Time-Dependent Capital Budgeting with Multiple Criteria

Kathrin Klamroth* Margaret M. Wiecek

Department of Mathematical Sciences
Clemson University
Clemson, SC
USA

Abstract

In this paper we introduce a time-dependent multiple criteria model of capital budgeting and propose a dynamic-programming-based solution approach to finding all the efficient solutions defined as sequences of projects that are consecutively performed and bring benefit to a company. An illustrative example is enclosed.

1 Introduction

Capital budgeting is a well known problem in managerial economics. The problem concerns a company confronted with a variety of possible investment projects and a fixed capital budget independent of the investment decisions. The cost and the revenue associated with every project are assumed to be known. The objective is to select from among the projects the particular projects that lead to the highest earnings for the company.

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*On leave from the Department of Mathematics, University of Kaiserslautern, Kaiserslautern, Germany.

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The problem has been given a lot of attention by economists, management scientists, industrial engineers, operations researchers, and mathematicians, and the related literature is very rich. Among many others, Weingartner [24] studied capital budgeting in the context of mathematical programming.

The traditional theory of capital budgeting uses a single objective, usually in the form of a maximization of company's revenues. In the late nineteen sixties and early nineteen seventies researchers proposed to extend the traditional model with multiple objectives, as the particular projects can be selected with respect to more than a single objective, see Ansoff [1], Carsberg [5], and Bromwich [4]. The other objectives could include appreciation, sustainability, readiness, etc. all yielded by the selected investment projects. Capital budgeting models with multiple objectives identify the so called efficient projects. Trading one efficient project for another results in improvement of at least one objective and simultaneous deterioration of at least one other.

Hawkins and Adams [9] proposed a goal programming model of capital budgeting. Bhaskar [3] more formally recognized multiple objective functions and proposed a generalized goal programming approach. Capital budgeting with multiple objectives was also studied by Lee and Lerro [14], Ignizio [11], Thanassoulis [17], Corner et al. [6], and others. An interactive procedure using a multiple criteria linear integer model was proposed by Gonzalez et al. [8]. More recent papers proposed multiple criteria capital budgeting models extended with additional features such as time preferences (see Vetschera [21]), stochastic and dynamic elements (see Turney [18]), risk (see Lin [15]), and multiple decision makers (see Kwak et al.[13]).

In the framework of mathemtical programming, a typical capital budgeting model with multiple criteria is based on the multiple criteria knapsack problem (MCKP), a known combinatorial optimization problem with applications in many other areas such as transportation planning, conservation biology, packaging and loading.

The bi-criteria knapsack problem (BCKP) was studied by Rosenblatt and Sinuany-Stern [16] whose work was continued later by Eben-Chaime [7]. Several recent papers of Ulungu and Teghem [19, 20] and Visée et al. [23] dealt with the BCKP or the MCKP. Villarreal and Karwan [22] were perhaps the only ones who proposed dynamic programming (DP) approaches to the MCKP with multiple constraints.

In this paper, we propose a time dependent capital budgeting model with multiple criteria. We assume that the vector of objective functions is composed of time-dependent functions. The resulting model applies to the decision situation when the particular projects have to be selected subject to a known and fixed budgetary constraint so that time-dependent criteria such as revenue, appreciation, and others are maximized while the time needed to perform the selected projects is minimized. In comparison to the MCKP that identifies the efficient projects with respect to all the objective functions, the TDMCKP yields efficient sequences of the projects that should be consecutively performed. We follow upon a DP model presented by Villarreal and Karwan [22] and propose a DP approach to the TDMCKP. While adapting this model to the time-dependent case, we modify the forward approach of the time-dependent multiple criteria dynamic programming introduced by Kostreva and Wiecek [12].

In Section 2 we present the TDMCKP and the DP-based solution approach is developed in Section 3. We illustrate the model and the approach with a tri-criteria example in Section 4 and conclude the paper in Section 5.

