Quantum gravity from the point of view of locally covariant QFT

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[1] Based on the joint work with Klaus Fredenhagen and Romeo Brunetti

Outline of the talk

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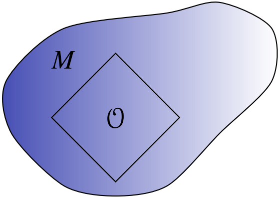

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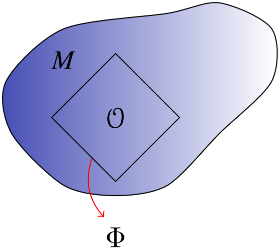

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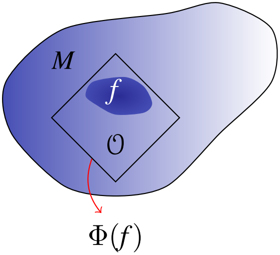

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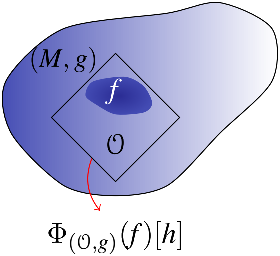

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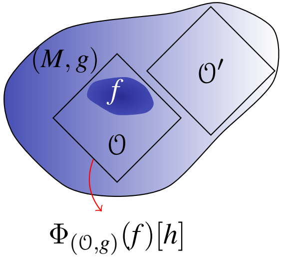
- Diffeomorphism transformation: move our experimental setup to a different region $\mathcal{O}'$.



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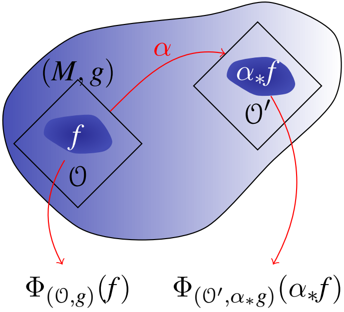

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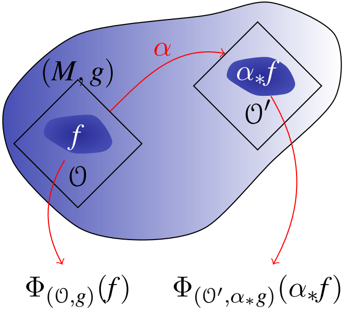

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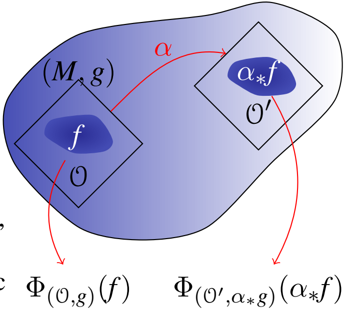

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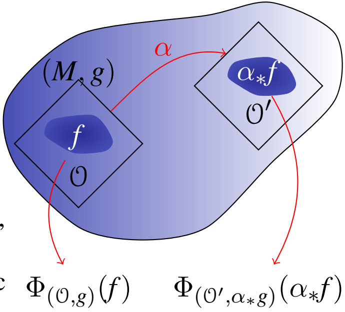
  - **Vec** with (small) topological vector spaces as objects and injective continuous homomorphisms of topological vector spaces as morphisms.



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- We define a contravariant functor $\mathfrak{E} : \mathbf{Loc} \to \mathbf{Vec}$, which assigns to a spacetime the corresponding configuration space and acts on morphisms $\chi : \mathcal{M} \to \mathcal{N}$ as $\mathfrak{E}\chi = \chi^* : \mathfrak{E}(\mathcal{N}) \to \mathfrak{E}(\mathcal{M})$.

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- In a similar way we define a covariant functor $\mathfrak{E}_c$: **Loc** $\to$ **Vec** by setting $\mathfrak{E}\chi = \chi_*$, where:

$$\chi_* h \doteq \begin{cases} (\chi^{-1})^* h(x) & , \quad x \in \chi(M), \\ 0 & , \quad \text{else} \end{cases}$$

Functionals and dynamics

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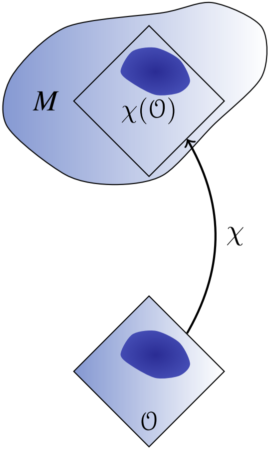
- For GR the action takes the form:

$$S_{(M,g)}(f)[h] \doteq \int R[\tilde{g}]f \, \mathrm{d}\, \mathrm{vol}_{(M,\tilde{g})}, \quad \tilde{g} = g + h.$$

Fields as natural transformations

- In the framework of locally covariant field theory [Brunetti-Fredenhagen-Verch 2003], fields are natural transformation between certain functors. Let $\Phi \in \mathrm{Nat}(\mathfrak{D}, \mathfrak{F})$, where $\mathfrak{D}$ is the functor of test function spaces $\mathfrak{D}(\mathcal{M}) = \mathcal{C}_c^\infty(M)$ (one could substitute $\mathfrak{F}$ with a functor to the category of Poisson or C^*-algebras).



