Institute of Applied Mathematics
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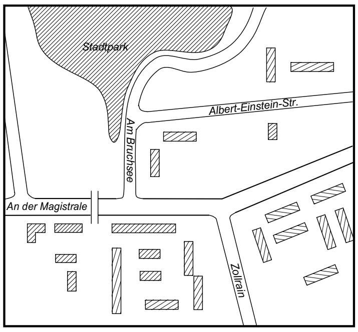
Location Analysis
WS 2004/2005
Homework 5

To be discussed in the tutorial on January 27, 2005.

16. The city council of Halle, Germany, plans to build a playground in a densly populated residential area of the city. Taking into account 18 appartment buildings in the respective area, the playground should be centrally located such that the maximum Euclidean distance from any of the blocks to the playground is minimized.



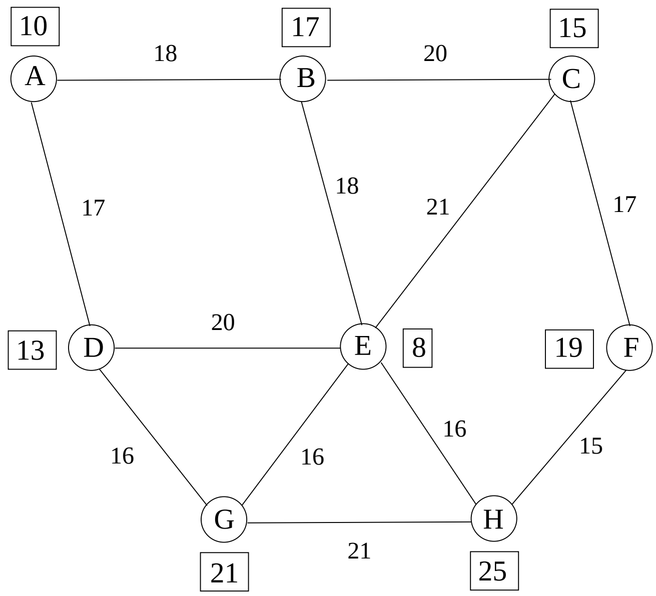
The coordinates of the entrances of the appartment blocks are specified in the following table:

Building number	Coordinates
1	$(1.5, 5)^\top$
2	$(2.5, 4)^\top$
3	$(3, 5)^\top$
4	$(3, 3)^\top$
5	$(5, 3)^\top$
6	$(5.5, 5.5)^\top$
7	$(6, 4)^\top$
8	$(6, 3.5)^\top$
9	$(6, 3)^\top$
10	$(6.5, 4.5)^\top$
11	$(7, 3.5)^\top$
12	$(10, 4.5)^\top$
13	$(10.5, 6)^\top$
14	$(11.5, 5.5)^\top$
15	$(5.5, 8.5)^\top$
16	$(6.5, 9)^\top$
17	$(9.5, 12.5)^\top$
18	$(11, 12)^\top$

Find the optimal center location for the playground using the Elzinga-Hearn Algorithm.

17. Find a mathematical programming formulation for the weighted multi-facility center problem with Euclidean distances $m/P/\bullet/l_2/\max$ and formulate the Karush-Kuhn-Tucker (KKT-) optimality conditions for this problem.

18. Consider the following network. The numbers beside each node enclosed in a box (e.g., $\boxed{10}$) are the demands associated with this node.



(a) Write out the objective function and constraints for the set covering model for this network when the coverage distance is 18. Assume that facilities can only be located on the nodes of the network.

(b) Which candidate sites can be excluded from the formulation? Which rows (corresponding to the need to cover specific demands) can be excluded?

(c) After you have reduced the problem as suggested in part (b), are there any sites at which you must locate facilities? If so, where? Why? If any sites are now forced into the solution, which demand nodes are now covered?

(d) Write out the remaining covering problem. That is, write out the constraint matrix for the remaining candidate sites and uncovered demand nodes.

(e) What is the linear programming relaxation solution to the problem?

(f) What is the solution to the (original) set covering problem? Specifically, where do you locate facilities and what is the objective function value?

19. There are often multiple alternate optima for the set covering location problem. This suggests that we can append secondary objectives to the problem to select from among the alternate optima to the primary objective problem (that of minimizing the number of facilities needed to cover all demands) a solution that best attains some secondary objective.

(a) For the network shown in Problem 18, write out as many alternate optima as possible to the set covering problem with a coverage distance of 18.

(b) Formulate the following problem:

Primary objective: Minimize the number of facilities needed to cover all demand *nodes* at least once with a coverage distance D_c^1.

Secondary objective: Maximize the number of *demands* (as opposed to nodes) that are covered at least twice with a coverage distance D_c^2 (which may be different from D_c^1).

(c) Suggest a means of solving this problem.

(d) Solve the problem for the network shown in Problem 18 using coverage distances of $D_c^1 = 18$ and $D_c^2 = 18$.