

Regeln zur bestimmten Divergenz

$$i) \quad |a_n| \rightarrow \infty \Leftrightarrow \frac{1}{a_n} \rightarrow 0$$

$$a_n \rightarrow 0 \Leftrightarrow \left| \frac{1}{a_n} \right| \rightarrow \infty$$

$$ii) \quad a_n \rightarrow \infty, b_n \rightarrow b \stackrel{b \in \mathbb{R}}{\neq}$$

$$\Rightarrow a_n + b_n \rightarrow \infty \quad \text{und} \quad a_n b_n \rightarrow \begin{cases} \infty, & \text{falls } b > 0 \\ -\infty, & \text{" } b < 0 \end{cases}$$

($b = 0 \rightarrow$ keine Aussage)

$$iii) \quad a_n \rightarrow \infty, b_n \rightarrow \infty$$

$$\Rightarrow a_n + b_n \rightarrow \infty, \quad a_n b_n \rightarrow \infty$$

$$\text{Bsp 1)} \quad \lim_{n \rightarrow \infty} \left(n + \frac{n+1}{n-1} \right) = \lim_{n \rightarrow \infty} \left(\underbrace{n}_{\rightarrow \infty} + \underbrace{\frac{1 + \frac{1}{n}}{1 - \frac{1}{n}}}_{\rightarrow 1} \right) = \infty$$

$$\begin{aligned} 2) \quad \lim_{n \rightarrow \infty} \frac{n^2}{1-n} &= \lim_{n \rightarrow \infty} \frac{n^2}{n \cdot \left(\frac{1}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n}{\frac{1}{n} - 1} \\ &= \lim_{n \rightarrow \infty} n \cdot \frac{1}{\underbrace{\frac{1}{n} - 1}_{\rightarrow -1}} = -\infty \end{aligned}$$

!! Achtung bei " $\infty \cdot 0$ "

$$\lim_{n \rightarrow \infty} \underbrace{n \cdot \frac{1}{n^2}}_{\rightarrow " \infty \cdot 0 "} = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow \infty} \underbrace{n^2 \cdot \frac{1}{n}}_{" \infty \cdot 0 "} = \lim_{n \rightarrow \infty} n = \infty$$

$$\lim_{n \rightarrow \infty} \underbrace{2n \cdot \frac{3}{n}}_{" \infty \cdot 0 "} = \lim_{n \rightarrow \infty} (2 \cdot 3) = 6$$