2 A Model

Given a set of n projects of interest to a company, let $\{x_1, \ldots, x_n\}$ be a set of elements representing the projects and let $S := \{1, \ldots, n\}$ be the related index set.

We assume that only one project can be performed at a time and that during a decision process (that starts at time zero) some projects will be selected at consecutive times in order to be performed. Every sequence of projects to be performed corresponds to a sequence $x := \{x_{j(r)}\}_{r=1}^{p}$ of elements $x_{j(r)}, r = 1, \ldots, p$, where $j(r) \in S := \{1, \ldots, n\}$.

Given a fixed available budget b, we model the budgetary constraint as

$$a(x) \leq b, \tag{1}$$

where $a(x)$ is the function defined as

$$a(x) = a(\{x_{j(r)}\}_{r=1}^{p}) := \sum_{r=1}^{p} a_{j(r)} \tag{2}$$

and $a_{j(r)}$ is the cost coefficient of the project $x_{j(r)}, j(r) \in S$. We additionally assume that the cost coefficients $a_i, i \in S$, and the budget b are positive integers.

Consequently, the set X of all the feasible sequences of projects of the TDMCKP is defined as $X := \{x : a(x) \leq b\}$, where each sequence x satisfies

$$x \in \{\{x_{j(r)}\}_{r=1}^{p} \ : \ p \in I\!N, \ \ j(r) \in S, r = 1, \ldots, p\}. \tag{3}$$

Note that due to the fact that all the cost coefficients are positive integers, all the feasible solutions in X are finite. Namely we get that $p \leq b$ for all $\{x_{j(r)}\}_{r=1}^{p} \in X$.

Given m objective functions $f_i(x), i = 1, \ldots, m$, of interest to the company, we assume that f_1 measures the time needed to accomplish the projects and therefore it should be minimized while the other functions $f_i, i = 2, \ldots, m$, represent the criteria such as earnings, revenue, appreciation, etc., that should be maximized. Each of the functions is a real-valued and time dependent function of x. We define the vector objective function as

$$f(x) := [f_1(x), f_2(x), \ldots, f_m(x)]^T := \sum_{r=1}^{p} c_{j(r)}(t^r), \tag{4}$$

where (4) involves a vector sum and t is a continuous variable, $t \geq 0$, representing the time, that can be calculated as

$$\begin{aligned} t^1 &= 0, \\ t^{s+1} &= t^s + c^1_{j(s)}(t^s), \qquad s = 1, \ldots, p. \end{aligned} \tag{5}$$

For every $j \in S$, $c_j(t) = [c^1_j(t), \ldots, c^m_j(t)]^T$ is a vector objective related to choosing the project x_j at time t. Elements $c^i_j(t)$, $i = 1, \ldots, m$, $j = 1, \ldots, n$, are defined to be real-valued functions of time t and are not assumed to be continuous. In particular, for every $j \in S$, $c^1_j(t)$, $j = 1, \ldots, n$, is a positive function measuring the time needed to accomplish the project x_j if its realization has started at time t, and the other components $c^i_j(t)$, $i = 2, \ldots, m$, represent the earnings, revenue, appreciation, etc., generated by selecting the project x_j if its realization has started at time t.

According to (5), the first project $x_{j(1)}$ in a feasible sequence of projects is chosen at time $t^1 = 0$ and its realization takes $c^1_{j(1)}(t^1)$ time. Then the next project $x_{j(2)}$ is selected at time $t^2 = t^1 + c^1_{j(1)}(t^1)$ and it is accomplished at time $t^2 + c^1_{j(2)}(t^2)$, and so on.

We also assume that there is no waiting time between choosing and performing two consecutive projects in a sequence of projects, i.e. once a project $x_{j(s)}$ has been selected, another project $x_{j(s+1)}$ is being selected right after.

