

Location Analysis, Handout 4: Multi-Facility Center Problems,

$m/P/\bullet/l_p/\max$

LP-Based Algorithm 5.2. for $m/P/\bullet/l_\infty/\max$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought;
positive weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$
and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$.

Step 1: For $k = 1, 2$ do: Solve the LP

$$\begin{array}{ll} \min & z_k \\ \text{s.t.} & \left. \begin{array}{l} -x_{ik} + \frac{1}{w_{1,ij}}z_k \geq -a_{jk} \\ x_{ik} + \frac{1}{w_{1,ij}}z_k \geq a_{jk} \end{array} \right\} \quad \forall i \in \{1, \dots, m\}, j \in \{1, \dots, n\} \\ & \left. \begin{array}{l} -x_{ik} + x_{rk} + \frac{1}{w_{2,ir}}z_k \geq 0 \\ x_{ik} - x_{rk} + \frac{1}{w_{2,ir}}z_k \geq 0 \end{array} \right\} \quad \forall i, r \in \{1, \dots, m\}, i < r \end{array}$$

and determine the optimal solution $x_{1k}^*, \dots, x_{mk}^*, z_k^*$.

Step 2: Set $X_i^* := (x_{i1}^*, x_{i2}^*)$, $i = 1, \dots, m$, $X^* := (X_1^*, \dots, X_m^*)$,
and $z^* := \max\{z_1^*, z_2^*\}$.

Output: Optimal solution X^* of $m/P/\bullet/l_\infty/\max$ with objective value z^* .

Algorithm 5.3. for $m/P/\bullet/l_1/\max$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought;
positive weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$
and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$.

Step 1: Find an optimal solution $\tilde{X}$ of the corresponding problem of type
 $m/P/\bullet/l_\infty/\max$ with existing facility locations at $T^{-1}(a_1), \dots, T^{-1}(a_n)$.

Output: Optimal solution $X^* := T(\tilde{X})$ of $m/P/\bullet/l_1/\max$.

Single-Facility Center Problems, $1/P/\bullet/l_p/\max$

Algorithm 5.7. for $1/P/\bullet/l_\infty/\max$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; positive weights $w_j, j = 1, \dots, n$.

Step 1: Determine $z^* := \max\{|z_{jl}^k| : j, l \in \{1, \dots, n\}, j < l; k \in \{1, 2\}\}$, where

$$z_{jl}^k := \frac{w_j w_l}{w_j + w_l} (a_{jk} - a_{lk}), \quad j, l \in \{1, \dots, n\}, j < l; k \in \{1, 2\}.$$

Step 2: Set $\mathcal{X}^* := [A_1^-(z^*), A_1^+(z^*)] \times [A_2^-(z^*), A_2^+(z^*)]$, where for $k = 1, 2$

$$\begin{aligned} A_k^+(z) &:= \min_{j=1, \dots, n} A_{jk}^+(z), & A_{jk}^+(z) &:= a_{jk} + \frac{1}{w_j} z, & j &= 1, \dots, n \\ A_k^-(z) &:= \max_{j=1, \dots, n} A_{jk}^-(z), & A_{jk}^-(z) &:= a_{jk} - \frac{1}{w_j} z, & j &= 1, \dots, n \end{aligned}$$

Output: Set of optimal solutions $\mathcal{X}^*$ of $1/P/\bullet/l_\infty/\max$, optimal objective value z^* .

Algorithm 5.8. for $1/P/w_j = 1/l_\infty/\max$

Input: Existing facility locations $a_1, \dots, a_n \in \mathbb{R}^2$.

Step 1: Determine

$$\begin{aligned} a_1^+ &:= \min_{j=1, \dots, n} a_{j1} & a_1^- &:= \max_{j=1, \dots, n} a_{j1} \\ a_2^+ &:= \min_{j=1, \dots, n} a_{j2} & a_2^- &:= \max_{j=1, \dots, n} a_{j2} \end{aligned}$$

Step 2: Determine

$$z_1 := \frac{a_1^- - a_1^+}{2}, \quad z_2 := \frac{a_2^- - a_2^+}{2}$$

and $z^* := \max\{z_1, z_2\}$.

Step 3: Set $\mathcal{X}^* := [a_1^- - z^*, a_1^+ + z^*] \times [a_2^- - z^*, a_2^+ + z^*]$.

Output: Set of optimal solutions $\mathcal{X}^*$ of $1/P/w_j = 1/l_\infty/\max$, optimal objective value z^* .