Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Linear and Network Optimization
Exercise 9

Please return your solutions by Wednesday, June 11th, 10:00 a.m., in the mailbox No. 5.

Problem 1 (5 points)
Show that the inverse of the matrix $\bar{A}\bar{A}^T$ exists, where

$$\bar{A} := \begin{pmatrix} A \cdot D \\ 1 \cdots 1 \end{pmatrix}, \qquad D := \begin{pmatrix} x_1^k & & 0 \\ & \ddots & \\ 0 & & x_n^k \end{pmatrix},$$

A is a regular $m \times n$-matrix and $\underline{x}^k > \underline{0}$ is an n-vector that satisfies $A\underline{x}^k = \underline{0}$.

Problem 2 (15 points)
Consider the following LP:

$$\begin{array}{rrcl} \min & x_2 & & \\ \text{s.t.} & x_1 + x_2 - 2x_3 & = & 0 \\ & x_1 + x_2 + x_3 & = & 1 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(a) Verify that the assumptions of Karmarkar's projective algorithm are satisfied for this LP.

(b) Apply two iterations of Karmarkar's projective algorithm to this LP, starting with $\underline{x}^0 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^T$.

(c) After 6 iterations the solution $\underline{x}^6 = (0.606509, 0.060158, \frac{1}{3})^T$ is obtained and the algorithm terminates. (Why?) Use this solution to determine an optimal basic feasible solution of the LP.