LEIBNIZIAN TRACES IN H. WEYL'S *PHILOSOPHIE DER MATHEMATIK UND NATURWISSENSCHAFT*

ERHARD SCHOLZ †

1. Introduction

After a phase of radical mathematical innovations between 1918 and 1925, often with strong repercussions in physics (from foundations of analysis, via general relativity, differential geometry and unified field theory to the representation theory of Lie groups), Hermann Weyl turned toward writing his contribution *Philosophie der Mathematik und Naturwissenschaft* on the philosophy of mathematics and natural sciences, in the sequel abbreviated PMN (Weyl 1927, Weyl 1949), for the *Handbuch der Philosophie* (Baeumler/Schroeter 1927). It was a time of reorientation for him with regard to foundations of mathematics and to the question how mathematics may contribute to the understanding of the external (natural) world. The phase of his most radical interventions into the foundations mathematics in a constructivist perspective from 1916 to 1919 and an intuitionist one, 1919 to 1922, was just lying behind him. Likewise a period was closed (1918 to 1922), in which he was convinced to be able to unify the two most recent pillars of mathematical physics, Einstein's geometric theory of gravity (general relativity) and Hilbert's attempts for a of a dynamistic field theoretic explanation of matter ("foundations of physics", Mie-Hilbert theory). Weyl proposed his *purely infinitesimal geometry*, a generalization of Riemannian geometry, and used it for formulating a geometrically unified field theory, the first in a series of attempted unified classical field theories which followed (Vizgin 1994, Goenner 2004, Goldstein 2003). His turn towards the study of "infinitesimal symmetries" during this work brought him into the research in the representation of Lie groups (1923 to 1925) which are generally estimated as the most important mathematical research work of his whole career (Hawkins 2000).

After this outburst of scientific activity for about eight years, which was already deeply permeated by philosophical motivations, Weyl took the task of writing his *Handbuch* article as a chance for rethinking much of his earlier philosophical convictions. Later he would like to talk about this kind of reflection by using the good old German word

† scholz@math.uni-wuppertal.de
University Wuppertal, Department C, Mathematics and Natural Sciences, and Interdisciplinary Center for Science and Technology Studies

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Besinnung which denotates a contemplative kind of reflection rather than an analytic one (Weyl 1954). In this sense, he allowed himself an interlude of holding back own thoughts and activities with regard to the "new quantum mechanics" of Heisenberg, Jordan, Born and Dirac, although he was well aware of it and started to have some forward pointing ideas about it (Scholz 2006). For more than half a year, between summer 1925 and early 1926, he delved into reading philosophical literature far beyond his earlier interests in this field, which had been concentrated around the philosophies of Kant, Poincaré, Mach, Husserl, Fichte, listed in this time-order, which lived from his close communication with his wife Helene, a Husserl scholar, and Fritz Medicus, a Zürich expert in the philosophy of German idealism and a personal friend of his (Sieroka 2007, Sieroka 2008, Sieroka 2009*a*). Apparently he used the chance to carefully read in Leibniz' works, among others, and to quote them extensively. H. Breger observed already years ago that, surprisingly, Leibniz became the by far most frequently quoted author in Weyl's PMN, with 79 entries, against the next ones, Kant: 43, and Newton: 41 (Breger 1986). At no other occasion, neither earlier nor later, Weyl ever referred so strongly and explicitly to Leibniz as a philosopher.

It has to be added that Weyl did not aim at a systematic exposition of Leibniz's thought (nor did he with any other philosopher). He rather used the references to philosophers' works interwoven into the development of his own ideas on the philosophy of the mathematical sciences. This did not have merely the function of ornaments by classical text fragments, but rather served the purpose of an examination of Leibnizian thoughts and a debate of them in the light of more recent developments in the object sciences considered by him.

This paper does not pretend to give anything like a systematic evaluation of Weyl's way of presenting Leibniz either. That would be a task for a professional philosopher (if that were attractive to her). Coming from a background in the history of mathematics, I just want to present those aspects which apparently made Leibniz so important for Weyl in the middle of the 1920s.