We formulate the time-dependent multiple criteria knapsack problem (TDMCKP) as:

$$\begin{aligned} \text{vmax}^* \quad & f(x) = [f_1(x), f_2(x), \ldots, f_m(x)]^T \\ \text{s.t.} \quad & a(x) \leq b. \end{aligned} \tag{6}$$

As we are interested in maximizing the objective functions $f_i(x)$, $i = 2, \ldots, m$ and in minimizing the time simultaneously, the operator $vmax^*$ in (6) denotes the maximization of $[-f_1(x), f_2(x), \ldots, f_m(x)]^T$, i.e.

$$\text{vmax}^* \, [f_1(x), f_2(x), \ldots, f_m(x)]^T := \text{vmax} \, [-f_1(x), f_2(x), \ldots, f_m(x)]^T. \tag{7}$$

We also find it convenient to consider the TDMCKP with the right-hand-side $k = 1, \ldots, b$ of the budgetary constraint and denote this problem by k-TDMCKP. A feasible solution $x := \{x_{j(r)}\}_{r=1}^{p}$ of the k-TDMCKP is a sequence of projects $x_{j(r)}, r = 1, \ldots, p, p \leq k$, so that $j(r) \in S$ and $\sum_{r=1}^{p} a_{j(r)} \leq k$ for $k = 1, \ldots, b$.

Solving (6) is understood as generating its efficient (Pareto) solutions (sequences of projects). A feasible solution $\hat{x} \in X$ is said to be an efficient solution of (6) if there is no other feasible solution $x \in X$ such that

$$f_1(x) \leq f_1(\hat{x}) \qquad \text{and} \qquad \forall i \in \{2, \ldots, m\} \;\; f_i(x) \geq f_i(\hat{x}) \tag{8}$$

with at least one strict inequality in (8).

Let $\mathcal{X}_e$ denote the set of efficient solutions of (6) and let $\mathcal{Y}_e$ denote the image of $\mathcal{X}_e$ in the objective space, that is $\mathcal{Y}_e = f(\mathcal{X}_e)$, where $f = [f_1, \ldots, f_m]^T$. $\mathcal{Y}_e$ is referred to as the set of nondominated criterion vectors of the efficient solutions (sequences of projects) of (6).

3 A dynamic programming approach

Let a set of states Q be defined as

$$Q := \{q(0), q(1), \ldots, q(b)\},$$

where the initial state is defined to be empty (independently from the cost function a and the objective functions)

$$q(0) = \emptyset$$

and the state $q(k)$, $k = 1, \ldots, b$, represents all the feasible solutions of the k-TDMCKP, i.e.

$$q(k) := \{\{x_{j(r)}\}_{r=1}^{p} \;:\; p \in I\!N,\;\; j(r) \in S, r = 1, \ldots, p,\;\; \sum_{r=1}^{p} a_{j(r)} = k\}.$$

In other words, a state represents all the feasible sequences of projects such that the total cost of each sequence is equal to a partial budget $k, 1 \leq k \leq b$.

Since there may occur nondominated solutions in all the states, the set of final states Q_F is given by

$$Q_F := \{q(0), q(1), \ldots, q(b)\}.$$

The decision of adding a project x_j to a solution sequence $x \in q(k)$ results in an increase of the right-hand side k by a_j and thus corresponds to a transition of x from the state $q(k)$ to the state $q(k+a_j)$. Observe that with this definition of the states, the original problem is represented as a loop-free sequential decision process, i.e. a process whose states can be indexed from 0 to b, so that a transition from a state $q(k)$ always occurs to a state $q(l)$ such that $k < l$, for any $k, l = 0, 1, \ldots, b$ (see [10]).

Without loss of generality we assume that the system is in the state $q(0)$ at time $t = 0$.

For all $j = 1, \ldots, n$, $c_j^1(t) > 0$ represents the time needed to make a transition from a state $q(k)$ to a state $q(k + a_j)$ for $k = 0, 1, \ldots, b - a_j$ given that the system is in the state $q(k)$ at time t, that is the time needed to accomplish the project x_j if it has started at time t. The transition to the state $q(k + a_j)$ is completed at time $t + c_j^1(t)$, which corresponds to the fact that the project x_j is accomplished at time $t + c_j^1(t)$.

