Monomial characters of finite solvable groups

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Let G be a finite group, let $\mathrm{Irr}(G)$ be the set of irreducible complex characters and denote by $\mathrm{Irr}_m(G)$ the set of monomial irreducible characters of G. We show that, if G is solvable, then in many situations the set $\mathrm{Irr}_m(G)$ gives us plenty of information about the group G.

First we consider consider nonvanishing elements. An element $g \in G$ is said to be nonvanishing if $\chi(g) \neq 0$, for all $\chi \in \mathrm{Irr}(G)$. We denote the set of all nonvanishing elements by $\mathcal{N}(G)$. In [1] Isaacs, Navarro and Wolf conjectured that if G is a solvable group, then $\mathcal{N}(G) \subseteq \mathbf{F}(G)$. Here we consider monomial-nonvanishing elements i.e. elements $g \in G$ such that $\chi(g) \neq 0$, for all $\chi \in \mathrm{Irr}_m(G)$. If $\mathcal{N}_m(G)$ is the set of all monomial-nonvanishing elements we show that, under suitable conditions and if G is solvable, then we also have $\mathcal{N}_m(G) \subseteq \mathbf{F}(G)$.

Then we consider a modular version of the above problem. If p is a prime and g is a p-regular element of G we say that g is p-nonvanishing if $\varphi(g) \neq 0$, for all $\varphi \in \mathrm{IBr}_p(G)$. Similarly we say that a p-regular element g is p-monomial-nonvanishing if $\varphi(g) \neq 0$, for all $\varphi \in \mathrm{IBr}_{p,m}(G)$. If $\mathcal{N}_p(G)$ is the set of all p-nonvanishing elements and $\mathcal{N}_{p,m}(G)$ is the set of all p-monomial-nonvanishing elements, then we show that:

(i) If G is supersolvable, then $\mathcal{N}_p(G/\mathbf{O}_p(G)) \subseteq \mathbf{Z}(\mathbf{F}(G/\mathbf{O}_p(G)))$.

(ii) If G is a group of odd order and p is the smallest prime divisor of $|G|$, then $\mathcal{N}_{p,m}(G) \subseteq \mathbf{F}_{p'}(G)$, where $\mathbf{F}_{p'}(G) = \bigcap_{q \neq p} \mathbf{O}_{q'q}(G)$.

References

[1] I. M. Isaacs, G. Navarro, T. R. Wolf, Finite group elements where no irreducible character vanishes, J. Algebra **222** (1999), 413–423.