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Multiple Precision Interval Arithmetic:

Comparing some Packages

Passo Fundo/Brazil, August 2004

Overview

1. Types of packages
2. Languages and Libraries
3. Packages for dedicated Scientific Computing Software Environments

Some Packages

Types of Packages

- Libraries for Programming Languages
  - C/C++: C-XSC, MPFI, GMP-XSC
- Packages for Scientific Computing Environments
  - Matlab: Intlab
  - Maple: `intpakX`

Objectives

Libraries:

- Extend mathematical capabilities of the language
- Efficiency
- Keep features of the language

Scientific Computing Packages:

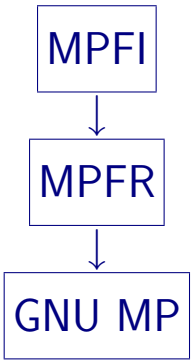
- Support special scientific computing features (symbolical computations, matrix computations)
- Convenience
- Make use of graphics

Libraries

MPFI

MPFI:

- Interval extension of the MPFR library
- based on GNU Multiple precision library



MPFR: Extension of GMP with correct rounding (suitable as a base for interval computation)

Authors of MPFI: Nathalie Revol, Fabrice Rouillier, ENS Lyon, France

Libraries: MPFI

MPFI Overview

- C library compliant to IEEE 754 floating point arithmetic
- Arbitrary precision variables ($\leftrightarrow$ arbitrary number of mantissa digits)
- Real arithmetic
- Correct rounding (taken from MPFR)

Libraries: MPFI

MPFI Features

- All computations done via C functions
- Full set of basic operations and standard functions available
- Set operations and some special functions also available
- String I/O

Libraries: MPFI

Examples of functions

- Initialization and assignment:

  `int mpfi_init_set (mpfi_t rop, mpfi_t op)`

  Initializes `rop` and sets its value from `op`, outward rounded so that `op` belongs to `rop`. The precision of `rop` will be taken from the active default precision, as set by `mpfr_set_default_prec`. The returned value indicates whether none, one or both endpoints are exact.

- Setting the precision:

  `int mpfi_set_prec (mpfi_t x, mp_prec_t prec)`

  Resets the precision of `x` to be exactly `prec` bits. The previous value stored in `x` is lost. It is equivalent to a call to `mpfi_clear(x)` followed by a call to `mpfi_init2(x, prec)`, but more efficient.

Libraries: MPFI

Examples of functions

- Basic operations:

```
int mpfi_add      (mpfi_t rop, mpfi_t op1, mpfi_t op2)
int mpfi_add_d    (mpfi_t rop, mpfi_t op1, double op2)
```

Carries out the appropriate addition.

$\longrightarrow$ Separate versions for different parameter types

MPFI: Interval features added to multiple precision library

Libraries

GMP-XSC

- Also based on GMP
- Package designed to serve special purposes in integral computation
- Designed as a future add-on for the XSC languages
- Faster than MPI, but no full set of operations

Author: Knut Petras, TU Braunschweig, Germany

Libraries

XSC languages

- Pascal and C/C++ Libraries, C-XSC extending "standard" C/C++ floating point arithmetic
- Fixed multiple precision called *staggered precision* (using n floating point variables)
- Real and complex arithmetic, vector and matrix computations
- Correct rounding

Authors: W. Krämer et al., Universities of Wuppertal and Karlsruhe, Germany

Libraries: C-XSC

C-XSC Features

- Full set of basic operations and standard functions
- Special functions and string I/O available
- Overloaded C/C++ operators $\rightarrow$ Denotation of expressions does not need to be changed
- Exact scalar product via *dotprecision* accumulator

C-XSC: Multiple precision added to interval library

Libraries: C-XSC

C-XSC Example

```
stagprec=6;                  // set staggered precision

str1 >> RndNext >> x1;       // string input and rounding

l_imatrix L_IA1(dim,dim),L_IA2(dim,dim),L_IR(dim,dim);
                             // construct staggered variables

L_IR=L_IA1*L_IA2;            // Matrix multiplication
                                with staggered data
```

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Packages

Intlab

- Interval package for Matlab (all Matlab Code)
- Fast real/complex interval computations based on BLAS routines
- Rudimentary multiple precision arithmetic (according to the author)

Author: S. Rump, TU Hamburg-Harburg

Packages: Intlab

Intlab Features

Intlab Multiple precision arithmetic:

- „long“ data type
- „Automatic“ choice of operators ($\approx$ Overloading)
- Reduced set of operations: basic operations (`+`,`-`,`*`,`/`), exponential and power function
- No vectors/matrices possible

Packages

`intpakX`

- Interval package for Maple (all Maple code)
- Arbitrary precision: Built-in Maple feature (precision can be set to an arbitrary value)
- Real arithmetic and complex disc arithmetic
- Correct rounding
- Some built-in applications

Authors: W. Krämer, I. Geulig, M. Grimmer, Universities of Wuppertal and Karlsruhe, Germany

Packages: intpakX

intpakX Features

- Full set of basic operations and standard functions for real arithmetic
- Reduced set of operations for complex disc arithmetic
- Special (but no overlaoded) operators $\longrightarrow$ Denotation of expressions similar to pseudocode
- Graphical output of the included applications (see detailed intpakX description)

Comparison - Timing

Matrix multiplications:

Data Type/Matrix Size	15 Digits	90 Digits	540 Digits
Maple float			
20×20	0.86	1.85	6.86
intpakX interval			
20×20	20.16	23.38	63.59

C-XSC:

Size	imatrix	l_imatrix (2 reals)	l_imatrix (6 reals)
20x20	0.07	0.15	0.68
200x200	63.70	132.38	663.07

GMP-XSC:

Size	15	30	90	540 Digits
20x20	0.07	0.09	0.09	0.09
200x200	79.10	81.60	86.20	121.28

Comparison - Timing

Maple:

	Maple float (90 Digits)	intpakX int. (90 Digits)	ratio
$sin(x)$	4.63	19.42	4.1
$exp(x)$	2.60	4.20	1.6

C-XSC:

	interval	l_interval (2 reals)	l_interval (6 reals)
$sin(x)$	0.0014	17.61	57.20
$exp(x)$	0.0012	17.74	78.78

GMP-XSC:

	15	30	90 Digits
$sin(x)$	0.22	0.30	0.74
$exp(x)$	0.16	0.23	0.52

References/Contact

MPFI:
http://perso.ens-lyon.fr/nathalie.revol/software.html

GMP-XSC:
http://www.tu-bs.de/~petras/software.html

XSC languages:
http://www.xsc.de

intpakX:
http://www.math.uni-wuppertal.de/wrswt/software/intpakX/
or from Waterloo Maple on
http://www.mapleapps.com/powertools/ResearchApplication.shtml.

Intlab:
http://www.ti3.tu-harburg.de/~rump/intlab/index.html