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Location Analysis
WS 2007/2008
Homework 3

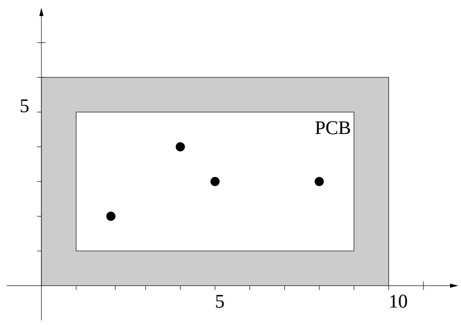
To be discussed in the tutorial on November 29, 2007.

9. Show that Lemma 3.5 is "tight" in the sense that an example with two or more local minima can be constructed if the radius is $\pi/4 + \varepsilon$, where $\varepsilon > 0$.

10. Find an algorithm to solve problems of the type $1/P/R/l_2^2/\sum$ where the forbidden region R is a non-convex polyhedron given by its corner points $(y_1, \dots, y_L)$.

    Apply the algorithm to an example problem with existing facility locations $a_1 = (1;1)$, $a_2 = (1;4)$, $a_3 = (2;1)$, $a_4 = (4;1)$, $a_5 = (4;4)$, weights $w_1 = 2$, $w_2 = 1$, $w_3 = 1$, $w_4 = 2$, $w_5 = 4$ and forbidden region R with corner points (in clockwise order) $y_1 = (0;0.4)$, $y_2 = (0;5)$, $y_3 = (7;5)$, $y_4 = (7;3)$, $y_5 = (5;3)$, $y_6 = (5;0.4)$.

11. Consider a PCB of the size $[1;9] \times [1;5]$. A circular part has to be placed at each of the locations $(2;2)$, $(4;4)$, $(5;3)$ and $(8;3)$. This is done using a robot arm which needs a time proportional to $\max\{|x_1 - y_1|, |x_2 - y_2|\}$ to move from a point $X = (x_1; x_2)$ to another point $Y = (y_1; y_2)$. Find an optimal location for a container containing the circular parts that has a security distance of 1 to the PCB using the boundary search algorithm. It is sufficient to search for the solution along the upper boundary of the forbidden region, that is, along the segment $\{(x_1; x_2) : x_1 \in [0;10], x_2 = 6\}$.



12. Prove the following result:
    Let $R \subset \mathbb{R}^2$ be closed and convex, and let $\mathcal{X}^* \subseteq \text{int}(R)$ for $1/P/\bullet/l_1/\sum$. Then there is an optimal solution X_R^* of $1/P/R/l_1/\sum$ such that

    $$X_R^* \in \partial R \quad \text{and} \quad X_R^* \in \mathcal{G}_{l_1}$$

    where

    $$\begin{aligned} \mathcal{G}_{l_1} &= \big\{(x_1, x_2) \in \mathbb{R}^2 : x_1 = a_{j_1}, j \in \{1, \dots, n\}\big\} \\ &\quad \cup \big\{(x_1, x_2) \in \mathbb{R}^2 : x_2 = a_{j_2}, j \in \{1, \dots, n\}\big\} \end{aligned}$$

    is the construction line grid with respect to the existing facility locations at $a_1, \dots, a_n$.

13. Solve the problem from exercise 11. using the construction line algorithm for $1/P/R/l_1/\sum$.