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Linear and Network Optimization
Handout 1
Example for pivot operations in a simplex tableau

$$
\text{(LP)} \qquad \begin{array}{lrcrcrcl}
\min & -2x_1 - 3x_2 - 4x_3 \\
\text{s.t.} & & & 2x_2 & + & 3x_3 & \leq & 5 \\
 & x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\
 & x_1 & + & 2x_2 & + & 3x_3 & \leq & 7 \\
 & & & & & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

Transformation into standard form:

$$
\begin{array}{lrcrcrcrcrcrcl}
\min & -2x_1 - 3x_2 - 4x_3 \\
\text{s.t.} & & & 2x_2 & + & 3x_3 & + & x_4 & & & & & = & 5 \\
 & x_1 & + & x_2 & + & 2x_3 & & & + & x_5 & & & = & 4 \\
 & x_1 & + & 2x_2 & + & 3x_3 & & & & & + & x_6 & = & 7 \\
 & & & & & & & & & & & x_1, \ldots, x_6 & \geq & 0
\end{array}
$$

Starting tableau:

$$
T = \begin{array}{|c|cccccc|c|}
\hline
1 & -2 & -3 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 2 & 3 & 1 & 0 & 0 & 5 \\
0 & 1 & 1 & 2 & 0 & 1 & 0 & 4 \\
0 & 1 & 2 & 3 & 0 & 0 & 1 & 7 \\
\hline
\end{array}
$$

Simplex tableau with respect to the basis $B = \{4, 5, 6\}$:

$$
T(B) = \begin{array}{|c|cccccc|c|}
\hline
1 & -2 & -3 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 2 & 3 & 1 & 0 & 0 & 5 \\
0 & 1 & 1 & 2 & 0 & 1 & 0 & 4 \\
0 & 1 & 2 & 3 & 0 & 0 & 1 & 7 \\
\hline
\end{array}
$$

Pivot operations:

$$T(B) \quad = \quad \begin{array}{|c|cccccc|c|} \hline 1 & -2 & -3 & -4 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 2 & \boxed{3} & 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 2 & 0 & 1 & 0 & 4 \\ 0 & 1 & 2 & 3 & 0 & 0 & 1 & 7 \\ \hline \end{array}$$

$$\longrightarrow \quad \begin{array}{|c|cccccc|c|} \hline 1 & -2 & -\frac{1}{3} & 0 & \frac{4}{3} & 0 & 0 & \frac{20}{3} \\ \hline 0 & 0 & \frac{2}{3} & 1 & \frac{1}{3} & 0 & 0 & \frac{5}{3} \\ 0 & \boxed{1} & -\frac{1}{3} & 0 & -\frac{2}{3} & 1 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 & 2 \\ \hline \end{array}$$

$$\longrightarrow \quad \begin{array}{|c|cccccc|c|} \hline 1 & 0 & -1 & 0 & 0 & 2 & 0 & 8 \\ \hline 0 & 0 & \boxed{\frac{2}{3}} & 1 & \frac{1}{3} & 0 & 0 & \frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & 0 & -\frac{2}{3} & 1 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} & -1 & 1 & \frac{4}{3} \\ \hline \end{array}$$

$$\longrightarrow \quad \begin{array}{|c|cccccc|c|} \hline 1 & 0 & 0 & \frac{3}{2} & \frac{1}{2} & 2 & 0 & \frac{21}{2} \\ \hline 0 & 0 & 1 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -1 & 1 & \frac{1}{2} \\ \hline \end{array}$$

We can conclude that $\underline{x} = (\frac{3}{2}, \frac{5}{2}, 0, 0, 0, \frac{1}{2})^T$, i.e., $x_1 = \frac{3}{2}$, $x_2 = \frac{5}{2}$ and $x_3 = 0$, is an optimal solution of the LP with optimal objective value $\underline{c}\,\underline{x} = \frac{21}{2}$.