Standort-Optimierung

5. Übung

Sommersemester 2010

Bergische Universität Wuppertal
Fachbereich C – Angewandte Mathematik / Optimierung und Approximation
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Besprechung der Aufgaben: Dienstag 1. Juni 2010

Aufgabe 17:

a) Prove Lemma 4.7, i.e., show that

$$\max_{\substack{x=(x_1,\ldots,x_m)\\ x_i\in\mathbb{R}^2,\ i=1,\ldots,m}} \{WMH(x) - WM(x)\} \le \sqrt[p]{2}\sqrt{\epsilon}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} w_{1,ij} + \sum_{i=1}^{m-1}\sum_{r=i+1}^{m} w_{2,ir}\right)$$

holds.

b) Motivate and verify, that

$$x_{rk}^{(l+1)} = \frac{SX_{rk}^{(l)} + SA_{rk}^{(l)}}{S_{1rk}^{(l)} + S_{2rk}^{(l)}}$$

with

$$SX_{rk}^{(l)} = \sum_{i=1}^{m} \frac{w_{2ri}x_{ik}^{(l)}}{\left(((x_{r1}^{(l)} - x_{i1}^{(l)})^2 + \epsilon)^{\frac{p}{2}} + ((x_{r2}^{(l)} - x_{i2}^{(l)})^2 + \epsilon)^{\frac{p}{2}}\right)^{\frac{p-1}{p}} \cdot ((x_{rk}^{(l)} - x_{ik}^{(l)})^2 + \epsilon)^{\frac{2-p}{2}}}$$

$$SA_{rk}^{(l)} = \sum_{j=1}^{n} \frac{w_{1ri}a_{jk}}{\left(((x_{r1}^{(l)} - a_{j1})^2 + \epsilon)^{\frac{p}{2}} + ((x_{r2}^{(l)} - a_{j2})^2 + \epsilon)^{\frac{p}{2}}\right)^{\frac{p-1}{p}} \cdot ((x_{rk}^{(l)} - a_{jk})^2 + \epsilon)^{\frac{2-p}{2}}}$$

$$S_{1rk}^{(l)} = \sum_{j=1}^{n} \frac{w_{1ri}}{\left(((x_{r1}^{(l)} - a_{j1})^2 + \epsilon)^{\frac{p}{2}} + ((x_{r2}^{(l)} - a_{j2})^2 + \epsilon)^{\frac{p}{2}}\right)^{\frac{p-1}{p}} \cdot ((x_{rk}^{(l)} - a_{jk})^2 + \epsilon)^{\frac{2-p}{2}}}$$

$$S_{2rk}^{(l)} = \sum_{i=1}^{m} \frac{w_{2ri}}{\left(((x_{r1}^{(l)} - x_{i1}^{(l)})^2 + \epsilon)^{\frac{p}{2}} + ((x_{r2}^{(l)} - x_{i2}^{(l)})^2 + \epsilon)^{\frac{p}{2}}\right)^{\frac{p-1}{p}} \cdot ((x_{rk}^{(l)} - x_{ik}^{(l)})^2 + \epsilon)^{\frac{2-p}{2}}}$$

can be used in an iterative procedure to solve a problem of type $m/P/\bullet/l_p/\sum$ (as in the Weiszfeld algorithm).

Aufgabe 18:

Consider a problem of type $2/P/\bullet/l_1/\sum$ with existing facilities $a_1 = (3,1)$, $a_2 = (2,2)$, $a_3 = (1,4)$ and $a_4 = (5,7)$ and weights $w_{1,11} = 1$, $w_{1,12} = 2$, $w_{1,13} = 5$, $w_{1,14} = 1$, $w_{1,21} = 5$, $w_{1,22} = 1$, $w_{1,23} = 3$, $w_{1,24} = 2$ and $w_{2,12} = 2$.

a) Formulate the problem as an LP (analogous to Section 4.2).

b) Solve the problem with an algorithm or solver of your choice and draw the solution.

c) Find an alternative LP formulation for problems of type $m/P/\bullet/l_1/\sum$. (Hint: It may help to have a look at Section 5.1.1.)

d) Apply your result of c) to the above example and compare it with the formulation from a).