2. General reflections on modern mathematics

Weyl's own position with regard to the foundations of mathematics and the recent developments in mathematical logic, axiomatics and set theory was still shaped by a constructivist perspective with strong intuitionistic sympathies (Feferman 2000, van Dalen 2000, Sieroka 2009*a*). For a general exposition of the philosophy of mathematics to a broader audience he had to express himself more balanced than he used to do the years before (Hesseling 2003); and in fact he wrote a short first chapter in PMN on mathematical logic and axiomatic, in which Leibniz appeared as a figure of peripherel reference, not much more.[1]

Weyl assigned a more interesting role to Leibniz in his discussion of *modern axiomatics*. After introducing the modern axiomatic method and the role of models for investigating consistency and independence of an axiomatic system, with full acknowledgement of Hilbert's "ingenious construction of suitable arithmetical models" (Weyl 1949, 22), Weyl presented an axiomatically founded mathematical theory as a "logical mould (Leerform) of possible sciences" (ibid. 25), a formulation which he liked and repeated frequently. That gave him the chance to call upon Leibniz as an inspirator of a development in this direction:

> Leibniz takes some decisive steps towards the realization of *mathesis universalis* in the sense here indicated and clearly understood by him. The theory of groups above all, that shining example of 'purely intellectual mathematics' belongs within the framework of his *ars combinatoria*. (Weyl 1949, 27)

This quote shows also that Weyl did not intend a historical reconstruction of Leibnizean thought, but rather read him in a presentist perspective (group theory as part of *ars combinatoria* etc.).

In his discussion of *number* (natural, rational) and *continuum* (real numbers), Weyl of course presented Dedekind cuts and nested interval constructions of the reals, but did not withhold his constructivist sympathies with respect to the ontology of such infinities. The determination of localizations in a continuum stood, for Weyl, in the tension between "the real" and "the ideal" and could be understood as paradigmatic for gaining (ideal) knowledge of (real) things. He insisted that a "real thing" can never be given, but has to be "unfolded" by an infinitely continued process (here he referred to Husserl's "inner horizon").

> For this reason it is impossible to posit the real thing as existing, closed and complete in itself. (Weyl 1949, 41)

In this contect, the concept of continuum was pivotal in driving "toward epistemological idealism". Here he could again cite:

[1]For instance, Weyl quoted from Leibniz's letters to Clarke:

> Leibniz speaks of a '. . . relation between L and M, without consideration as to which member is preceding or succeeding, which is the subject or object. . . . One cannot say that both together, L and M, form the subject for an *accidens*; . . . It must be said, therefore that the relation . . . is something outside of the subjects; but since it is neither substance nor *accidens* it musts be something purely ideal, which is nevertheless worthy of examination.' (Leibniz 5th letter to Clarke, §47) (Weyl 1949, 4f.).

> Leibniz, among others, testifies that it was the search for a way out of the 'labyrinth of the continuum' which first suggested him the conception of space and time as orders of the phenomena. 'From the fact that a mathematical solid cannot be resolved into primal elements it follows immediately that it is nothing real but merely an ideal construct designating only a possibility of parts'. (Correspondence Leibniz de Volder, Leibniz Phil. Schr. II, p. 268). (Weyl 1949, 41)

Referring to Leibniz's introduction of *monads* as an attempted path towards giving a metaphysical foundation to the world of phenomena, Weyl continued with a Leibnizian argument on the continuum:

> 'Within the ideal or the continuum the whole precedes the parts ... The parts are here only potential; among the real (i.e. substantial) things, however, the simple precedes the aggregates, and the parts are given actually and prior to the whole ... '(letter to Remond, Phil. Schr. III, 622) (Weyl 1949, 41)

Such a view of the continuum as a whole which had to be stipulated in the intuition, rather than being postulated formally by means of transfinite set theory, was particular close to his own semi-intuitionist understanding of the continuum (Breger 1986). It prepared the way to a discussion of the two controversial points of view (set theory versus intuitionism) in the next two subsections of his book.

