Implementation of [2]

The programs Freebound1.p, Freebound2.p, Freebound3.p and Freebound4.p consider a free boundary problem which is defined as follows:

Let $f : [0, \infty) \times \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ be a function and $y_0 > 0$. Then the ordinary free boundary problem is defined as follows:

$$\left.\begin{array}{rl} \text{Find } c \in \mathbf{R} \quad \text{and} & y(\cdot) : [0, \infty) \rightarrow \mathbf{R} \text{ with} \\ y''(x) = f(x, y(x), y'(x)) & \text{if } x \in [0, c], \\ y(x) > 0 & \text{if } x \in [0, c), \\ y(x) = 0 & \text{if } x \in [c, \infty), \\ y'(c) = 0, & y(0) = y_0. \end{array}\right\} \quad (1)$$

Example 1: Freebound1.p considers

$$f(x, y, y') = \sqrt{1 + (y')^2}.$$

Example 2: Freebound2.p considers

$$f(x, y, y') = 4y + 4e^{2x}.$$

Example 3: Freebound3.p considers

$$f(x, y, y') = y + arctan(xy') + \pi.$$

Example 4: Freebound4.p considers

$$f(x, y, y') = \sqrt{1 + y^2}.$$

These examples have got a unique solution $(\tilde{c}, \tilde{y}(\cdot))$. Either this solution can be calculated by hand very easily or the conditions of Theorem 3.1 in [3] are fulfilled which guarantee the unique solvability. All algorithms were implemented using PASCAL-XSC (see [1]). The input of Freebound1.p, Freebound2.p, Freebound3.p and Freebound4.p, respectively, is the initial value y_0 and n which is the number of components where the solution $\tilde{y}(\cdot)$ is included by an interval. The output of Freebound1.p, Freebound2.p, Freebound3.p and Freebound4.p, respectively, is an inclusion of $\tilde{c}$ of the corresponding free boundary problem.

Remark. There are techniques that improve the running time. Those implementations will come soon.

References

[1] Klatte, R., Kulisch, U., Neaga, M., Ratz, D. and Ullrich, Ch.: *PASCAL-XSC - Language Reference with Examples*, Springer-Verlag, Berlin, 1992.

[2] Schäfer, U.: An enclosure method for free boundary problems based on a linear complementarity problem with interval data, *Numer. Funct. Anal. Optim.*, **22** (2001), pp. 991-1011.

[3] Schäfer, U.: Unique solvability of an ordinary free boundary problem , *Rocky Mt. J. Math.*, to appear.