der Quantentheorie. Hier muß man scharf scheiden zwischen dem verborgenen physikalischen Vorgang, der nur durch den Symbolismus der Quantenphysik erfaßbar ist, auf den aber auch mit solchen Worten wie Elektron, Proton, Wirkungsquantum usw. hingewiesen wird, und der tatsächlichen Beobachtung und Messung. Über die letztere müssen wir nach Bohr sprechen in der anschaulich verständlichen Sprache der klassischen Physik; oder sollte man besser sagen: in der Sprache des täglichen Lebens?"

two levels of language. Although he did not share the Copenhagen-Göttingen conviction that quantum mechanics was a "complete theory", an epistemological construct introduced by W. Heisenberg in the battle against critics like Einstein and Schrödinger, Weyl came to the conclusion that the conceptual two levels might probably constitute an unresolvable opposition in quantum physical knowledge, different from the transformation which relativity had achieved for the pre-relativistic concepts.

> It may very well be that we can never dispense of our natural understanding of the world and the language in which it is expressed, perhaps a little purified and enlightened by classical physics, and that the symbolism of quantum physics will never be able to offer a substitute for it. In this case we would have here a true dialectic which cannot be resolved/lifted by any historical development .... (Weyl 1948, 341)[26]

He observed an analogous constellation in the foundations of mathematics, brought about by Hilbert's foundational program. There we need, as Weyl stated in agreement with Hilbert, "signs, real signs, written on paper by the pen or on the blackboard by chalk" (Weyl 1948, 341), not ideas or forms of pure consciousness, but concrete signs, in some material realization. Any attempt to dissolve these material signs by means of a physical analysis of the chalk as constituted by "charged and uncharged elementary particles" would lead to a resolution into quantum physical symbolism. This would obviouosly result in a "ridiculous circle", as Weyl pinpointed:

> ...[T]hese symbols are, in the end, again concrete signs written in chalk on a blackboard. You realize the ridiculous circle ....

Weyl drew the natural conclusion, that this circle can only be avoided

> ..., if we accept the way in which we understand things and people dealing with them in everyday life as an irreducible foundation. (Weyl 1948, 342) [27]

We can see in this argument a slighty ironical allusion to the "vicious circle" which Weyl had struggled with, three decades earlier, in the foundations of mathematics. By this formulation he had denoted, following Poincaré and

[26]"Aber es mag bei alledem doch dabei zu bleiben haben, daß wir das natürliche Weltverständnis und die Sprache, in der dieses Verständnis sich ausspricht, vielleicht eine wenig gereinigt und geklärt durch die klassische Physik, nimmer entbehren können und der Symbolismus der Quantenphysik keinen Ersatz dafür zu bieten vermag. Dann handelte es sich um eine echte, durch keine historische Entwicklung aufzuhebende Dialektik ...."

[27]"...die Symbole aber sind letzten Endes wieder konkrete, mit Kreide auf die Tafel geschriebene Zeichen. Sie bemerken den lächerlichen Zirkel. Wir entrinnen ihm nur, wenn wir die Weise, in der wir im täglichen Leben die Dinge und Menschen, mit ihnen umgehend, verstehen, als ein unreduzierbares Fundament gelten lassen."

Russell, the impredicativity problem in the symbol construction of classical analysis. It had driven him, at that time, towards looking for ground in post-Kantian idealist philosophy. Now it was the reality problem of quantum physics which lead to a *ridiculous circle* in ontology, if one wanted to restrict the consideration to the realm of signs only. While the "vicious circle" in the foundations of analysis could be abandoned by a restriction to constructive practices expressed in a semi-formalized arithmetical language, i.e., inside mathematics proper, the "ridiculous circle" could only be broken if one accepted everyday practices and natural language, not only as a practical basis but even as an "irreducible foundation" of science, as Weyl stated with an incling of Göttingen foundationalism in his choice of words.

When Weyl referred to everyday practices, he did not think of Peircean pragmatism or anything alike. Even after fifteen years of life in the United States he continued to think in terms of European philosophy. In the years of the Second Grat War, and with another deep world crisis as a result of it, Weyl had turned towards the existential philosophy of M. Heidegger and K. Jaspers. In fact, in his Ascona talk Weyl explained how he saw the epistemic situation of quantum physics related to the existential constellation of the modern individual, described by Heidegger in *Sein und Zeit* (Heidegger 1928), "being thrown" into the world, no longer bound by some transcendent power.

