

14th Internet Seminar 2010/2011

Infinite-Dimensional Linear Systems Theory

The 14th Internet Seminar on Evolution Equations is devoted to mathematical systems theory. This field has its origin in engineering problems, such as controlling engines and telephone transmission. Applications still play an important role within systems theory and they have opened up many new research topics. In this course we concentrate on infinite-dimensional systems theory, wherein functional analysis is prominently present. Among others, infinite-dimensional systems included models described by partial differential equations. Apart from introducing the general description of infinite-dimensional systems, we treat some control problems, such as stabilization, and we apply the obtained results to physical models.

The lectures are at a beginning graduate level and assume only basic familiarity with functional analysis.

Organised by the German-Italian consortium “International School on Evolution Equations”, the annual Internet Seminars introduce PhD students and postdocs to varying subjects related to evolution equations. Each course consists of three phases. In Phase 1 (October–February), a weekly lecture is provided via the ISEM website. In Phase 2 (March–May), the participants form small international groups to work on projects. The projects complement the theory of Phase 1 and provide some applications of it. Phase 3 (June) consists of the final one-week workshop in Blaubeuren (Germany). The teams present their projects, and lectures will be delivered by leading experts in the field.

The 14th Internet Seminar 2010/2011 starts mid October 2010 with Phase 1 and ends with the one-week workshop in Blaubeuren (June 5-11, 2011) sponsored by the Volkswagen Foundation.

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Further information: <http://www.math.uni-wuppertal.de/~fa/isem.html>

Description of the course

A linear system is described by the (linear) differential equation

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), & t \geq 0, \quad x(0) = x_0 \\ y(t) &= Cx(t) + Du(t),\end{aligned}$$

where u is the input (control), x is the state, and y is the output (observations). These functions take their values in the linear space U , X , and Y , respectively. Furthermore, A , B , C , and D are linear mappings between the appropriate spaces.

The central theme within systems theory is to design u such that y and/or x has a desired behavior. A typical example is stabilization: Design an input u such that $x(t) \rightarrow 0$ for $t \rightarrow \infty$.

Motivated by the engineering applications, the construction of the control function u should be robust under small errors in the initial condition x_0 and the system parameters, A , B , C , and D . A control designed by feedback achieves this goal. Feedback means that we do not calculate u only based on x_0 , A , B , C , and D , but also on the current state. This results in a control of the form $u(t) = Fx(t)$, and the question of stabilization turns into a question of finding F .

If the spaces U , X , and Y are finite-dimensional, then the system is called finite-dimensional. Otherwise, we have an infinite-dimensional linear system. Since we assume no prior knowledge of systems theory, we begin with some of the highlights of finite-dimensional systems theory. Linear algebra and functional analysis play an important role in finite- and infinite-dimensional systems theory, respectively. The necessary background in functional analysis will be listed in the appendix and the references of the lecture notes.

Infinite-dimensional linear systems appear naturally when studying control problems for systems modelled by linear partial differential equations. Further, many physical models have a conserved quantity, like mass or energy. It is very beneficial to use this quantity in the design of a controller. This has led to the class of port-Hamiltonian systems, which is one of the classes we treat in more detail. Port-Hamiltonian systems with boundary control and observations can be treated with only a minimal of mathematical technicalities. Topics that will be studied in this course are:

- Controllability and stabilizability of finite-dimensional systems
- Strongly continuous semigroups
- Stabilizability for infinite-dimensional systems
- Boundary control for partial differential equations
- Transfer functions
- Stabilizability of port-Hamiltonian systems
- Well-posedness for linear infinite-dimensional systems