3. Hilbert's foundational program in the light of symbolic mathematics

In his PMN Weyl discussed both, Brouwer's intuitionist and Hilbert's formalist program, for the foundation of mathematics. Although he still sympathized with the intuitionist perspective, he was quite clear that the loss of the principle of excluded middle was akward for mathematics. He ended this passage by an often quoted remark:

> And the mathematician watches with pain the larger part of his towering edifice which he believed to be built of concrete blocks dissolve into mist before his eyes. (Weyl 1949, 54)

That was a positive motif for turning toward Hilbert's foundational program which Weyl called *symbolic* rather than formalist mathematics. He discussed Hilbert's foundational enterprise for arithmetics relatively open-minded, including references to recent progress made by J. von Neumann (Weyl 1927, 49), in the later English edition also to P. Bernays and W. Ackermann (Weyl 1949, 60). As long as finite sequences of proof derivations were considered only, everything worked well and was acceptable also from the intuitionist standpoint. But clearly Hilbert's proof theory aimed at more. For this reason he introduced a specific transfinite logical axiom rule, which should suffice to safeguard the transfinite parts of set theoretic mathematics, so Hilbert hoped, by a purely logical analysis of the deductive structure it allowed for. In 1926 Weyl remarked that von Neumann had recently shown the consistency of those parts of mathematics which treats the "series of all natural numbers as a closed totality of existing objects", comparable to his own point of view in *Das Kontinuum* of 1918 (Weyl 1918), i.e., as long as countable transfinite sets are concerned. Weyl added that the more complicated case of the transfinite dealing with "the totality of all possible sets of numbers", i.e., the uncountable transfinite, was still wide open.

> Only the realization of the consistency proof, or at least the attempts at it, disclose to us the highly sophisticated (verzwickt) logical structure of mathematics, its maze (Gewirr) of circular back references which do not allow to survey whether they might not lead to blatant contradictions. (Weyl 1927, 1st. ed., 49)

In the English translation more than twenty years later, he felt no need to soften his argument; rather to the contrary he reminded the reader of Gödel's results in 1931 which, according to Weyl, "precipitated a catastrophe" for Hilbert's proof theoretic program (Weyl 1949, 61).

But that was not even decisive for Weyl's view of the achievements of Hilbert's approach. Leibniz reentered Weyl's reflections, perhaps to relativize both, Hilbert's achievements and Weyl's own former sceptical reaction to it.

> The described symbolism evidently attacks again, in a refined form, the task which Leibniz had set himself with his 'general characteristics' and *ars combinatoria*. But is it really more than a bloodless ghost of the old analysis that confronts us here? Hilbert's mathematics may be a pretty game with formulas, more amusing even than chess; but what bearing does it have on cognition, since its formulas admittedly have no material meaning by virtue of which they could express intuitive truth? The subject of mathematical investigation, according to Hilbert, is the concrete symbols themselves. (Weyl 1949, 61)

The "old analysis" of Leibniz and others had still developed a symbolic enterprise which aimed at a better understanding of the relationships and laws of nature. For Weyl, the "symbolic" character of mathematics contained more than just its formal side. He adressed the reader:

> . . . This last remark reminds us that it is the function of mathematics to be at the service of the natural sciences . . . (ibid.).

For Weyl, the "symbolic" consisted of more than the syntactical structure; it aimed at more than a formalist game, and its justification presupposed more than a formal analysis of consistency. It ought, according to Weyl, "furnish" knowledge, and knowledge contained somehow a claim for "truth".

> It seems that we have to differentiate carefully between phenomenal knowledge and insight . . . and theoretical construction. Knowledge furnishes truth, its organ is 'seeing' in the wides sense. Though subject to error it is essentially definitive and unalterable. Theoretical construction seems to be bound only to one strictly formulable rational principle, that of concordance (. . .) which in mathematics (. . .) reduces to consistency. In connection with physics we have to discuss in greater detail the question. (Weyl 1949, 61f.)

The situation changes, if on turns toward physics and the role of mathematics in physical knowledge. Then more than mere consistency is at stake and mathematics acquires a specific role in a "symbolical representation" of material objectivity, the "transcendent", as Weyl liked to say in counterposition to the "immanent" cognitive reality of the symbols.[2]

4. A dialogue partner for understanding modern physics

In his passage on physical questions in PMN Weyl referred to Leibniz at different occasions: purely infinitesimal ("near") geometry, orientation of space and time, matter, and the topical complex of causality, law, chance, and freedom.