Heidegger's philosophizing started from the "Dasein (being there)" of man, awoken to the consciousness of the self, who deals with things in unrefined terms of everyday knowledge. Weyl now described the turn towards scientific knowledge in Heideggerian terms: The latter kind of "objective" or "scientific" knowledge presupposes a radical detachment from everyday knowledge. He remarked that Heidegger had turned from the question of how to "prove" the existence of an objective, external world to the philosophical question of

> ... why the 'Dasein' as a being-in-the world has a tendency to bury the external world in nothingness from the viewpoint of the theory of knowlege, and then to prove it afterwards by indirect argumentation (Weyl 1948, 343f.).[28]

Weyl rephrased the Heideggerian experience of the individual of being thrown without last resorts to a transcendent reality in the following terms:

> After the original phenomenon of being-in-the-world has been suppressed, ones tries to glue the remaining isolated subject to

[28]"Zum Problem der Außenwelt bemerkt darum Heidegger, daß man nicht zu beweisen hat, daß und wie eine Außenwelt existiert, sondern aufzuzeigen, warum Dasein als Sein-in-der-Welt eine Tendenz hat, die Außenwelt erkenntniskritisch ins Nichts zu begraben und dann sie nachträglich indirekt zu beweisen."

> the torn patches of the world; but it remains patchwork . . . (Weyl 1948, 344).[29]

In this way Weyl argued that the disruption of the modern existential condition, Heidegger's "Geworfenheit (being thrown)" is mirrored in the epistemic structure of the modern mathematical sciences.

He was not satisfied with this state of affairs and indicated that the stipulation of a "natural understanding of the world" as it worked in everyday life was in itself highly problematic and worth of "further inquiry" (ibid.). In fact, he continued to think about these questions. In his article on symbolism in the mathematical sciences, quoted already above (Weyl 1953), he extended the perspective to a broader view on cultural philosophies, without negating his allegiance to the existential philosophy of Jaspers and Heidegger.

Now he turned to authors who had dealt with the role of language, signs, and their symbolic function, the representation by signs: W. von Humboldt, H. von Helmholtz, H. Noack, K. Voßler and others. He quoted the first volume of Cassirer's theory of symbolic forms and Wittgenstein's *Tractatus*.[30] Wittgenstein's youth work appeared a bit strange to him, because of its solipsistic perception of language. In contrast to the argumentation deployed in the *Tractatus* he insisted that "the existential origin" and and task of language has to be looked for, at first instance, in *communication* (Weyl 1953, 527).

Weyl characterized the *symbol* as a sign, different in function from names and images by carrying meaning in the scientific or wider communication and as an object that has to be prepared and worked with in intellectual practices. The latter aspect made it comparable to tools in the material practices of artisans. He was glad that mathematics uses written signs reproducible, in principle, without limits.

> Visible configurations of a certain stability are used as signs (rather than sounds or clouds of smoke; at least persistent as is necessary for the execution of certain operations on them).(Weyl 1953, 528f.)[31]

In contrast to an idealist view, which he exemplified at this occasion by a reference to his former ally Brouwer, Weyl sided here with Hilbert and

[29]"Nachdem man das ursprüngliche Phänomen des Seins-in-der-Welt unterdrückt hat, sucht man das zurückgebliebene isolierte Subjekt mit den abgerissenen Weltfetzen wieder zusammenzuleimen; aber es bleibt ein Flickwerk."

[30](Humboldt 1836, Helmholtz 1887, Noack 1936, Vossler 1925, Cassirer 1922, Wittgenstein 1922)

[31]"Als Zeichen dienen sichtbare Gebilde von einer gewissen Beständigkeit (nicht etwa Schälle und Rauchwolken; zum mindesten so lange standhaltend, als zur Ausführung der an ihnen vorzunehmenden Operationen benötigt wird)." Note the slightly alienated allusion to Faust's declaration "*Namen* sind Schall und Rauch (*names* are sound and smoke)" (J.W. von Goethe, my emphasis).

emphasized the tool character of the signs. Here the "concrete activities of people" comes into the play and allows to adopt even an "anthropic" perspective with respect to mathematical knowledge.

