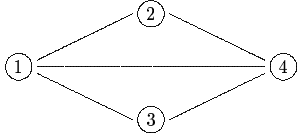
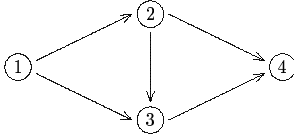
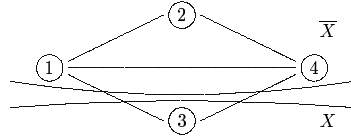
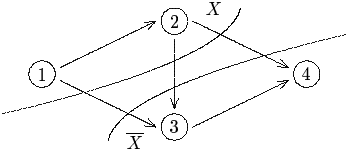


	graph G	digraph G
		
nodes	$N = \{1, 2, 3, 4\}$	$N = \{1, 2, 3, 4\}$
arcs	$A = \{[1, 2], [1, 3], [1, 4], [2, 4], [3, 4]\}$	$A = \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)\}$
degree	$d(1) = 3$	$d(3) = 3$
indegree		$d^-(3) = 2$
outdegree		$d^+(3) = 1$
path: not simple	$P = (1, 2, 4, 1, 3), l = 4$	$P = (1, 2, 4, 3, 2, 4), l = 5$
simple	$P = (1, 2, 4, 3), l = 3$	$P = (1, 2, 4, 3), l = 3$
dipath		$P = (1, 2, 3, 4)$
cycle	$C = (1, 2, 4, 1)$	$C = (1, 2, 3, 1)$
dicycle		does not exist
G acyclic?	no	yes
connectedness	is connected	is connected
disconnecting set of arcs	$Q = \{[1, 3], [3, 4], [1, 4]\}$	$Q = \{(1, 2), (1, 3), (3, 4)\}$
cut	$Q = \{[1, 3], [3, 4]\} = (X, \bar{X})$ with $X = \{3\}, \bar{X} = \{1, 2, 4\}$	$Q = \{(1, 3), (2, 3), (2, 4)\} = (X, \bar{X})$ with $X = \{1, 2\}, \bar{X} = \{3, 4\};$ $Q^+ = Q, Q^- = \emptyset$
		

subgraph		
spanning subgraph		
spanning tree T of G		
leaf nodes of T	$\{2, 3, 4\}$	$\{1, 4\}$
incidence matrix	$\mathcal{N} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$	$\mathcal{N} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$
adjacency matrix	$\mathcal{H} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$	$\mathcal{H} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Trees and spanning trees have the following properties:

Lemma: Let $T = (N, A(T))$ be a spanning tree of $G = (N, A)$. Then:

- If $|N| \geq 2$, then T has at least two leaf nodes.
- T has exactly $n - 1$ arcs.
- Every pair of nodes in T is connected by exactly one path.
- $T + a := (N, A(T) \cup \{a\})$ contains exactly one cycle ($\forall a \in A \setminus A(T)$).
- If $a \in A \setminus A(T)$ and if C is the uniquely defined cycle in $T + a$, then $T + a \setminus \tilde{a} := (N, A(T) \cup \{a\} \setminus \{\tilde{a}\})$, $\forall \tilde{a} \in C$, is a spanning tree.
- $T \setminus a := (N, A(T) \setminus \{a\})$ consists of two subtrees $(X, A(X))$ and $(\bar{X}, A(\bar{X}))$. Furthermore, $Q = (X, \bar{X})$ is a cut in G ($\forall a \in A(T)$).
- If $a \in A(T)$ and if $Q = (X, \bar{X})$ is the uniquely defined cut in G (cf. (f)), then $T \setminus a + \tilde{a} := (N, A(T) \setminus \{a\} \cup \{\tilde{a}\})$, $\forall \tilde{a} \in (X, \bar{X})$, is a spanning tree.