We saw already that Weyl's turn towards PMN happened after several years of great activities in the mathematical sciences and at the end of a phase of changing perspectives. In 1918 he had invented and proposed his "purely infinitesimal geometry", today one would call it a *scale gauge geometry* which, in contrast to Riemannian geometry, excluded the possibility of a direct comparison of metrical quantities at different points in the manifold. Beginning in 1920 he gave up, step by step, his immediate hope for a geometrically unified field theory of electromagnetism and gravity and a Mie-Hilbert type dynamistic matter theory built upon it. But this did not mean a withdrawal of the conviction that the geometrical invention of a "purely infinitesimal" scale gauge geometry was justified and continued to be valuable. Between

[2]More in (Scholz 2005*b*).

1921 and 1923 he developed a philosophically founded, but mathematically formulated program of a conceptual foundation of a most general metrical infinitesimal geometry, with some Kantian inklings, the "mathematical analysis of the problem of space"(Weyl 1923).[3]

In 1926 Weyl quoted Leibniz as supporter and advisor

> As the true lawfulness of nature, according to Leibniz's continuity principle, finds its expression in laws of nearby action, connecting only the values of physical quantities at space-time points in the immediate vicinity of one another, so the basic relations of geometry should concern only infinitely closely adjacent points ('near- geometry' as opposed to 'far-geometry'). Only in the infinitely small we may expect to encounter the elementary and uniform laws, hence the world must be comprehended through its behaviour in the infinitely small. (Weyl 1949, 86)

In 1918 he had other, in particular field theoretic reasons to demand such a perception of geometry, but in any case such a view stood in good agreement with a Leibnizian perspective, which could be appealed for in order to foster such a view in geometry.

In his discussion of the relativity of space and time, Weyl shortly mentioned the respective views of Aristotle, Descartes, Galilei and Leibniz, while he discussed the position of Newton, "the absolutist", at length. To illustrate the relationist position of Leibniz he quoted from the third letter to Clarke:

> 'Under the assumption that space be something in itself, that it be more than merely the order of bodies among themselves, it is impossible to give a reason why God should have put the bodies (without tampering with their mutual distances and relative positions) just at this particular place and not somewhere else; for instance, why He should not have arranged everything in the opposite order by turning East and West about. If, on the other hand, space is nothing more than just the order and relation of things, if without the bodies it is nothing at all except the possibility of assigning locations to them, then the two states supposed above, the actual one and its transposition, are in no way different from each other. Their apparent difference is solely a consequence of our chimerical assumption of the reality of space itself. ...' (Weyl 1949, 97)

Weyl compared this view of Leibniz with Kant's famous argument for the transcendental ideality of space (*Prolegomena* §13 etc.) and sided

[3]Cf. (Scholz 2004*b*)

with Leibniz (Weyl 1949, 80, 97). He illustrated the point of difference by a striking metaphysical thought experiment. He proposed to assume that a "left hand", i.e. a specific chiral spatial object, was produced in "the first creative act of God". Kant's position would be that this would have introduced already the character of a "left" object, rather than a "right" one, even without any other object of comparison. This was a result of Kant's realization that, at his time, left- ore right-handedness were not definable conceptually, but could be discerned by pure intuition only. Moreover he seemed to assume the transcendental subject of "pure intuition" as timeless as God herself. Weyl did not accept this as a valid argument. According to him, only the comparison of the first object ("left hand") with a chiral object brought about in a "second act of creation" would allow to make any distinction at all:

> He [God, ES] would have changed the plan of the universe not in the first but in the second act, by bringing forth a hand which was equally rather than oppositely oriented to the first-created one. (Weyl 1949, 97, footnote)

Translated into mathematical terminology: *Pure space* being assumed orientable (at least locally), God's "first creative act" would select an orientation. Only after that, in the "second creative act", it makes sense to ask for a locally defined, oriented object to coincide or to break the orientation selected in the first step.[4]

In the discussion of the causal structure of relativity theory and its importance for the concept of time order, Weyl made an illuminating excursion to Leibniz again. He started to explain the modern (Einsteinian) causal order:

> Likewise any event happening at O has influence only upon the events at later world points; the past cannot be changed. That is to say, the stratification [of past and future and simultaneity] has a causal meaning; it determines the *causal connection of the world.* (Weyl 1949, 101)

Without any recontextualization he continued:

[4]Indirectly this metaphor may even shed light on the problem of symmetry break (also between matter and anti- matter) in the "early universe", discussed in modern elementary particle physics. Philosophically reflected people need not adhere to an interpretation of the "early universe" in the sense of scientific realism, which dominates the imagination of present mainstream physics so strongly, but may take it as what it is: speculative metaphysics in scientific guise. If "generation of the world" is not understood in a quasi natural-historic sense, but as *structural genesis* with only indirect relation to timelike developments in the material world, "God's first creative act" may be read as a rhetorical figure for the first step of (conceptual) structure generation in the formation of our scientific world picture.

