

Location Analysis, Handout 3: Multi-Facility Weber Problems, $m/P/\bullet/l_p/\Sigma$

Approximation Algorithm 4.3 for $m/P/\bullet/l_p/\Sigma$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought;
nonnegative weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$
and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$.

Step 1: For $i = 1, \dots, m$ do:
Determine an (approximate) optimal solution \tilde{X}_i for $1/P/\bullet/l_p/\Sigma$
with existing facility locations a_1, \dots, a_n and weights $w_{1,i1}, \dots, w_{1,in}$.

Step 2: Set $\tilde{X} := (\tilde{X}_1, \dots, \tilde{X}_m)$ and determine

$$\Delta(\tilde{X}) := \frac{WM_{new}(\tilde{X})}{WM_{ex.}(\tilde{X})} = \frac{\sum_{i=1}^{m-1} \sum_{r=i+1}^m w_{2,ir} l_p(\tilde{X}_i, \tilde{X}_r)}{\sum_{i=1}^m \sum_{j=1}^n w_{1,ij} l_p(\tilde{X}_i, a_j)}$$

Output: Approximate solution \tilde{X} of $m/P/\bullet/l_p/\Sigma$ and error bound $\Delta(\tilde{X})$.

Approximation Algorithm 4.4 for $m/P/\bullet/l_p/\Sigma$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought;
nonnegative weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$
and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$,
desired accuracy $\varepsilon > 0$.

Step 1: For $i = 1, \dots, m$ do:
Determine an (approximate) optimal solution \tilde{X}_i for $1/P/\bullet/l_p/\Sigma$
with existing facility locations a_1, \dots, a_n and weights $w_{1,i1}, \dots, w_{1,in}$.

Step 2: Set $\tilde{X} := (\tilde{X}_1, \dots, \tilde{X}_m)$ and determine

$$\Delta(\tilde{X}) := \frac{WM_{new}(\tilde{X})}{WM_{ex.}(\tilde{X})} = \frac{\sum_{i=1}^{m-1} \sum_{r=i+1}^m w_{2,ir} l_p(\tilde{X}_i, \tilde{X}_r)}{\sum_{i=1}^m \sum_{j=1}^n w_{1,ij} l_p(\tilde{X}_i, a_j)}$$

Step 3: If $\Delta(\tilde{X}) < \varepsilon$, STOP.

Step 4: For $i = 1, \dots, m$ do:
Determine an optimal solution \hat{X}_i of $1/P/\bullet/l_p/\Sigma$
with existing facility locations $\{a_1, \dots, a_n\} \cup \{\tilde{X}_k : k \in \{1, \dots, m\} \setminus \{i\}\}$
and weights $w_{1,i1}, \dots, w_{1,in}, w_{2,1i}, \dots, w_{2,(i-1)i}, w_{2,i(i+1)}, \dots, w_{2,im}$.

Step 5: Set $\hat{X} := (\hat{X}_1, \dots, \hat{X}_m)$.
If $\hat{X} = \tilde{X}$, STOP.
Otherwise, set $\tilde{X} := \hat{X}$ and goto Step 3.

Output: Approximate solution \tilde{X} of $m/P/\bullet/l_p/\Sigma$ and error bound $\Delta(\tilde{X})$.