The states defined above and the possible transitions between them yield a network whose nodes and arcs are defined by these states and transitions, respectively. This network does not have any circuits (a circuit is a path traversing through a node twice, see [2]) since the states and the transitions form a loop-free decision process. Associated with every arc of this network is a criterion vector $[c_j^1(t), \ldots, c_j^m(t)]$ related to adding the project x_j to a sequence of projects at time t.

Given the network, we are in the position to apply the forward approach of Kostreva and Wiecek [12]. They developed the forward approach to find the set of all nondominated (shortest) paths from a given source node to every other node in the network whose links carried a time-dependent vector cost. They considered a general network whose every node could be connected to every other node. The costs were assumed to be real-valued positive and monotone increasing functions of time. These assumptions were necessary to establish the Principle of Optimality for Dynamic Multiple Objective Networks.

Given the special structure of our network, we may relax and change some of their assumptions due to the fact that the feasibility constraint of the TDMCKP yields a circuit-free network and that our problem involves maximization rather than minimization. In general, we can allow all the objective functions to be positive and/or negative functions of time no matter whether we pose a maximization or a minimization problem. In both cases the optimal objective function (vector) value will be necessarily bounded as we have a finite number of states (nodes), a finite number of transitions (arcs), and no circuits in the network. However, as we have chosen $c_j^1(t), j = 1, \ldots, n$ to represent the time, we require these functions to be positive, while the other functions $c_j^i(t), i = 2, \ldots, m, j = 1, \ldots, n$, representing general criteria of interest may be of any sign. In fact, a transition from a state $q(k)$ at a given time t_1 to a state $q(l)$ may for some criterion $i, i \in \{2, \ldots, m\}$, yield an objective value $c_j^i(t_1) < 0$, which means that adding the project x_j at time t_1 to a sequence of projects is not beneficial at all and this project will certainly not contribute to an efficient sequence. But the same project x_j could be added at a time $t_2, t_2 \neq t_1$, so that the corresponding objective value would be positive, $c_j^i(t_1) > 0$, and make the project x_j competitive.

The following assumption is necessary for the principle of optimality for the TDMCKP to hold.

Assumption 1 *For all $t_1, t_2 \geq 0$, if $t_1 \leq t_2$, then*

(a) $t_1 + c_j^1(t_1) \leq t_2 + c_j^1(t_2)$ for all $j = 1, \ldots, n$, and

(b) $c_j^i(t_1) \geq c_j^i(t_2)$ for all $i = 2, \ldots, m$, and $j = 1, \ldots, n$.

Assumption 1 (a) requires that if a project x_j is initiated at time t_1 or at a later time t_2, then with the earlier start time it has to be accomplished earlier than with the later start time. In other words, the earlier a project is started, the earlier it has to be completed. We observe that if $c_j^1(t), j = 1,\ldots,n$, are monotone increasing functions of time, then this assumption holds. Assumption 1 (b) simply requires that the other components of the objective functions $c_j^i(t)$ for all $i = 2,\ldots,m$ and $j = 1,\ldots,n$ be monotone decreasing functions of time. In the context of the model this implies that (for example) the revenue generated by a project x_j decreases in time, or in other words, the later the project is initiated the less revenue it brings. We believe that both assumptions naturally fit into the model as they mathematically interpret the commonly made assertions in capital budgeting.

Let $f(x) := f(\{x_{j(r)}\}_{r=1}^p)$ be a nondominated criterion vector of the k-TDMCKP accomplished at time t^{p+1} that can be computed using (5). Let $G(q(k)) = \{f(x) : x \in q(k)\}$ be the set of all the nondominated criterion vectors of the k-TDMCKP.

The principle of optimality for dynamic multiple criteria networks established in [12] adapted to the time-dependent capital budgeting network model yields the following theorem.

