BERGISCHE UNIVERSITÄT WUPPERTAL

G S S I

Interacting particle systems in thermodynamic models: From Monday 26, 10:00 a.m. to Friday 30 January, 15:00 p.m. 2015 at the Gran Sasso Science Institute (GSSI) at L'Aquila (Italy).

Titles and Abstracts of speakers:

1) Prof. Dr. Sergio Albeverio, University of Bonn

Stochastic Quantization Equation and critical fluctuations of Van der Waals-Maxwell gas (DFG Project (RU791/2-1) with A. De Masi, E. Presutti, B. Rüdiger 2012-2015).

Abstract:

For the Van der Waals -Maxwell gas the macroscopic dynamic was derived by De Masi -Orlando -Presutti- Triolo [DOPT1], and is described by a pseudo -differential equation, i.e. a differential equation with a non local character, being a consequence of the long range interaction of particles described by Van der Waals.

Fluctuations around the deterministic dynamics are described by random processes. De Masi -Orlandi -Presutti -Triolo [DOPT1], [DOPT2] proved that scaling the same interacting particle dynamical model according to a "central limit theorem" linear fluctuations appear around the deterministic dynamic. In fact at any temperature and any dimension linear fluctuations are described by an Ornstein -Uhlenbeck stochastic process (OU-process), i.e a stochastic process which solves a linear Stochastic Differential Equation (SDE) with Gaussian noise.

In this project started in 2012 we show that non linear fluctuations coexist at the critical temperature in a different time and space scaling. This was proven by two of the investigators (in collaboration) for the one -dimensional model in [BPRS] (finite volume) and [FR] (finite and infinite volume), while the 2-dimensional case was still not solved. We observe that in a mean field model a critical mean field temperature T_m exists also for the one dimensional macroscopic model, while a critical temperature T_c^ϵ described by the Dobrushin-Lanford-Ruelle (DLR) - theory for the corresponding microscopic model exists in dimension 2 and is a perturbation

of the mean field critical temperature T_m^ϵ, the perturbation $T_c^\epsilon - T_m$ vanishing in the limit from the microscopic to macroscopic scale, when $\epsilon \to 0$. We show that critical fluctuations converge at the temperature T_c^ϵ to the solution of the stochastic quantization equation (SQE), having as invariant measure the ϕ_2^4 Euclidean field. Existence and uniqueness of a strong solution of the SQE was proven by G. Da Prato and A. Debussche [DD]. Interesting is that the Wick power appearing in the stochastic quantization equation and related to the ϕ_2^4 Euclidean field are expected to be generated by the perturbation of the temperature $T_c^\epsilon - T_m$. This was conjectured previously by one of the investigators in [GLP].

References

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[$DOPT2$] A. De Masi, E. Orlandi, E. Presutti, L.Triolo, Glauber evolution for Kac potentials: II. Fluctuations. Nonlinearity 9 (1996), 27 -51.
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[GLP] G.Giacomin, J.Lebowitz, E.Presutti; Deterministic and stochastic hydrodynamic equations arising from simple microscopic model systems. Collection: Stochastic partial differential equations: six perspectives, 107-152, Math. Surveys Monogr., 64, Amer.Math.Soc., Providence 1999.

2) Prof. Dr. Stefano Bonaccorsi, University of Trento

A Probabilistic Representation for Solutions to High Order Heat-type Equations.

Abstract:

The talk is based on a joint work with Sonia Mazzucchi (to appear on SPA 2015); we propose a new probabilistic construction of the solution of higher order initial value problems that is based on the scaling limit of a family of random walks in the complex plane.

3) Prof. Dr. Jinky Bornales, Physics Department, Mindanao State University-IIT, Iligan (with Profs. Oliveira and Streit)

Varadhan renormalization and gap regularization in terms of white noise.

Abstract:

Self-intersection local times of Brownian motion are increasingly singular in higher dimensions. They require regularizations to be well-defined, and counter terms to obtain a finite limit when the regularizations are removed. Since self-crossings of paths at short time intervals are at the origin of this singularities, various regularizations have been proposed to suppress this effect. The gap regularization disregards all self-intersections within time intervals less than a given cut-off Λ. We compute the multiple Wiener integral decomposition of the gap-regularized local time, show how it leads to a stronger version of Varadhan's rate of convergence argument, and point out the relevance to models of chain polymer conformations.

4) Prof. Dr. Emanuele Caglioti, Sapienza University of Rome

Long time behavior of solutions of Vlasov-like Equations.

5) Dr. Diana Conache, University of Bielefeld

The Uniqueness Problem for Gibbs Fields.

