Location Analysis WS 2004/2005 Homework 3

To be discussed in the tutorial on December 9, 2004.

9. Find an algorithm to solve problems of the type $1/P/R/l_2^2/\sum$ where the forbidden region R is a non-convex polyhedron given by its corner points (y_1, \ldots, y_L) .

Apply the algorithm to an example problem with existing facility locations $a_1 = (1; 1)$, $a_2 = (1; 4)$, $a_3 = (2; 1)$, $a_4 = (4; 1)$, $a_5 = (4; 4)$, weights $w_1 = 2$, $w_2 = 1$, $w_3 = 1$, $w_4 = 2$, $w_5 = 4$ and forbidden region R with corner points (in clockwise order) $y_1 = (0; 0.4)$, $y_2 = (0; 5)$, $y_3 = (7; 5)$, $y_4 = (7; 3)$, $y_5 = (5; 3)$, $y_6 = (5; 0.4)$.

10. Consider a PCB of the size $[1;9] \times [1;5]$. A circular part has to be placed at each of the locations (2;2), (4;4), (5;3) and (8;3). This is done using a robot arm which needs a time proportional to max $\{|x_1 - y_1|, |x_2 - y_2|\}$ to move from a point $X = (x_1; x_2)$ to another point $Y = (y_1; y_2)$. Find an optimal location for a container containing the circular parts that has a security distance of 1 to the PCB using the boundary search algorithm. It is sufficient to search for the solution along the upper boundary of the forbidden region, that is, along the segment $\{(x_1; x_2) : x_1 \in [0; 10], x_2 = 6\}$.



11. Prove Lemma 3.21:

Let $R \subset \mathbb{R}^2$ be closed and convex, and let $\mathfrak{X}^* \subseteq \operatorname{int}(R)$ for $1/P/ \bullet /l_1 / \sum$. Then there is an optimal solution X_R^* of $1/P/R/l_1 / \sum$ such that

$$X_R^* \in \partial R$$
 and $X_R^* \in \mathcal{G}_{l_1}$

where

$$\mathfrak{G}_{l_1} = \{ (x_1, x_2) \in \mathbb{R}^2 : x_1 = a_{j_1}, j \in \{1, \dots, n\} \} \\
\cup \{ (x_1, x_2) \in \mathbb{R}^2 : x_2 = a_{j_2}, j \in \{1, \dots, n\} \}$$

is the construction line grid with respect to the existing facility locations at a_1, \ldots, a_n .

12. Solve the problem from exercise 10. using the construction line algorithm for $1/P/R/l_1/\sum$.