Theorem 1 *Principle of Optimality for the TDMCKP.*

Under Assumption 1, an efficient sequence of projects $x^p = \{x_{j(r)}\}_{r=1}^p$ of the k-TDMCKP accomplished at time t^{p+1} has the property that each subsequence of projects $x^s = \{x_{j(r)}\}_{r=1}^s, 1 \le s < p$ accomplished at time $t^{s+1}, t^{s+1} \le t^{p+1}$, is an efficient sequence of the $(\sum_{r=1}^s a_{j(r)})$-TDMCKP.

Theorem 1 results in the following recursive equations for $t^p > 0$:

$$
\begin{aligned}
G(q(0)) &= \{\underline{0}\} \\
G(q(k)) &= \mathrm{vmax}^*\{f(\{x_{j(r)}\}_{r=1}^p) : \{x_{j(r)}\}_{r=1}^p \in q(k)\} \\
&= \mathrm{vmax}^*\{f(\{x_{j(r)}\}_{r=1}^{p-1}) + c_{j(p)}(t^p) : \\
&\qquad\qquad f(\{x_{j(r)}\}_{r=1}^{p-1}) \in G(q(k - a_{j(p)})),\ j(p) \in S,\ k - a_{j(p)} \ge 0\}, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 1,\ldots,b,
\end{aligned}
$$

where the operation $vmax^*$ computes the nondominated criterion vectors according to (7) in the set whose every element is a vector sum of a nondominated criterion vector of the efficient sequence of projects of the $(k - a_{j(p)})$-TDMCKP accomplished at time t^p, and the criterion vector $c_{j(p)}(t)$ evaluated at time t^p.

Since all the states are final, the set of all the nondominated criterion vectors $\mathcal{Y}_e$ is obtained as the vector-maximum of the union of the sets $G(q(k))$, $k = 1, \ldots, b$; i.e.

$$\mathcal{Y}_e = \text{vmax}^* \bigcup_{k=1,\ldots,b} G(q(k)).$$

Note that in each step of the recursion two or more nondominated criterion vectors may correspond to different efficient sequences of projects, however composed of the same projects, which shows that different criterion vectors can be achieved while choosing the same projects to a sequence but at different times. We discuss this and similar situations in Section 4.

4 Example

We now present a didactic example of the time-dependent capital budgeting problem with three criteria ($m = 3$).

Assume there are four projects ($n = 4$) of interest to the company. The fixed budget equals 3 and the cost coefficients a_j, $j = 1, \ldots, 4$ of each project are given by

$$a_1 = 1,\ a_2 = 2,\ a_3 = 1,\ a_4 = 1.$$

The criteria include the time of performing the projects to be minimized, and the revenue and appreciation yielded by the projects to be maximized. The objective vectors $c_j(t)$, $j = 1, \ldots, 4$ related to each project are defined as

$$c_1(t) = \begin{bmatrix} 1 \\ 10 - t^2 \\ 40 - t \end{bmatrix},\ c_2(t) = \begin{bmatrix} 2 \\ 70 - 2t^2 \\ 10 - t \end{bmatrix},\ c_3(t) = \begin{bmatrix} t + 1 \\ 20 - 30t^2 \\ 20 - 2t \end{bmatrix},\ c_4(t) = \begin{bmatrix} 2t + 1 \\ 30 \\ 10 - 2t \end{bmatrix}.$$

The resulting TDMCKP has the following form:

$$\begin{array}{ll} \text{vmax}^* & f(x) = [f_1(x), f_2(x), f_3(x)]^T \\ \text{s.t.} & a(x) \leq 3. \end{array} \tag{9}$$

The possible transitions between states for this example problem are represented by the arcs in the network given in Figure 1. The objective vector $c_j(t) = [c_j^1(t), c_j^2(t), c_j^3(t)]^T$ of each transition and the corresponding variable x_j are identified for each arc and denoted by the vector $[j, c_j]$.

