Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
Prof. Dr. K. Klamroth

08.05.2003

Algorithm 2.24: 2-Phase Method

(Input) LP $\min\{\underline{c}\,\underline{x} : A\underline{x} = \underline{b},\ \underline{x} \geq \underline{0}\}$

(1) **Transformation of the system $A\underline{x} = \underline{b}$:**

a) Multiply all those equations $A^i\underline{x} = b_i$ by (-1), for which $b_i < 0$.

b) Identify those equations $A^i\underline{x} = b_i$ for which a variable $x_{s(i)}$ exists that occurs only in this equation and for which $a_{i\,s(i)} > 0$. Transform these equations to

$$\frac{1}{a_{i\,s(i)}} A^i\underline{x} = \frac{b_i}{a_{i\,s(i)}}.$$

Let $I \subseteq \{1, \ldots, m\}$ be the index set of the corresponding equations.

c) Introduce an artificial variable $\hat{x}_i$ for all $i \in \overline{I} := \{1, \ldots, m\} \setminus I$, i.e. replace $A^i\underline{x} = b_i$ by $A^i\underline{x} + \hat{x}_i = b_i$. Denote the new constraint matrix by $\tilde{A}$ and the extended solution vector by $\tilde{\underline{x}}$.

(2) **Phase 1 of the simplex method**

a) Set $(\tilde{\underline{x}}_B, \tilde{\underline{x}}_N)$ with

$$\tilde{x}_{B(i)} := \begin{cases} x_{s(i)} = \frac{b_i}{a_{i\,s(i)}} & \text{if} \quad i \in I \\ \hat{x}_i = b_i & \phantom{\text{if}} \quad i \in \overline{I} \end{cases}$$

If $\overline{I} = \emptyset$, goto Step (3).

b) Find an optimal solution $\tilde{\underline{x}}^*$ of the LP

$$\min\left\{\sum_{i\in\overline{I}} \hat{x}_i : \tilde{A}\tilde{\underline{x}} = \underline{b},\ \tilde{\underline{x}} \geq \underline{0}\right\}.$$

c) If $\sum_{i\in\overline{I}} \hat{x}_i^* > 0$ (STOP),
the LP $\min\{\underline{c}\,\underline{x} : A\underline{x} = \underline{b},\ \underline{x} \geq \underline{0}\}$ is infeasible.

d) If $\sum_{i\in\overline{I}} \hat{x}_i^* = 0$,
pivot all artificial variables $\hat{x}_i$ out of the basis (if they are not yet non-basic variables).

e) Remove all those columns corresponding to artificial variables from the last tableau and replace the auxiliary objective function $\sum_{i\in\overline{I}} \hat{x}_i$ by the original objective function $\underline{c}\,\underline{x}$.

f) Apply elementary row operations to the resulting tableau such that $t_{0\,B(i)} = 0$ for all basic variables $x_{B(i)}$.

(3) **Phase 2 of the simplex method**
Apply the simplex method to the tableau found in Step (2f).