Abstract:

We present a generalized and improved version of the Dobrushin-Pechersky criterion for Gibbs fields on graphs. Moreover, we show how this result can be applied in the case of interacting particle systems in continuum.

6) Prof. Dr. Giuseppe Da Prato (SNS), Work in progress with Arnaud Debussche

Existence of $P_t D\varphi$ when φ is continuous and P_t is the transition semigroup of a dissipative SPDE.

Abstract:

We are concerned with SPDEs in a separable Hilbert space H of the form

$$dX = (AX + F(X))dt + dW_t, \quad X(0) = x, \tag{1}$$

where $A : D(A) \subset H \to H$ is self-adjoint, $F : D(F) \subset H \to H$ is m-dissipative and W is a cylindrical Wiener processs in H.

We shall assume that equation (1) has a unique solution $X(t, x)$, and that the corresponding transition semigroup $(P_t)_{t\geq 0}$ has a unique invariant measure ν. A typical example is provided by reaction-diffusion equations where F is a polynomial with negative leading coefficient.

Under suitable additional assumptions, P_t is regularity improving and the following Bismut-Elworthy-Li Formula (BEL), holds

$$\langle DP_t\varphi(x), h\rangle = \frac{1}{t}\mathbb{E}\big[\varphi(X(t,x))\int_0^t X_x(s,x)h\cdot dW(s)\big],\quad \varphi\in C_b(H),\ h\in H.$$

In this talk we show that when $\varphi \in C_b(H)$, $\langle P_t D\varphi(x), h\rangle$ depending only on φ and h. We do not use the Malliavin calculus.

This formula is then exploited to prove the Fomin differentiability of the invariant measure ν.

7) Dr. Cristina Di Girolami, University of Pescara

Stochastic calculus for non-semimartingales in Banach spaces, an infinite dimensional PDE and some stability results.

Abstract:

This talk develops some aspects of stochastic calculus via regularization for processes with values in a general Banach space B. A new concept of quadratic variation which depends on a particular subspace is introduced. An Itô formula and stability results for processes admitting this kind of quadratic variation are presented. Particular interest is devoted to the case when B is the space of real continuous functions defined on $[-T, 0]$, $T > 0$ and the process is the window process $X(\cdot)$ associated with a continuous real process X which, at time t, it takes into account the past of the process. If X is a finite quadratic variation process (for instance Dirichlet, weak Dirichlet), it is possible to represent a large class of path-dependent random variable h as a real number plus a real forward integral in a semiexplicite form. This representation result of h makes use of a functional solving an infinite dimensional partial differential equation. This decomposition generalizes, in some cases, the Clark-Ocone formula which is true when X is the standard Brownian motion W. Some stability results will be given explicitly. This is a joint work with Francesco Russo (ENSTA ParisTech Paris).

8) Prof. Dr. Hanno Gottschalk, University Wuppertal (based on joint work with Sergio Albeverio and Minoru Yoshida)

Cluster expansions for particle system beyond pair interaction.

Abstract:

We introduce a class of interactions for particle systems that is motivated from quantum field theory in low dimensions, namely the sine-gordon field theories. These interactions are non-pair interactions. For the case of particle systems, we show thermodynamic stability of such systems via cluster expansions (in arbitrary dimension). To this aim, we represent the interacting system as a Widom-Rowlinson type of model associated with a (formal) Potts model at a temperature which is purely imaginary. Nontheless, the cluster expansion for correlation functionals can be formulated for such formal models and the DLR-equations for their projected

counterparts can be established. Some remarks on the scaling limit and relations to Euclidean quantum field theory are given.

9) Dr. Peng Jin , University Wuppertal (joint work with Ch. Trabelsi and B. Rüdiger)

On ergodicity of some affine processes.

Abstract:

Affine processes are Markov processes for which the logarithm of the characteristic function of the process is affine with respect to the initial state. Typical examples of affine processes include the classical Ornstein-Uhlenbeck process and Feller diffusion. In the state space $\mathbb{R}_+$ affine processes are actually extension of the continuous-state branching processes with immigration. In this talk we present some ergodic results for some affine processes with the state space $\mathbb{R}_+$.

10) Prof. Dr. Yuri Kondratiev, University of Bielefeld

Markov statistical dynamics and kinetic equations.

Abstract:

We will discuss an approach to modeling of real world phenomena by interacting particle systems in the continuum. In many cases, evolutions of considered systems may be described as Markov stochastic processes. Related statistical dynamics are evolutions of macroscopic states associated with the forward Kolmogorov (or Fokker-Planck) equations for these Markov processes. The statistical dynamics can be described in terms of corresponding hierarchical chains of evolution equations for correlation functions. These chains admit mesoscopic scalings that leads to kinetic evolution equations for densities of considered systems.

