Location Analysis, Handout 3: Multi-Facility Weber Problems, $m/P/\bullet/l_p/\sum$

Approximation Algorithm 4.3 for $m/P/\bullet/l_p/\sum$

Input: Existing facilities $a_1,\ldots,a_n \in I\!R^2$; number m of new facilities sought; nonnegative weights $w_{1,ij}$, $i=1,\ldots,m$, $j=1,\ldots,n$ and $w_{2,ir}$, $i=1,\ldots,m-1$, $r=i+1,\ldots,m$.

Step 1: For $i=1,\ldots,m$ do:
Determine an (approximate) optimal solution $\tilde{X}_i$ for $1/P/\bullet/l_p/\sum$ with existing facility locations $a_1,\ldots,a_n$ and weights $w_{1,i1},\ldots,w_{1,in}$.

Step 2: Set $\tilde{X} := (\tilde{X}_1,\ldots,\tilde{X}_m)$ and determine

$$\Delta(\tilde{X}) := \frac{WM_{new}(\tilde{X})}{WM_{ex.}(\tilde{X})} = \frac{\sum_{i=1}^{m-1}\sum_{r=i+1}^{m} w_{2,ir}l_p(\tilde{X}_i,\tilde{X}_r)}{\sum_{i=1}^{m}\sum_{j=1}^{n} w_{1,ij}l_p(\tilde{X}_i,a_j)}$$

Output: Approximate solution $\tilde{X}$ of $m/P/\bullet/l_p/\sum$ and error bound $\Delta(\tilde{X})$.

Approximation Algorithm 4.4 for $m/P/\bullet/l_p/\sum$

Input: Existing facilities $a_1,\ldots,a_n \in I\!R^2$; number m of new facilities sought; nonnegative weights $w_{1,ij}$, $i=1,\ldots,m$, $j=1,\ldots,n$ and $w_{2,ir}$, $i=1,\ldots,m-1$, $r=i+1,\ldots,m$, desired accuracy $\varepsilon > 0$.

Step 1: For $i=1,\ldots,m$ do:
Determine an (approximate) optimal solution $\tilde{X}_i$ for $1/P/\bullet/l_p/\sum$ with existing facility locations $a_1,\ldots,a_n$ and weights $w_{1,i1},\ldots,w_{1,in}$.

Step 2: Set $\tilde{X} := (\tilde{X}_1,\ldots,\tilde{X}_m)$ and determine

$$\Delta(\tilde{X}) := \frac{WM_{new}(\tilde{X})}{WM_{ex.}(\tilde{X})} = \frac{\sum_{i=1}^{m-1}\sum_{r=i+1}^{m} w_{2,ir}l_p(\tilde{X}_i,\tilde{X}_r)}{\sum_{i=1}^{m}\sum_{j=1}^{n} w_{1,ij}l_p(\tilde{X}_i,a_j)}$$

Step 3: If $\Delta(\tilde{X}) < \varepsilon$, STOP.

Step 4: For $i=1,\ldots,m$ do:
Determine an optimal solution $\hat{X}_i$ of $1/P/\bullet/l_p/\sum$ with existing facility locations $\{a_1,\ldots,a_n\}\cup\{\tilde{X}_k : k\in\{1,\ldots,m\}\setminus\{i\}\}$ and weights $w_{1,i1},\ldots,w_{1,in},w_{2,1i},\ldots,w_{2,(i-1)i},w_{2,i(i+1)},\ldots,w_{2,im}$.

Step 5: Set $\hat{X} := (\hat{X}_1,\ldots,\hat{X}_m)$.
If $\hat{X} = \tilde{X}$, STOP.
Otherwise, set $\tilde{X} := \hat{X}$ and goto Step 3.

Output: Approximate solution $\tilde{X}$ of $m/P/\bullet/l_p/\sum$ and error bound $\Delta(\tilde{X})$.