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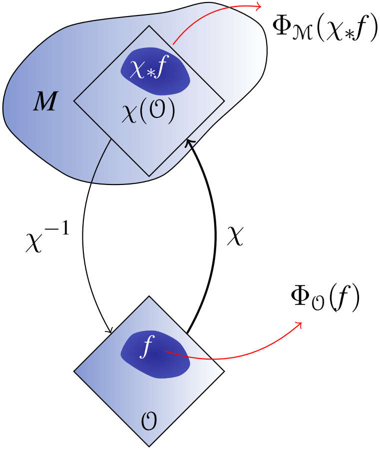

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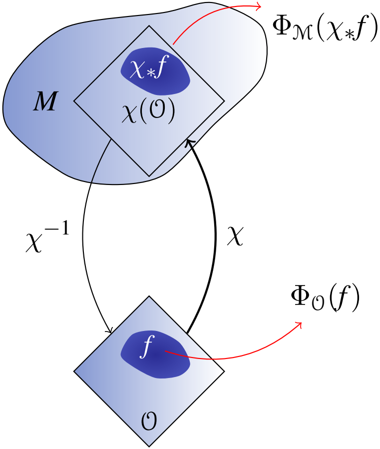
- In classical gravity we understand physical quantities not as pointwise objects but rather as something defined on all the spacetimes in a coherent way.



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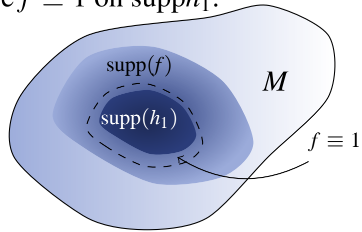

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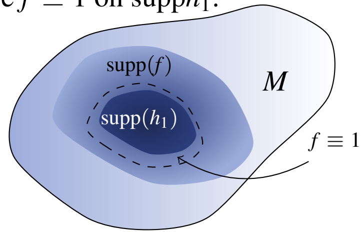

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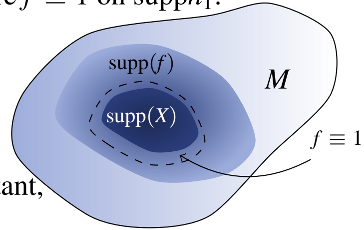

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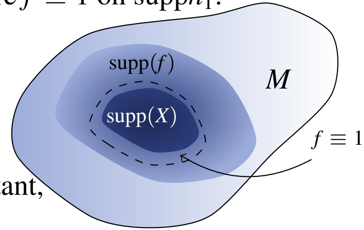

- Let $\mathfrak{E}_S(M)$ denote the space of solutions of field equations. We want to characterise the space of functionals on $\mathfrak{E}_S(M)$ which are invariant under all the local symmetries of S: invariant on-shell functionals $\mathfrak{F}_S^{\text{inv}}(M)$. In a finite dimensional case this space has a clear homological interpretation.

Diffeomorphism invariance

- For GR symmetries are infinitesimal diffeomorphisms, i.e. elements of $\mathfrak{X}(\mathcal{M}) \doteq \Gamma_c(TM)$. Let us choose a sequence $\vec{\xi} = (\xi_{\mathcal{M}})_{\mathcal{M}\in\mathrm{Obj}(\mathbf{Loc})}$, $\xi_{\mathcal{M}} \in \mathfrak{X}(\mathcal{M})$.
- After applying the exponential map we obtain $\alpha_{\mathcal{M}} \doteq \exp(\xi_{\mathcal{M}})$.
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- Diffeomorphism invariance is the statement that $\vec{\xi}\Phi = 0$.
- Example: $\int R[\tilde{g}] f\, \mathrm{d}\,\mathrm{vol}_{(M,\tilde{g})}$ is diffeomorphism invariant, but $\int R[\tilde{g}] f\, \mathrm{d}\,\mathrm{vol}_{(M,g)}$ is not.