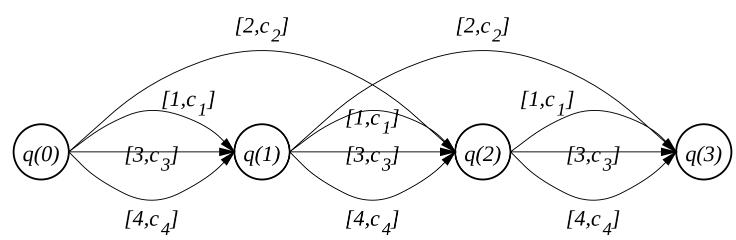

Figure 1: The vertices of this network represent the states of the DP-formulation for the example problem (9).

Applying the recursive equations developed in Section 3 we obtain the following sets of nondominated criterion vectors $G(q(k))$, $k = 0, 1, \ldots, 3$:

$$G(q(0)) = \{\underline{0}\}$$

$$G(q(1)) = \left\{ \begin{bmatrix} 1 \\ 10 \\ 40 \end{bmatrix}, \begin{bmatrix} 1 \\ 20 \\ 20 \end{bmatrix}, \begin{bmatrix} 1 \\ 30 \\ 10 \end{bmatrix} \right\}$$

$$G(q(2)) = \left\{ \begin{bmatrix} 2 \\ 19 \\ 79 \end{bmatrix}, \begin{bmatrix} 2 \\ 29 \\ 59 \end{bmatrix}, \begin{bmatrix} 2 \\ 39 \\ 49 \end{bmatrix}, \begin{bmatrix} 2 \\ 70 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ 47 \\ 28 \end{bmatrix}, \begin{bmatrix} 4 \\ 40 \\ 48 \end{bmatrix}, \begin{bmatrix} 4 \\ 50 \\ 28 \end{bmatrix}, \begin{bmatrix} 4 \\ 60 \\ 18 \end{bmatrix} \right\}$$

$$G(q(3)) = \left\{ \begin{bmatrix} 3 \\ 25 \\ 117 \end{bmatrix}, \begin{bmatrix} 3 \\ 35 \\ 97 \end{bmatrix}, \begin{bmatrix} 3 \\ 45 \\ 87 \end{bmatrix}, \begin{bmatrix} 3 \\ 78 \\ 49 \end{bmatrix}, \begin{bmatrix} 3 \\ 88 \\ 29 \end{bmatrix}, \begin{bmatrix} 3 \\ 98 \\ 19 \end{bmatrix}, \begin{bmatrix} 4 \\ 48 \\ 65 \end{bmatrix}, \begin{bmatrix} 5 \\ 54 \\ 54 \end{bmatrix}, \right.$$
$$\left. \begin{bmatrix} 7 \\ 49 \\ 85 \end{bmatrix}, \begin{bmatrix} 7 \\ 59 \\ 65 \end{bmatrix}, \begin{bmatrix} 7 \\ 69 \\ 55 \end{bmatrix}, \begin{bmatrix} 7 \\ 100 \\ 16 \end{bmatrix}, \begin{bmatrix} 13 \\ 70 \\ 50 \end{bmatrix}, \begin{bmatrix} 13 \\ 80 \\ 30 \end{bmatrix}, \begin{bmatrix} 13 \\ 90 \\ 20 \end{bmatrix} \right\}$$

The set of efficient solutions $\mathcal{X}_e$ and the set of nondominated criterion vectors $\mathcal{Y}_e$ of this example problem can thus be calculated as

$$\mathcal{X}_e = \{ \{\,\}, \{x_1\}, \{x_3\}, \{x_4\}, \{x_1x_1\}, \{x_3x_1\}, \{x_4x_1\}, \{x_2\}, \{x_1x_1x_1\}, \{x_3x_1x_1\},$$