> Here the mathematician, with his formulae made up of signs, does not work so differently from the carpenter in his workshop with wood and plane, saw and glue. (Weyl 1953, 529)[32]

Thus in Weyl's latest reflections we find that mathematical symbols are understood within the context of a communicative practice which has strong parallels to material practices, in the way how the symbols are handled, and with multiple links to the other scientific and technical activities. Nevertheless, the main goal of the practices is to establish meaning and semantical connections between them and the world beyond the signs. The signs offer the material for a symbolic [re-]construction of some "objective world", like in relativity and/or quantum mechanics, where the problematic can be most strikingly be studied and exemplified. Here two points are important to realize, according to Weyl:

(i) The symbol is neither taken from "the given (dem Gegebenen)" nor is it a part of the reality represented by it.

(ii) The symbolic construct is *neither* a reality lying at the base of the appearances, *nor* has the bound to the observable been cut.

To make clear that (i) and the first part of (ii) ("neither ...") stand in stark constrast to classical notions, Weyl presented the example of a light ray. C. Huygens "could with good consciousness still say that a monochromatic light ray *in reality* consists of an oscillation of the light ether ...". The modern physicist, on the other hand, represents the ray by a "formula, in which a certain symbol F, called electromagnetic field strength, is expressed by an arithmetically constructed function of four other symbols x, y, z, t " (Weyl 1953, 529). A plane wave solution of the Maxwell equation is obviously a symbolic contruct indicating something in the world, but is neither part nor "lying behind" or "at the base" of the optical or, more broadly, electromagnetic observations.

The symbolic construct is neither arbitrary nor self-relying in its meaning. The second part of (ii) is established in an interdisplinary exchange between the sciences. Weyl argued:

> Of course, the bond between the symbol and the given in the observation need not be cut; the physicist understands how the

[32]"Da geht der Mathematiker nicht viel anders mit seinen aus Zeichen gebauten Formeln um wie der Tischler in seiner Wertkstatt mit Holz und Hobel, Säge und Leim."

symbolism is 'meant', when he confronts the laws expressed in it with his experience. (ibid.)[33]

We see that Weyl tried, as much as he could, to distantiate himself from classical metaphysics, in particular its reference to the kind of transcendent reality as it was stipulated there. He resolutely refused, however, to cut the bonds to all kinds of metaphysics. He rather substituted strong references to symbolical and material practices in place of the old realism.

Moroever, at the core of his mind and heart he remained a believer in some kind of Eckehardt-Fichtean God and, as we added, a Spinozean one, accessible through the experiences of the self and the symbolic cognition of some "objective" or "transcendent" reality established in scientific practices. This point is mainly important for a proper and respectful historical understanding of our protagonist. Weyl's turn towards scientific material practices as the most important base for the realism inherent in symbolic knowledge, has to be considered the essential feature of his mature and late work. It seems justified to use the denotation *symbolic realism* for such an approach. We may have reasons to relate to it in our own reflection and work.

4. 'Symbolic realism' as an ingredient of a trans-modern philosophy of practice

In his symbolic realism, Weyl insisted upon *meaning* acquired in the complex context of scientific, technical and social practices, as an important ingredient of mathematical knowledge. It is is not inscribed uniquely, ex-ante, and forever in the symbolical knowledge of the mathematical sciences. Just to the contrary, this meaning is constituted in a permanent process of elaboration, communication, and usage of mathematical knowledge. It is multi-faceted, in enduring change, and often arrived at ex-post, i.e., long after corresponding mathematical structures have been established and studied as such.

Such a view does not lead to arbitrariness and allows to avoid a formalist or neo-positivist reduction of mathematics. The endowment of meaning to symbolic knowledge and its diverse possible usages is deeply bound to goal oriented practices of science, technics, education, sometimes arts, and culture more broadly. In this way, it is embedded in a social and cultural discourse on values which should be part of a *philosophy of practice*. That goes well with radical agnosticism in questions of *fundamental* ontology, inherited from the criticism of the enlightenment and its modern continuators.[34]

Such a philosophy of practice can stand to its metaphysical components, withoug being in danger to fall back into an unbroken, or even naive con-

[33]"Natürlich braucht darum das Band zwischen Symbol und wahrnehmungsmäßig Gegebenem nicht durchschnitten zu werden; der Physiker versteht, wie der Symbolismus 'gemeint' ist, wenn er die in ihm niedergelegten physikalischen Gesetze an der Erfahrung prüft."