Physical interpretation

- Let us fix $\mathcal{M}$. A test tensor $f \in \mathfrak{Tens}_c(\mathcal{M})$ corresponds to a concrete geometrical setting of an experiment, so for each $\mathcal{M} \in \mathrm{Obj}(\mathbf{Loc})$, we obtain a functional $\Phi(f)$, which depends covariantly on the geometrical data provided by f.

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New insight

Classical (or quantum) fields generate physical quantities, but a concrete observable quantity is obtained by evaluation on a test tensor. New concept: evaluated fields.

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- Let $\mathcal{F}$ denote the subspace of $\mathcal{C}^\infty(\mathrm{Diff}_c(\mathcal{M}), \mathfrak{F}(\mathcal{M}))$ generated by elements of the form Φ_f with respect to the pointwise product.

BV complex

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- The space of on-shell functionals is a quotient of $\mathcal{F}$ by the ideal generated by $S'_{\mathcal{M}}(\tilde{g})$ and can be described as a homology of the Koszul-Tate complex.

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  where $C \in \mathfrak{X}(M)$ is the ghost and γ is the Chevalley-Eilenberg differential, which acts on $\mathcal{BV}$ via infinitesimal diffeomorphism transformations along the ghost fields C. For $\Phi \in \mathcal{F}$ we have

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- Note that $H^0(s^\Psi, \alpha_\Psi(\mathcal{BV})) = H^0(s, \mathcal{BV}) = \mathcal{F}_S^{\mathrm{inv}}$.

Equations of motion and Poisson bracket

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- Using this input, we define the free Poisson bracket on $\mathcal{BV}$

$$\{F, G\}_0^g \doteq \left\langle F^{(1)}, \Delta_g G^{(1)} \right\rangle \qquad \Delta_g = \Delta_g^R - \Delta_g^A \,,$$

Deformation quantization

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- The deformation quantization of $(\mathcal{BV}_{\mu c}, \{.,.\}_0^g)$ can be performed in the standard way, by introducing a $\star$-product:

$$(F \star_H G) \doteq m \circ \exp(\hbar \Gamma_{\omega_H})(F \otimes G) \,,$$

where $\Gamma_{\omega_H} \doteq \int dx\, dy \omega_H(x,y) \frac{\delta}{\delta\varphi(x)} \otimes \frac{\delta}{\delta\varphi(y)}$ and $\omega_H = \frac{i}{2}\Delta_g + H$ is the Hadamard 2-point function (satisfies the linearized EOM's in both arguments and the μSC).

Deformation quantization

- For a fixed $\mathcal{M}$ we have a family of algebras $\mathfrak{A}_H(\mathcal{M}) = (\mathcal{BV}_{\mu c}[[\hbar, \lambda]], \star_H)$, numbered by possible choices of H. We can define $\mathfrak{A}(\mathcal{M})$ to be an algebra consisting of families (F_H), such that $F_H = e^{\frac{\hbar}{2}\Gamma'_{H-H'}} F_{H'}$, where $\Gamma'_{H-H'} \doteq \int dx\, dy (H - H')(x, y) \frac{\delta^2}{\delta\varphi(x)\delta\varphi(y)}$ and the star product is given by

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- This leads to a deformation quantization $(\mathfrak{A}(\mathcal{M}), \star)$ of the space of fields.

Interaction

- In the next step we have to introduce the interaction, i.e. consider the algebras $\mathfrak{A}_H(\mathcal{M}) = (\mathcal{BV}_{\mu c}[[\hbar, \lambda]], \star_H)$ and define on them the renormalized time-ordered products $\cdot_{\mathcal{T}_H}$ by the Epstein-Glaser method.

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- Interacting fields are obtained from free ones by the Bogoliubov formula:

$$(R_V(\Phi))_{\mathcal{M}}(f) \doteq \frac{d}{dt}\Big|_{t=0} \mathcal{S}(V^g)^{\star -1} \star \mathcal{S}(V^g + t\Phi_{\mathcal{M}}(f))\,.$$

Quantum observables

- In the framework of [K. Fredenhagen, K.R., CMP 2013], the gauge invariance of the S-matrix is guaranteed by the so called quantum master equation (QME):