$\{x_4x_1x_1\}, \{x_1x_2\}, \{x_3x_2\}, \{x_4x_2\}, \{x_4x_3x_1\}, \{x_4x_4x_1\}, \{x_1x_1x_4\},$
$\{x_3x_1x_4\}, \{x_4x_1x_4\}, \{x_2x_4\}, \{x_1x_4x_4\}, \{x_3x_4x_4\}, \{x_4x_4x_4\}\}$,

$$\mathcal{Y}_e = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 10 \\ 40 \end{bmatrix}, \begin{bmatrix} 1 \\ 20 \\ 20 \end{bmatrix}, \begin{bmatrix} 1 \\ 30 \\ 10 \end{bmatrix}, \begin{bmatrix} 2 \\ 19 \\ 79 \end{bmatrix}, \begin{bmatrix} 2 \\ 29 \\ 59 \end{bmatrix}, \begin{bmatrix} 2 \\ 39 \\ 49 \end{bmatrix}, \begin{bmatrix} 2 \\ 70 \\ 10 \end{bmatrix}, \right.$$
$$\begin{bmatrix} 3 \\ 25 \\ 117 \end{bmatrix}, \begin{bmatrix} 3 \\ 35 \\ 97 \end{bmatrix}, \begin{bmatrix} 3 \\ 45 \\ 87 \end{bmatrix}, \begin{bmatrix} 3 \\ 78 \\ 49 \end{bmatrix}, \begin{bmatrix} 3 \\ 88 \\ 29 \end{bmatrix}, \begin{bmatrix} 3 \\ 98 \\ 19 \end{bmatrix}, \begin{bmatrix} 4 \\ 48 \\ 65 \end{bmatrix}, \begin{bmatrix} 5 \\ 54 \\ 54 \end{bmatrix},$$
$$\left. \begin{bmatrix} 7 \\ 49 \\ 85 \end{bmatrix}, \begin{bmatrix} 7 \\ 59 \\ 65 \end{bmatrix}, \begin{bmatrix} 7 \\ 69 \\ 55 \end{bmatrix}, \begin{bmatrix} 7 \\ 100 \\ 16 \end{bmatrix}, \begin{bmatrix} 13 \\ 70 \\ 50 \end{bmatrix}, \begin{bmatrix} 13 \\ 80 \\ 30 \end{bmatrix}, \begin{bmatrix} 13 \\ 90 \\ 20 \end{bmatrix} \right\}.$$

Note that in this example different criterion vectors are achieved by choosing the same projects to a solution sequence at different times. For example, the solution sequences $\{x_4x_2\}$ and $\{x_2x_4\}$ with the criterion vectors $[3, 98, 19]^T$ and $[7, 100, 16]^T$ are both nondominated. Similarly, the solution sequences $\{x_4x_4x_1\}$, $\{x_4x_1x_4\}$ and $\{x_1x_4x_4\}$ with the criterion vectors $[5, 54, 54]^T$, $[7, 69, 55]^T$ and $[13, 70, 50]^T$ are all nondominated.

Furthermore we observe that shorter times are achieved when only one or two projects are performed which may not be of high priority to the company. On the other hand, none of the sequences includes all the projects, at most three projects can be selected in any case, and project 2 seems to be the least popular in all the sequences.

In order to make a final decision what projects should be selected, the company would have to specify additional preferences. For example, if the preference was to perform three different projects, two sequences $\{x_4x_3x_1\}$ and $\{x_3x_1x_4\}$ would be the candidates for the final optimal solution. The decision maker would have to choose between the criterion vectors $[4, 48, 65]$ and $[7, 59, 65]$. The vectors show that performing project 4 at the beginning rather than at the end saves time, yields less revenue, and keeps appreciation at the same level. The final decision would be then between the time and the revenue.

5 Conclusions

We developed the time-dependent multiple criteria knapsack problem (TDM-CKP) and used it to model time-dependent capital budgeting with multiple criteria. The novelty of the formulation comes from the fact that the solution set of the problem includes efficient sequences of projects to be performed consecutively over time. We believe that the new model significantly enhances the traditional multiple criteria knapsack model and could be applied in many decision making situations involving capital budgeting.

Future research should focus on developing more complex models such as time-dependent models with multiple constraints or multi-period models.

References

[1] H. I. Ansoff. *Corporate Strategy.* Penguin, 1968.

