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# Linear and Network Optimization

## Exercise 2

Please return your solutions by Wednesday, April 23rd, in the mailbox No. 5.

**Problem 1** (6 points)

(a) Solve the following LP graphically:

$$\begin{array}{lrcrcrcl} \max & x_1 & + & 2x_2 & + & 3x_3 & & \\ \text{s.t.} & x_1 & & & + & 2x_3 & \leq & 3 \\ & & & x_2 & + & 2x_3 & \leq & 2 \\ & & & & & x_i & \geq & 0, \quad i = 1, 2, 3. \end{array}$$

(b) Determine the basic solutions for all bases of this LP.

**Problem 2** (6 points)

(a) Find a linear program in two variables for each of the following cases and draw the feasible region $P$ (if possible):

(i) $P$ is unbounded

(ii) $P$ contains only one point

(iii) $P$ is empty.

(b) Consider the following statement: $P$ unbounded $\Longrightarrow$ the objective function value is unbounded. Is this true or false? Please explain.

**Problem 3** (4 points)
The basic idea of the simplex method is to move from one vertex of the polyhedral feasible set to an adjacent vertex until an optimal vertex is reached. On this and on the following exercise-sheet we will analyse this property.
For this purpose, prove Theorem 1 on the back of this sheet based on the definition of convex sets given in Definition 1 (on the back of this sheet).

**Problem 4** (4 points)
We have seen that there exists a *unique* basic feasible solution for every basis of $A$. Prove the following converse statement or give a counterexample:
For every basic solution there exists a *unique* corresponding basis.

**Definition 1** A set $X \subseteq \mathbb{R}^n$ is *convex* if the linear segment joining any two points in the set lies entirely in the set, i.e.

$$\forall \underline{x}, \underline{y} \in X : \qquad \lambda \underline{x} + (1-\lambda)\underline{y} \in X \qquad \forall \lambda \in [0,1].$$

**Definition 2** (Alternative definition of extreme points).
An *extreme point* (or a *vertex*) of a convex set $X \subseteq \mathbb{R}^n$ is a point $\underline{x} \in X$ such that $\underline{x}$ cannot be represented in the form

$$\underline{x} = \lambda \underline{x}_1 + (1-\lambda)\underline{x}_2,\ 0 < \lambda < 1$$

with $\underline{x}_1$, $\underline{x}_2 \in X$, $\underline{x}_1 \neq \underline{x}_2$.

**Theorem 1** The feasible set $P := \{\underline{x} \in \mathbb{R}^n : A\underline{x} = \underline{b}, \underline{x} \geq \underline{0}\}$ is convex.