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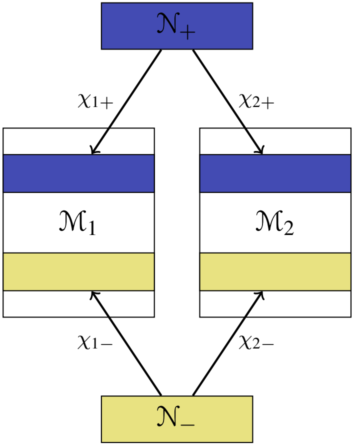
where $\triangle_{V^g}$ is the anomaly.
- If the QME holds, then gauge invariant quantum observables are recovered as the 0th cohomology of the quantum BV operator $\hat{s} \doteq R_V^{-1} \circ \{.,S_0\} \circ R_V$. Equivalently,

$$\hat{s}\Phi_{\mathcal{M}}(f) = \{., {S_0}^g + V^g\} + \Phi_{\mathcal{M}}(\pounds_C f) - i\hbar\,\triangle_{V^g}\left(\Phi_{\mathcal{M}}(f)\right).$$

Relative Cauchy evolution

- Let $\mathcal{N}_+$ and $\mathcal{N}_-$ be two spacetimes that embed into two other spacetimes $\mathcal{M}_1$ and $\mathcal{M}_2$ around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.



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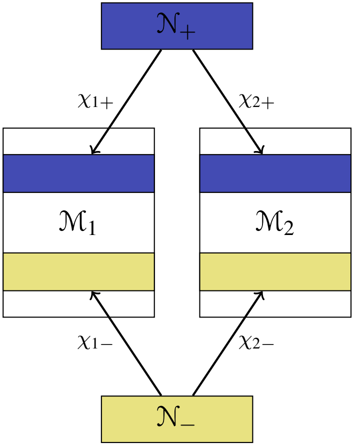

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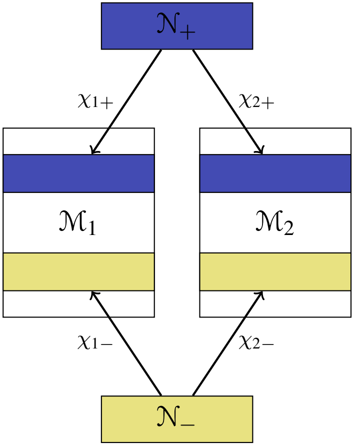
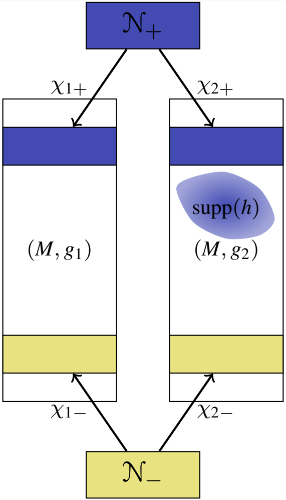


Background independence

- Let $\mathcal{M}_1 = (M, g_1)$ and $\mathcal{M}_2 = (M, g_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\text{supp}(h) \cap J^+(\mathcal{N}_+) \cap J^-(\mathcal{N}_-) = \emptyset$,



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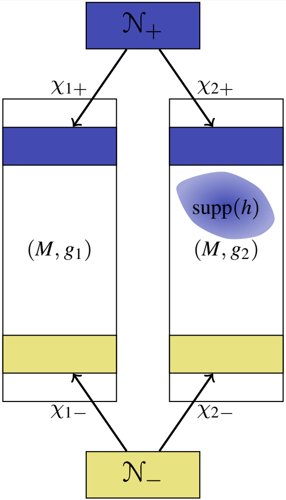

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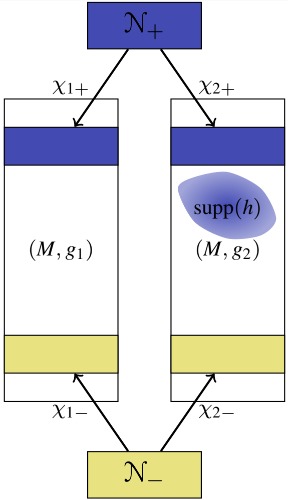
- The infinitesimal version of the background independence is a condition that $\Theta_{\mu\nu} = 0$.



Background independence

Theorem [Brunetti, Fredenhagen, K.R. 2013]

The functional derivative $\Theta_{\mu\nu}$ of the relative Cauchy evolution can be expressed, on-shell, as

$$\Theta_{\mu\nu}(\Phi_{\mathcal{M}_1}(f)) \overset{o.s.}{=} [R_{V_1}(\Phi_{\mathcal{M}_1}(f)), R_{V_1}(T_{\mu\nu})]_\star ,$$

where $T_{\mu\nu}$ is the stress-energy tensor of the extended action and one can define the time-ordered products in such a way that $T_{\mu\nu} = 0$ holds, so the interacting theory is background independent.

Conclusions

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- We have shown, using the relative Cauchy evolution, that our theory is background independent, i.e. independent of the split into free and interacting part.

Thank you for your attention!