> This was recognized by Leibniz, who explains in his 'Initia rerum mathematicarum metaphysica' (Math. Schriften VII, p. 18). 'If of two elements which are not simultaneous one comprehends the cause of the other, then the former is considered as *preceding*, the latter as *succeeding*.' (ibid.)

Weyl thus invoked Leibniz just as if he were a contemporaneous (or time-less) dialogue partner who could be asked for advice in questions pertaining to most recent modern physics.

With respect to the concept of matter Weyl was, of course, fond of Leibniz's dynamical (quasi "active") characterization of the latter.

> Leibniz (opposing Descartes) has emphatically stressed the dynamic character of inertia as a tendency to resist deflecting forces; for instance, in a letter to de Volder (Philosophische Schriften, II, p. 170) he writes, 'It is one thing if something merely retains its state until some event happens to change it – a circumstance which may occur if the subject is completely indifferent with respect to either state; it is another thing and signifies much more if the subject is not indifferent but possesses a power, an inclination as it were, to retain its state and to resist the cause of change.' (Weyl 1949, 105)

After his own failed attempt to unify forces and the hope for a (Mie-Hilbert type) dynamist derivation of matter structures, Weyl returned to a more cautious position. In 1926 he spoke of an unreducible duality of matter and (interaction) field, at least for the moment, and referred to Newton as "entirely dominated by this dualism". Of course this could not be convincing for him, and he used Leibniz as an early protagonist of the contrasting position:

> The classical philosopher of the dynamical conception of the world is, however, Leibniz. To him, what is real in motion does not lie in the change of position as such, but in the moving force. 'La substance est un être capable d'action, une force primitive' transspatial and immaterial. ...
> ... The ultimate element is the *monad*, an indecomposable unit without extension, from which the force bursts forth as a transcendental power. Only with regard to the distribution of the monads in space, which itself is merely a *phaenomenum bene fundatum*, is the body described as an *extended* agent. Pure activity, however, is all; preestablished harmony takes the place of such reciprocal effects as we think are carried by the field from particle to particle. (Weyl 1949, 174)

Weyl then explained the field actions of matter in general relativity and expressed the hypothesis that mass is established by the "flux of the gravitational field which a particle sends through an enveloping shell" (p. 175). He even indicated his idea that, maybe, matter is lying "beyond" the singularities of spacetime strucutre.

> Indeed general relativity does not prescribe the topology of the world, and it may therefore happen that the world has unattainable 'fringes' not only toward the infinite but also inwardly. In line with Leibniz's idea, the material particle, although embedded in a spatial environment from which its field effects take their start, would itself then be a *monad* existing beyond space and time. (Weyl 1949, 175)

Weyl continued with a reinterpretation of his own in the context of modern field theory. There charge cannot be localized pointwise; it is rather given by a density, such that integrals over closed surfaces indicate that the latter surrounds a charge. Comparable to a neokantian view, the splitting of space and time was for him a question of the subject, but unlike Kant, no transcendental one, but an empirical one, bound to matter.[5]

He made a surprising move from Leibniz, via his own thoughts about a matter concept compatible with general relativity, to Schelling:

> Schelling, partially under the influence of Leibniz has expressed ideas which vaguely anticipate this development [toward a general relativist concept of matter in the Weylian sense, ES]. 'Thus there ought to be discernible in experience something', he says on p. 21 of his 'Erster Entwurf der Naturphilosophie' (1799; *Sämtliche Werke*, III, p. 21, Cotta, 1858) 'which without being in space, would be principle of all spatiality.' This 'natural monad' is not itself matter but action, 'for which there is no measure but its own product.' (Weyl 1949, 176).

