Lösung Aufgabe 3 Übungsblatt VIII

Annahme

$X \sim \exp(\lambda)$; $\lambda = 1$

Zu berechnen:

$$\mathbb{E}[X(X-1)^2] = \mathbb{E}[X^3 - 2X^2 + X]$$

$$\mathbb{E}[X] \overset{1)}{=} \sum_{k\in\mathbb{N}} k \frac{e^{-\lambda}\lambda^k}{k!} = \lambda \left(= \sum_{k\in\mathbb{N}} e^{-\lambda} \frac{\lambda^{k-1}}{k!} \right)$$

$$\mathbb{E}[X^2] \overset{2)}{=} \sum_{k\in\mathbb{N}} k^2 \frac{e^{-\lambda}\lambda^k}{k!} = \sum_{k\in\mathbb{N}} k \frac{e^{-\lambda}\lambda^k}{(k-1)!}$$

$$= \sum_{k\in\mathbb{N}} (k+1) \frac{e^{-\lambda}\lambda^{k+1}}{k!} = \lambda \sum_{k\in\mathbb{N}} (k+1) \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= \lambda \sum_{k\in\mathbb{N}} k \frac{e^{-\lambda}\lambda^k}{k!} + \lambda \sum_{k\in\mathbb{N}} \frac{e^{-\lambda}\lambda^k}{k!} = \lambda(\lambda+1)$$

$$= \lambda^2 + \lambda \quad \text{wegen } 1)$$

$$\mathbb{E}[X^3] = \sum_{k\in\mathbb{N}} k^3 \frac{e^{-\lambda}\lambda^k}{k!} = \sum_{k\in\mathbb{N}} k^2 \frac{e^{-\lambda}\lambda^k}{(k-1)!}$$

$$= \lambda \sum (k+1)^2 \frac{e^{-\lambda}\lambda^k}{k!} = \lambda \sum (k^2+1+2k) \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= \lambda(\lambda^2+\lambda+1+2\lambda) = \lambda^3+\lambda^2+\lambda+2\lambda^2$$

wegen 2) und 1)

$$\mathbb{E}[X^3-2X^2+X] = \lambda^3+\lambda^2+\cancel{\lambda}+2\cancel{\lambda^2}-2(\lambda^2+\lambda)\cancel{+\lambda}$$