WORKSHOP DÜSSELDORF/WUPPERTAL: ALGEBRA AND GEOMETRIE SS 2015 TOPIC: BRAUER GROUPS

ORGANIZERS: HORNBOSTEL, KLOPSCH, ORLIK,
PERRIN, REINEKE, SCHRÖER, ZIBROWIUS

Facts

As usual, we convene on thursday, alternatingly in Düsseldorf and Wuppertal, this semester with six meetings althogether, each with two 60-minutes-talks. The idea is to go through some basic sources on Brauer groups, and have some more research-oriented talks as well.

Meeting	Date	Where	Talk	Speaker
1	16.4.	BUW	1	Steffen Kionke
	16.4.	BUW	2	Falk Beckert
2	7.5.	HHU	3	Thorsten Weist
	7.5.	HHU	4	Sven Meinhardt
3	21.5.	BUW	5	Marcus Zibrowius
	21.5.	BUW	6	Jens Hornbostel
4	11.6.	HHU	7	Richard Gonzales Vilcarromero
	11.6.	HHU	8	Markus Reineke
5	25.6.	BUW	9	Leif Zimmermann
	25.6.	BUW	10	Saša Novaković
6	16.7.	HHU	11	Magdalena Boos
	16.7.	HHU	12	TBA

Düsseldorf: Room 25.22.03.73, Time 16:30.
Wuppertal: Floor F13, Room TBA, Time 16:00.

Introduction

In this workshop we want to study some texts on of *Brauer groups*. These groups can be viewed as analogous of the Picard group in degree two, and play an important role in various branches of algebra, geometry and arithmetic. They appear, for examples, in representation theory, moduli problems, and class field theory. The main goal is that everybody gains basic working knowledge.

Roughly speaking, the Brauer group $\mathrm{Br}(R)$ of a ring R is an abelian torsion group, whose elements are represented by twisted forms of matrix algebras $\mathrm{Mat}_n(R)$, and where addition comes from the tensor product of algebra. The designation refers to Richard Brauer (1901–1977), who is famous for his work on representations of finite groups and simple algebras. To my knowledge, the first textbook occurrence of the term "Brauer group" might be [Deuring 1935].

A thorough account for the theory of central simple algebras appears in [Bourbaki 1958]. Brauer groups for rings were introduced by Auslander and Goldman [1960]. The definite treatment, emphasizing the role schemes and étale cohomology, was given by Grothendieck [1968] in a book-length series of three papers. Other notable sources are the hands-on approach of Farb and Dennis [1993], and the short but dense lecture notes of Saltman [1999].

In this workshop, however, our goals are more modest. After a group-theoretical warm-up, we start with the first two chapters of the beautiful book of Gille and Szamuely [2006], which treat quaternion algebras and the definitions of Brauer groups over fields. We then continue with Chapter IV in Milne [1980], which defines Brauer groups for rings and relates them to étale cohomology. Finally, we read Artin's expository lecture [1982], coming to geometric interpretations in terms of Brauer–Severi varieties.

First Meeting

Talk 1. Steffen Kionke: The Schur multiplier. Source: [Karpilovsky 1987], Section 2.3 or others. Discuss the notion of projective representations $G \to \mathrm{PGL}(V)$, and explain that the obstruction to lift a projective representation to a linear representation is an element in $H^2(G, K^\times)$. The latter group is also called the Schur multiplier. Explain some computations of the Schur multiplier of your choice.

Talk 2. Falk Beckert: Quaternion algebras. Source: [Gille and Szamuely 2006], Chapter 1. Introduce the notion of quaternion algebras $(a,b)_K$ and their tensor products. Also, discuss the associated smooth curve $C \subset \mathbb{P}^2$ of degree two, and the Theorem of Witt.

Second Meeting

Talk 3. Thorsten Weist: Central simple algebras and Brauer groups. Source: [Gille and Szamuely 2006], Sections 2.1–2.3. Explain the notion of central simple algebras (CSA) and discuss their structure, using the Theorem of Wedderburn.

Talk 4. Sven Meinhardt: The Brauer group. Source: [Gille and Szamuely 2006], Chapter 2, Sections 2.4–2.5. Define the Brauer group for fields, and discuss the notion of cyclic algebras.

Third Meeting

Talk 5. Marcus Zibrowius: The Brauer groups for rings and schemes. Source: [Milne 1980], Chapter IV, pp. 136–146. Explain the notion of Azumaya algebras, which should be regarded as families of CSAs. Give a very brief discussion of étale cohomology, and explain the inclusion $\mathrm{Br}(X) \subset H^2(X, \mathbb{G}_m)$. Skip the gerbe-part if you feel uncomfortable with it.

Talk 6. Jens Hornbostel: Motivic cohomology (Research talk)

Fourth Meeting

Talk 7. Richard Gonzales Vilcarromero: The cohomological Brauer group. Source: [Milne 1980], Chapter IV, pp. 146–154. Discuss the relation between the Brauer group and the cohomological Brauer group. Mention Gabber's result and de Jong's proof [2006], without going into details.

Talk 8. Markus Reineke: Brauer groups for quiver moduli (Research talk).

Fifth Meeting

Talk 9. Leif Zimmermann: Brauer–Severi varieties. Source: [Artin 1982], see also [Gille and Szamuely 2006], Section 5.3. Discuss the notation of Brauer–Severi varieties and their relation to CSAs.

Talk 10. Saša Novaković: Vector bundles on Brauer–Severi varieties (Research talk).

Sixth Meeting

Talk 11. Magdalena Boos: TBA (Research talk).

Talk 12: TBA (Research talk).

Literatur

Artin 1982: Brauer–Severi varieties. In: F. van Oystaeyen, A. Verschoren (eds.), Brauer groups in ring theory and algebraic geometry, pp. 194-210, Springer, Berlin-New York.

Auslander and Goldman 1960: The Brauer group of a commutative ring. Trans. Amer. Math. Soc. 97, 367–409.

Bourbaki 1958: Algèbre. Chapitre 8. Modules et anneaux semi-simples. Hermann, Paris.

de Jong 2006: A result of Gabber. Preprint, http://www.math.columbia.edu/ dejong/

Deuring 1935: Algebren. Springer, Berlin.

Farb and Dennis 1993: Noncommutative algebra. Graduate Texts in Mathematics 144. Springer, New York.

Gille and Szamuely 2006: Central simple algebras and Galois cohomology. Cambridge University Press, Cambridge.

Grothendieck 1968: Le groupe de Brauer. In: J. Giraud (ed.) et al.: Dix exposés sur la cohomologie des schémas, pp. 46–189. North-Holland, Amsterdam.

Karpilovsky 1987: The Schur multiplier. London Math. Soc. Monogr. 2. Clarendon Press, New York.

Milne 1980: Étale cohomology. Princeton University Press, Princeton.
Saltman 1999: Lectures on division algebras. American Mathematical Society, Providence, RI.