Weyl continued with a sketch of Schelling's "construction" of continuous spacefilling matter, "a shapeless fluid – which we today would replace by the field." Once one started to consider most recent structures of mathematical physics frome the point of view of this Schellingian scheme, one could also transfer it one stage further, replacing classical fields by quantum fields ("product of activity"). The activity behind the (quantum) field, could just as well be characterized metaphysically by "natural monads", a mathematical representation of which would

[5]Weyl talked about "...the geometrico-physical basis for the splitting of the world into space and time which takes place within our consciousness, tied as it is to a material body" (Weyl 1949, 176).

have to be a quantum agens structure beyond space and time, giving rise to a spacefilling "shapless fluid", expressed on different levels by quantum fields, semiclassical fields or, in the classical limit, by classical fields (Sieroka 2009*b*). In any case, we can here see a clear impact of Leibnizean thought on Weyl's agency concept of matter (Sieroka 2007, Scholz 2004*a*).

Finally with respect to the challenging relationship between *causality/law* on the material side of the world and *freedom/purposiveness* on the humanistic side of the world, with the grey area of *chance* in between, Weyl developed a clarifying view of his own. Again he did not abstain from allusions to classical positions in philosophy, among others to Leibniz. After a short review of diverse positions, even including premodern world views, Weyl hinted at the contraposition between Hobbes and the latter's "first consistent modern theory of determinism in which natural law appears as the binding force" (rather than God's predetermination, Kismet etc.) and Descartes who "clung to the freedom of will, and (...) had to do so if the self-certainty of thinking guarantees truth as demanded by the principles of his philosophy" (Weyl 1949, 208). The contradiction between lawful determination in the realm of *res extensa* and self-determination according to clear ideas in the *res cogitans*, arising from the Cartesian approach, remained a philosophical evergreen in philosophy of the modern era.

Weyl explained:

> Two quotations from Leibniz may be given here. In an essay on freedom (*Lettres et opuscules inédits de Leibniz*, ed. Foucher de Careil, Paris 1854, p. 178 et seq.) he states 'that there may, or even must, be truths which no analysis can reduce to the identical truths or to the principle of contradiction, which, on the contrary, require an infinite series of reasons for their support; a series which is transparent to God only. And this is the essence of all that one considers free and accidental.' Further in his *Monadology (Philosophische Schriften*, VI, pp. 607–623; Section 79): 'The souls act according to the laws of final causes through appetences, means and ends. The bodies act according to the laws of efficient causes or motion. And these two realms, of final and efficient causes, are in mutual harmony.' (Weyl 1949, 209)

Between these and many more short and striking characterizations of contributions to this topic by philosophers and scientists of the last three centuries, Weyl framed his own view. The recent scientific insights into the unreducible stochastic features of "atomic events", (Weyl 1920), (Weyl 1949, 198), and into the causal structure of relativity, in which the causal future of an event x is not completely determined by its causal past, because the causal past of any future point is strictly wider than that of x (Weyl 1949, 210), undermined or even dissolved, according to Weyl, the "antinomy" of "knowing and being", which had been so acute in the early and classical modern period. Among those who had been struggling with this antinomy was Kant whose argumentation, how freedom of will was reconcilable with classical (Laplacian) determinism, could not achieve a convincing solution.

> Kant, according to the scientific situation of his time, agrees with this view as far as the world of space-time phenomena is concerned [classical determinism, ES], and he tries, by distinguishing between the phenomenal and the intelligible world, to give a transcendental solution of the conflict between natural causality and freedom of will. His solution, however, can hardly be carried through consistently and even remained obscure to himself to such a degree that he was unable to understand the changes in the character of a person. (Weyl 1949, 210)

Weyl did not claim to have a definite solution. As a philosophizing scientist, he looked for answers in better understanding of the scientific base of human nature, although he saw clearly that the scientific knowledge in biology and psychology was not sufficiently far developed to allow anything like a convincing scientific treatment of the question. He was critical of contemporary *vitalist* answers to these questions, expressed, e.g. by Hans Driesch who, by the way, was his coauthor in *Handbuch der Philosophie* with an essay on *Metaphysics of Nature* (Driesch 1927).