Aufgabe 19:

Problems of type $1/P/\bullet/l_1/\sum$ can be solved using a construction line algorithm, see, for example, Algorithm 3.20. Is this (i.e., solving the location problem by using a construction line grid) also possible for problems of type $m/P/\bullet/l_1/\sum$? Illustrate your answer with an example.

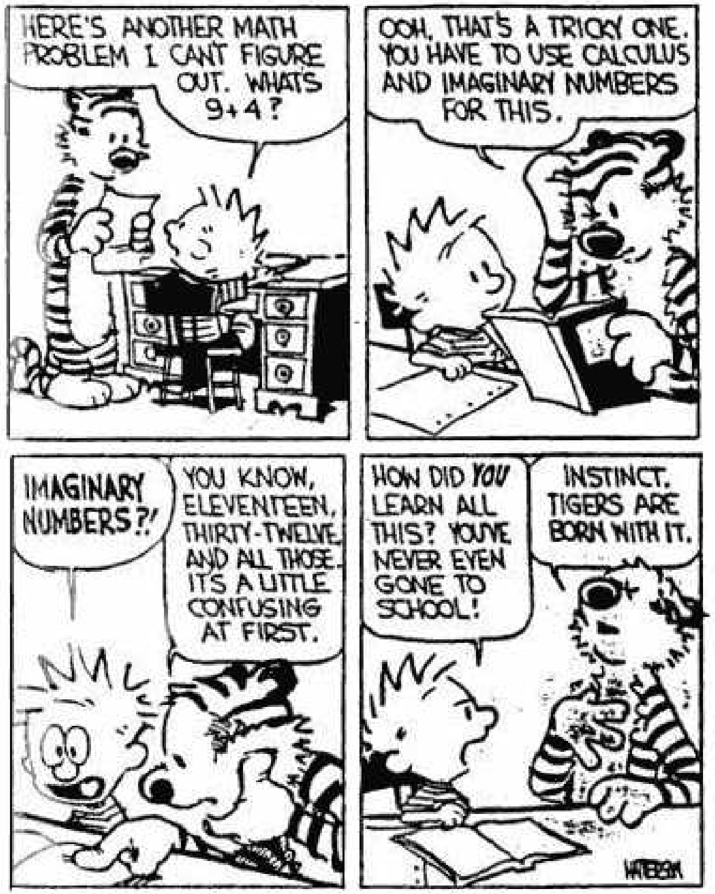
Aufgabe 20:

a) For $1/P/w_m = 1/l_1/\max$ we define:

$$m_1 := \max_{m\in M}\{a_{m1} + a_{m2}\},$$
$$m_2 := \max_{m\in M}\{a_{m1} - a_{m2}\}$$
$$m_3 := \max_{m\in M}\{-a_{m1} + a_{m2}\}$$
$$m_4 := \max_{m\in M}\{-a_{m1} - a_{m2}\}, \text{ and}$$
$$m := \max\{\frac{m_1 + m_4}{2}, \frac{m_2 + m_3}{2}\}.$$

Prove: All points on the straight line between the points $\frac{1}{2}(m_2 - m_4, -m_2 - m_4 + 2m)$ and $\frac{1}{2}(m_1 - m_3, m_1 + m_3 - 2m)$ are optimal.

b) Solve the instance of a center problem $1/P/w_m = 1/l_1/\max$ with existing facilities $a_1 = (0,0)$, $a_2 = (6,6)$, $a_3 = (4,0)$ and $a_4 = (6,4)$.



Bemerkung: Aktuelle Informationen zur Vorlesung und zu den Übungen finden Sie im Internet unter:

http://www.math.uni-wuppertal.de/opt/location_ss2010/