11) Prof. Dr. Andreas Klümper, University Wuppertal

Spectral properties and thermodynamics of quantum spin chains of Temperley - Lieb type.

Abstract:

The Temperley-Lieb (TL) algebra is of eminent importance for the study of quantum spin systems. A model with local interactions satisfying the TL relations is known to be 'TL equivalent' to the spin-1/2 Heisenberg chain with which it shares spectral properties. The usual 'weak' TL equivalence is restricted to open boundary conditions and does not yield information on the degeneracies of the spectral values. However, these data are essential for the physical understanding at least in two situations.

For identifying the conformal properties of a critical quantum chain, the finite-size data of low-lying excitations for periodic (!) boundary conditions have to be calculated. Furthermore, the thermodynamics of a quantum chain is determined by all energy eigenvalues including their degeneracies (!).

Here it is shown how the complete spectrum of TL models with periodic boundary conditions is obtained sectorwise from the spectrum of Heisenberg chains with twisted boundary conditions.

The applications are:
(i) the complete treatment of the thermodynamics of two 3-state quantum spin chains with su(3) and sl(2|1) symmetry (gapped and critical, resp.),
(ii) all scaling dimensions and logarithmic corrections for all low-lying excitations of the critical sl(2|1)-invariant spin chain corresponding to a Chalker-Coddington-network for the quantum spin Hall effect.

12) Prof. Dr. Xue-Mei Li, The University of Warwick

Limits of random ordinary differential equations on manifolds.

Abstract:

I will discuss the convergence of solutions to a family of ODE's on a manifold with random coefficients where the fast motion are Hormander type diffusions.

13) Prof. Dr. Alessandro Pellegrinotti, University di Roma Tre

Random walk in fluctuating in time random environment.

14) Prof. Dr. Mario Pulvirenti, Sapienza University of Rome

Derivation of the Fick's law in a low-density regime (in collaboration with G. Basile, A. Nota, F. Pezzotti).

Abstract:

We consider the Lorentz model in a slab with two mass reservoirs at the boundaries. We show that, in a low density regime, there exists a unique stationary solution for the microscopic dynamics for which the density profile is approximatively linear . In the same regime the macroscopic current in the stationary state is given by the Fick's law, with the diffusion coecient determined by the Green-Kubo formula.

15) Prof. Dr. Ludwig Streit, University of Bielefeld

a) Self-repelling fractional Brownian motion and polymer conformations - learning from each other.

Abstract:

Self-avoiding or self-repelling random paths, with motivation from their use in polymer physics, have been widely studied using the tools of mathematics, physics, and computer simulations. We highlight mathrmatical results, predictions from physics, and verifications by computer simulations, with emphasis on recent extensions of the models to fractional Brownian paths.

b) Brownian Motion - Past, Present, and Future.

Abstract:

In this talk for incoming students, we present the evolution of the Brownian motion concept in its historical setting, starting with Einstein, moving back to Boltzmann, Brown, and Buffon, then to Perrin, to fractals, to Wiener, Kolmogorov, and Bachelier which then brings us to the emergence of modern financial markets and to the computer driven, complex, high speed global village, and the challenges it poses to governance, a challenge that the young generation is urged to confront.

16) Prof. Dr. Padmanabhan Sundar, Louisiana State University (joint work with S. Albeverio and B. Rüdiger)

McKean-Vlasov equations with jumps and associated PDEs in statistical Physics.

Abstract:

McKean-Vlasov equations driven by compensated Poisson random measures are studied in the context of partial differential equations that arise in statistical physics. The existence of weak solutions to such stochastic systems is established, and the marginal (in time) distribution of any solution is shown to be unique. Probabilistic behavior of the solution, in particular cases, will be described. This is a joint work with S. Albeverio and B. Rüdiger.

17) Dr. Stefania Ugolini, University Milano

Bose-Einstein Condensation: an interacting particle system.

Abstract:

Taking advantage of Nelson's Stochastic Mechanics, we can associate a well defined interacting diffusions system to the ground-state of the N-body Hamiltonian for N trapped interacting Bose particles. Under the Gross-Pitaevskii (GP) scaling limit (with N going to infinity) we can prove a transition to chaos result for the symmetric probability measure associated with the ground state of the N-body Hamiltonian. Moreover, introducing a proper one-particle relative entropy, we can investigate its asymptotic behavior under the same GP scaling limit. In particular we prove an existence theorem for the probability measure associated with the GP functional and a related weak convergence result.

References

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