$B = B_{(0,\infty)}$ ← The 1. Definition with the family of valuations $(v_r)_{r\in(0,\infty)}$

Lemma:

It holds $(B^b)^{\varphi=\pi^d} = \begin{cases} E, & d = 0 \\ 0, & d \neq 0 \end{cases}$, $B^b = A_{inf}\left[\frac{1}{\pi}, \frac{1}{[\varpi]}\right]$, $\varpi \in \mathfrak{m}_{F^\flat}$

A_{inf}

proof:

look at two cases:

1. $d = 0$
2. $d \neq 0$

1. Case:

Let $f = \sum_{i \gg -\infty} [x_i]\pi^i \in (B^b)^{\varphi = id}$.

$\forall i \in \mathbb{Z}: \varphi(x_i) \overset{= x_i^q}{=} x_i$; i.e. $x_i \in \mathbb{F}_q$

$\Rightarrow f \in E$.

2. Case: Let $d \neq 0$, $0 \neq f \in B^{\varphi = \pi^d}$.

$\forall x \in \mathbb{R}: \quad q\, Newt(f)(x) = Newt(\varphi(f)(x))$

$= Newt(\pi^d f)(x)$

$= Newt(f)(x-d)$

$\Rightarrow$

$\forall n \in \mathbb{N}:$

$\forall n \in \mathbb{N}$:

$$q^n \, \mathrm{Newt}(f)(x) = \ldots = \mathrm{Newt}(f)(x - nd).$$

Two distinguished two subcases:

i.) $d > 0$

ii.) $d < 0$

<u>Ad i.)</u>: $q^n \, \mathrm{Newt}(f)(x) = \underbrace{\mathrm{Newt}(f)(x - nd)}_{= \infty}$

$\forall n \gg 0.$

$\lightning$ to $f \neq 0$

<u>Ad ii.)</u>: $\exists x_0 \ll 0$: $\mathrm{Newt}(f)(x_0 - nd)$

$= q^n \, \mathrm{Newt}(f)(x_0)$

$\geq q^n \cdot \mathrm{Newt}(f)(x)$

$\forall n \gg 0, \ \forall x \in \mathbb{R}.$

$\Longrightarrow \mathrm{Newt}(f)(x) = \infty \quad \forall x \in \mathbb{R}$

$\Rightarrow f = 0$ $\square$

Def:

For an open intervall $I \subset (0, \infty)$ and $f \in B_I$

we define $Newt_I^\circ(f)$ as the decreasing convex function whose Legendre-transformation is

$$h : \mathbb{R} \to \mathbb{R} \cup \{\pm\infty\}$$
$$r \mapsto \begin{cases} v_r(f), & r \in I \\ -\infty, & r \notin I \end{cases}$$

with

$$Newt_I(f) := \{(x, Newt_I^\circ(f)(x)) \in \Gamma(Newt_I^\circ(f)) \mid \exists!\, i \in \mathbb{N}_0 : x \in [i, i+1] : \lambda_i \in -I\}$$
$$\cap$$
$$\mathbb{R}^2$$

$\mathbb{R}^{<}$

λ_i is the slope of $\operatorname{Newt}^{\circ}_{I}(f)$
on the intervall $[i, i+1]$ for all
$i \in \mathbb{N}_0$.