

## Deligne-Lusztig varieties and representations of finite groups of Lie type

We want to study DL-varieties with an eye towards their role in representation theory of finite groups of Lie type. These varieties were introduced by Deligne and Lusztig [DL] in 1976 for the construction of Harish-Chandra induction with respect to non-rational parabolic subgroups. Moreover they give rise to a cohomological realization of irreducible representations of the corresponding finite group of Lie type.

### Sessions

#### 1. Session: 28.04.16, W'tal

Talk 1:  $\mathbb{F}_q$ -structures on varieties and algebraic groups, Martin Bender

Summarize [DiM, Section 3, p. 33–37], and explain [DiM, 3.17] (Classification of simply-connected simple groups and their possible  $\mathbb{F}_q$ -structures). See also [DuM] §6 and [C] §1.

Talk 2: Theorem of Lang(-Steinberg), Sven Meinhardt

Give a proof of the Lang(-Steinberg) Theorem (p. 38). Discuss its applications, p. 38-44 together with the mentioned examples. See also [DuM] §7.

#### 2. Session: 12.05.16, D'dorf

Talk 1: Harish-Chandra induction I, Lucas Ruhstorfer

Give the definition of Harish-Chandra induction [DiM, §4] and their basic properties. ( See also [DuM, §10] and [C, 9.1, 9.2].)

Talk 2: Harish-Chandra induction II, Steffen Kionke

Summarize [DiM, §6,7] and give a rough idea of the Mackey formula from section 5 (without proof). See also [DuM] §10.

#### 3. Session: 2.6.16, W'tal

Talk 1: DL-varieties, Stefan Schroer

Introduce (classical) DL-varieties and explain their basic properties. Treat the case of  $G = \mathrm{GL}_n(q) = \mathrm{GL}_n(\mathbb{F}_q)$  and that of Coxeter elements following [Du, §3] (with  $r = 1$  and  $w \in W$ ) and [DL, §1, §2].

Talk 2:  $\ell$ -adic cohomology, Leif Zimmermann

Present the basic properties of  $\ell$ -adic cohomology [Du, §4] , [C, 7.1, Appendix] and apply them to DL-varieties (Examples).

#### 4. Session: 23.6.16, D'dorf

Talk 1: Deligne-Lusztig theory I, Peter Arndt

Summarize [Du, §5, 5.1 - 5.3].

Talk 2: Deligne-Lusztig theory II

Summarize [Du, §5, 5.4 - 5.7].

### 5. Session: 14.7.16, W'tal

Talk 1: Lusztig series and classification of characters of finite groups of Lie type, Kevin Langlois

Summarize [Du, §7].

Talk 2: Examples, Alexander Samokhin

Describe the representation theory of  $G = \mathrm{SL}_2(\mathbb{F}_q)$  and  $G = \mathrm{Sp}_4(\mathbb{F}_q)$ , [Du, §7]. Evtl. describe the (unipotent) characters of  $\mathrm{GL}_n(\mathbb{F}_q)$  as in [DiM, Section 15.4].

## References

- [B] C. Bonnafé. *Representations de  $\mathrm{SL}_2(q)$* , Springer, 2011.
- [C] R. Carter. *Finite groups of Lie type, conjugacy classes and complex characters*, Wiley, 1985.
- [DL] P. Deligne, G. Lusztig. *Representations of reductive groups over finite fields*. Ann. of Math. **103** (1976), 103–161.
- [DiM] F. Digne, J. Michel. *Representations of finite groups of Lie type*, Cambridge, 1991.
- [Du] O. Dudas. *Introduction à la théorie de Deligne-Lusztig*, Cours de M2, Université Paris-Diderot, 2014. <https://webusers.imj-prg.fr/olivier.dudas/intro.pdf>
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