[2] M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali. *Linear Programming and Network Flows.* John Wiley, New York, 1990.

[3] K. Bhaskar. A multiple objective approach to capital budgeting. *Accounting and Business Research*, 9:25–46, 1979.

[4] M. Bromwich. *The Economics of Capital Budgeting.* Penguin, 1976.

[5] B. V. Carsberg. *Analysis for Investment Decisions.* Accountancy Age Books. Haymarket Publishing, 1974.

[6] J. L. Corner, R. F. Deckro, and R. W. Spahr. Multiple-objective linear programming in capital budgeting. *Advances in Mathematical Programming and Financial Planning*, 3:241–264, 1993.

[7] M. Eben-Chaime. Parametric solution for linear bicriteria knapsack models. *Management Science*, 42:1565–1575, 1996.

[8] J. J. Gonzalez, G. R. Reeves, and L. S. Franz. Capital budgeting decision making: An interactive multiple objective linear integer programming search procedure. *Advances in Mathematical Programming and Financial Planning*, 1:21–44, 1987.

[9] C. A. Hawkins and R. A. Adams. A goal programming model for capital budgeting. *Journal of Financial Management*, 1974.

[10] T. Ibaraki. Enumerative approaches to combinatorial optimization, part ii. In P. L. Hammer, editor, *Annals of Operations Research*, volume 11, pages 343–602. Baltzer, Basel, 1987.

[11] J. P. Ignizio. A approach to capital budgeting with multiple objectives. *The Engineering Economist*, pages 259–272, 1976.

[12] M.M. Kostreva and M.M. Wiecek. Time dependency in multiple objective dynamic programming. *Journal of Mathematical Analysis and Applications*, 173:289–307, 1993.

[13] W. Kwak, Y. Shi, H. Lee, and C.F. Lee. Capital budgeting with multiple criteria and multiple decision makers. *Review of Quantitative Finance and Accounting*, 7:97–112, 1996.

[14] S. M. Lee and A. J. Lerro. Capital budgeting for multiple objectives. *Management Science*, 36:1106–1119, 1974.

[15] T. W. Lin. Multiple-criteria capital budgeting under risk. *Advances in Mathematical Programming and Financial Planning*, 3:231–239, 1993.

[16] M.J. Rosenblatt and Z. Sinuany-Stern. Generating the discrete efficient frontier to the capital budgeting problem. *Operations Research*, 37:384–394, 1989.

[17] E. Thanassoulis. Selecting a suitable solution method for a multiple-objective programming capital budgeting problem. *Journal of Business Finance and Accounting*, 12:453–471, 1985.

[18] S. T. Turney. Deterministic and stochastic dynamic adjustment of capital investment budgets. *Mathematical Computation and Modelling*, 13:1–9, 1990.

[19] E.L. Ulungu and J. Teghem. Application of the two phases method to solve the bi-objective knapsack problem. Technical report, Department of Mathematics & Operational Research, Faculté Polytechnique de Mons, Belgium, 1994.

[20] E.L. Ulungu and J. Teghem. Solving multi-objective knapsack problem by a branch-and-bound procedure. In J.N. Climaco, editor, *Multicriteria Analysis*, pages 269–278. Springer-Verlag, 1997.

[21] R. Vetschera. Time preferences in capital budgeting - an application of interactive multiobjective optimization. *Methods of Operations Research*, 50:649–660, 1985.

[22] B. Villarreal and M. H. Karwan. Multicriteria integer programming: A (hybrid) dynamic programming recursive approach. *Mathematical Programming*, 21:204–223, 1981.

[23] M. Visée, J. Teghem, M. Pirlot, and E.L. Ulungu. Two-phases method and branch and bound procedures to solve the bi-objective knapsack problem. Technical report, Department of Mathematics & Operational Research, Faculté Polytechnique de Mons, Belgium, 1996.

[24] H. M. Weingartner. *Mathematical Programming and the Analysis of Capital Budgeting Problems.* Prentice-Hall, Englewood Cliffs, N. J., 1963.