Bergische Universität Wuppertal winter term 2023/24
Kay Rülling, Matthias Wendt

Differential forms and algebraic de Rham cohomology

Lecture 1, de Rham complex on manifolds:
12.10.@HHU (Matthias)
brief recap of definitions, differential forms, Poincaré lemma for $U \subseteq \mathbb{R}^n$, de Rham complex

Lecture 2, de Rham theorem on smooth manifolds, periods:
19.10.@BUW (Matthias)
differential forms and de Rham complex on manifolds, recap singular cohomology, de Rham theorem on smooth manifolds, remarks on periods via de Rham isomorphism.

Lecture 3, sheaf cohomology, de Rham theorem via hypercohomology:
26.10.@HHU (Kay)
definition of sheaf cohomology via injective resolutions, hypercohomology, reformulation of de Rham theorem as hypercohomology statement

Lecture 4, analytic de Rham and Dolbeault complex on complex manifolds:
2.11.@BUW (Kay)
complex manifolds, analytic Poincaré lemma, de Rham and Dolbeault complex

Lecture 5, harmonic forms and Hodge decomposition:
9.11.@HHU (Matthias)
Laplace operator, harmonic forms, Hodge decomposition on compact Kähler manifolds, examples and sample applications

Lecture 6, spectral sequences (general and Hodge-to-de Rham):
16.11.@BUW (Matthias)
brief introduction to spectral sequences, Hodge-to-de Rham spectral sequence and degeneration

Lecture 7, mixed Hodge structures:
23.11.@HHU (Matthias)
log forms, mixed Hodge structures, maybe some motivic ideas.

Lecture 8, algebraic de Rham cohomology:
30.11.@BUW (Kay)
Kähler differentials, algebraic de Rham cohomology, standard exact sequences, universal characterisation of de Rham complex as dga.

Lecture 9, Grothendieck's theorem "algebraic de Rham=analytic de Rham":
14.12.@BUW (Kay)
GAGA translation between algebraic and analytic settings, identification of algebraic and analytic de Rham cohomology

Lecture 10, Abel–Jacobi map and geometric applications:
21.12.@HHU (Matthias)
Jacobians and Abel–Jacobi maps, intermediate Jacobians, maybe non-algebraic complex tori, geometric applications (like Clemens–Griffiths cubic threefold)

Lecture 11, de Rham cohomology in characteristic $p > 0$, Cartier operator:
11.01.24@BUW (Kay)
pathological behavior of de Rham cohomology in positive characteristic, definition of the (inverse) Cartier operator, Cartier isomorphism

Lecture 12, Deligne–Illusie:
18.01.24@HHU (Kay)
lifting smooth schemes in positive characteristic over the Witt vectors, theorem of Deligne-Illusie on the degeneration of the Hodge-to-de Rham spectral sequence in positive characteristic

Lecture 13, applications to positive characteristic geometry:
25.01.24@HHU (Kay)
obstructions to lifting to characteristic 0, applications to vanishing results, examples...