> All these questions as to the essence of life and the possibility of spontaneous generation are premature and must rest until the day when the laws of life will be known to us to a much wider extent. (Weyl 1949, 215)

On the other hand future progress of the natural sciences, in particular biology and psychology, might lead to a deeper understanding of how the *open* lawfulness of the natural world, which expressed itself already on the foundational level by the stochastic nature of physical laws, may go in hand with organization, life, and even purposiveness of the soul and the intellect. For the moment, the former was perfectly consistent with the latter, but far from being able to "explain" it. Although thus, according to Weyl, the "body-soul problem" still belonged to the class of riddles which were unsolvable at the time, he was optimistic in principle:

> I do not believe that insurmountable difficulties will be encountered in any unprejudiced attempt to subject the entire reality, which undoubtedly is of a psycho-physical

> nature, to theoretical construction – provided the soul is interpreted merely as the aggregate of the real psychic acts in an individual. It is an altogether too mechanical conception of causality which view the mutual effects of body and soul as being so paradoxical that one would rather resort, like Descartes, to the occasionalistic intervention of God or, like Leibniz, to a harmony instituted at the beginning of time. (ibid.)

At the end of his PMN, Weyl came to the conclusion that there was a "general agreement regarding the most essential insights of natural philosophy as it is found among all those who approach the problem seriously and with a free and independent mind rather than in the light of traditional schemes", regardless of whether their background was in philosophy, in the sciences, or in mathematics. His own contributions had their "firm foundation in the first, mathematical part". He opened the last paragraph with an appeal:

> Exact natural sciences, if not the most important, is the most distinctive feature of our culture in comparison to other cultures. Philosophy has the task to understand this feature in its peculiarity and its singularity. (Weyl 1949, 216)

Leibniz had adressed this task, in his time and his way, in a highly elaborate and productive way. Weyl did so at a critical stage of the development of modern science in the early 20th century.

5. In place of a conclusion: Weyl's Leibniz

Philosophical considerations are difficult to resume. Let us, nevertheless, try a short glance back: Like any other of the scientists or philosophers of the 19th or 20th centuries Weyl adapted Leibniz to his own perspective. We have have seen this effect in topics like

- *mathesis universalis* realized in modern axiomatic mathematics,
- *characteristica generalis* and *ars combinatoria* realized in Hilbert's foundational approach to mathematics,
- *Ausdehnungslehre* and vector calculus considered as a variant of *analysis situs*,
- the discussion of the *relativity* of space,
- causality and *time order* in modern physics,
- and the *dynamical* character of inertia, including Weyl's own *agens theory of matter*, supported and upgraded by Leibnizian fragments.

This should not be read too critically. Weyl knew clearly what he did; he frankly admitted:

> In conclusion I want to emphasize once more that it has not been my intention to write a history of philosophical

> thought within the natural sciences. This would require much more comprehensive historical studies . . . [reference to Lasswitz and Cassirer, ES] . . . Primarily interested in mathematical research, I am wanting, in both time and love, for such work. (Weyl 1949, 216)

Moreover, we found three topics with an indirect or even a direct influence of Leibnizian motifs on Weyl's own work in mathematics or mathematical physics. The latter's view of the *continuum* concept and on *near*, or "purely infinitesimal", *geometry* carried Leibnizian traces (Breger 1986); although they were apparently indirectly transmitted by Weyl's reading of Husserl and Fichte after 1916 (Ryckman 2005, Scholz 2000, Scholz 2005*a*, Sieroka 2007, Sieroka 2008).

Weyl's transition from a field theoretic dynamistic concept of matter to his *agency theory of matter* was triggered by problems inside the mathematical "construction" of empirically adequate matter structures in the frame of classical field theories; but its reflection and its connection to wider philosophical topics seems to be enriched by our protagonist's intense studies of Leibniz during 1926, in additon to that of Fichte already in the years before (Sieroka 2007, Sieroka 2008, Scholz 2004*a*).

Finally Weyl's mature understanding of the nature of mathematics and its role in acquiring knowledge on the outside world, which, in lack of a better label, I have called *symbolic realism* elsewhere (Scholz 2005*b*), was apparently supported by his reading of Leibniz's metaphysics and the role of mathematics in it. In view of all this and in spite of all necessary caution, we may finally conclude that Leibnizian traces are to be found not only in Weyl's PMN. Some of them seem to have left perceivable, although not spectacular imprints on Weyl's work as a mathematical scientist.

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