[34]The discussion of domain specific "ontologies" is another question.

tinuation of traditional metaphysics which was driven into crisis and even into dissolution, with good reasons, by modernity. In this sense I propose to consider it as a *trans-modern* philosophy which is in the making. This term must not be confused with the label "postmodern/postmodernity" which has completely different meaning and connotations. It would go far beyond the goals of this contribution to fully discuss the connotations intended in this qualification as "trans-modern". It may suffice here to indicate that I intend this expression to include a conscious reference to the criticism of techno-scientific practices of high modernity, as spelt out by I. Illich in his quest for *conviviality*.[35]

Ivan Illich (1926 — 2002) attempted to shatter the naive belief in the conception of a uni-directional "progress" in modern society based on elaborate techno-scientific practices and industrial systems as such. He reminded that non-modern practices may be comparable, sometimes and in many places even preferable for a convenient satisfaction of social and cultural needs. He did not call for abandonment of technoscientific-industrial practices en gros, but demanded their critical evaluation and reorientation according to their potentials to contribute to achieve human goals in nature ("conviviality").

Surely, Illich's criticism of the conditions of life in the modern sector of our emerging world society went deeper than Weyl's and was written from the background of having lived through large parts of his life in another segment of the world than Weyl, and a generation later, but there is is no reason to dismiss such considerations as foreign to the Weylian perspective. H. Weyl experienced the cultural development and the social, military, and technical history of the first half of the 20th century as a deep and multiple crisis. He perceived the outcome of the second world war as a dangerous constellation for mankind as a whole. Like other scientists of the post-second-war period, he considered the development of nuclear weapons to be the watershed of a development of modern technoscience into a stage in which it started to have at least as much destructive powers, as it could serve as a potential for conviviality (an "improvement of the conditions of life" in more classical terms).

In the manuscript for a talk on *The development of mathematics since 1900*, given about 1949, Weyl warned that the seemingly abstract and detached knowledge of the mathematical sciences may have contributed to give so strong powers to human society that in a kind of revenge of the "Gods" it might lead to self-destruction, rather than to an improvement of the conditions of life. He referred here to a passage of Aristotle's metaphysics in which the quest for a kind of (metaphysical) "pure" knowledge, distached from human goals, is discussed, which is of use only for the gods themselves. Such a "stepping beyond" might be considered as a self-adulating conceit, comparable to what the ancient Greek called "hybris" and expected

[35](Illich 1973)

to be sanctioned by a "revenge of the gods".[36] Weyl transferred Aristotles' warning from metaphysics to mathematics and the mathematical sciences and argued strongly in favour of a symbolic knowledge which is aware of its cultural connectors and practical meanings.

> ... I am not so sure whether we mathematicians during the last decades have not 'stepped beyond' the human realm by our abstractions. (...) For us today the idea that the Gods from which we wrestled the secret of knowledge by symbolic construction will revenge our $\upsilon\beta\rho\iota\varsigma$ [hybris] has taken on a quite concrete form. For who can close his eyes against the menace of our self-destruction by science; the alarming fact is that the rapid progress of scientific knowledge is unparalleled by a congruous growth of man's moral strength and responsibility, which has hardly chance in historical time. (Weyl Ms 1949*a*, 7, English in original)

Another half a century later, deeper inside the transition to a world-wide society stricken by unjust social divisions of power, labour and resources, the nuclear menace persists, mitigated only slightly and probably only temporarily. In addition to it, much broader and multifarious corridors of destructive practices in nature and society endanger a decent and enduring development of humanity in our terrestrial mesocosmos. In this context, *our context*, it may be more than useful to take up Hermann Weyl's thoughts on a symbolical realism for the mathematical sciences and to fuse them with Ivan Illich's challenge to reorient *all* our practices in accordance with conviviality.

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[36]Aristotle: Metaphysics 982b. Cf. W.D. Ross' translation in (Aristotle 1984). Weyl's own translation of Aristotle's hybris warning was sharper than the one by Ross, compare the quote in (Scholz 2002).

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