Homework 1

Summer Term 2014

BERGISCHE UNIVERSITÄT WUPPERTAL

Fachbereich C - Mathematik und Naturwissenschaften
Arbeitsgruppe Optimierung und Approximation
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Problem 1.1:
Show that the Lorentz cone

$$\mathcal{L}^{n+1} = \left\{(x,t) \in \mathbb{R}^{n+1} \,:\, \|x\| \leq t\right\}$$

is self-dual, i.e., $(\mathcal{L}^{n+1})^* = \mathcal{L}^{n+1}$.

Problem 1.2:
Prove Lemma 3.4: Let $K, K_1, K_2 \subseteq \mathbb{R}^n$ be convex cones. Then

(1) K^* is a closed and convex cone

(2) $K_1 \subseteq K_2 \;\Rightarrow\; K_2^* \subseteq K_1^*$

(3) $\mathrm{int}(K^*) = \{y \in \mathbb{R}^n \,:\, y^{\mathrm{T}}x > 0 \,\forall x \in K \setminus \{0\}\}$

(4) $\mathrm{int}(K) \neq \emptyset \;\Rightarrow\; K^*$ pointed

(5) $(K^*)^* = \mathrm{cl}(K)$

(6) $\mathrm{cl}(K)$ pointed $\Rightarrow \mathrm{int}(K^*) \neq \emptyset$

Problem 1.3:
Prove Lemma 1.9: Let $A \in \mathcal{S}^n$. Then

$$A \succeq 0 \quad \Leftrightarrow \quad \forall B \in \mathcal{S}^n_{\succeq 0} \,:\, \langle A, B \rangle \geq 0.$$