Standort-Optimierung

3. Übung

Sommersemester 2010

Bergische Universität Wuppertal
Fachbereich C – Angewandte Mathematik / Optimierung und Approximation
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Aufgabe 9:
Justify the following properties of problems of type $1/S/\bullet/A/\sum$:

a) A point on S is a minimizer for $\sum_{j=1}^{n} w_j A(x, a_j)$ if, and only if, its antipode is a maximizer.

b) A point and its antipode with equal weights can be added to the problem without a change in the optimal location of the facility.

c) A point with weight w_j can be replaced by its antipode with weight $-w_j$ without changing the optimal location of the facility.

d) Every problem can be transformed to an equivalent problem that has only positive weights.

Aufgabe 10:
Derive the equations

$$\tan x_2 = \frac{\sum_{j=1}^{n} (w_j \cos a_{j1} \sin a_{j2}) / \sin A(x, a_j)}{\sum_{j=1}^{n} (w_j \cos a_{j1} \cos a_{j2}) / \sin A(x, a_j)}$$

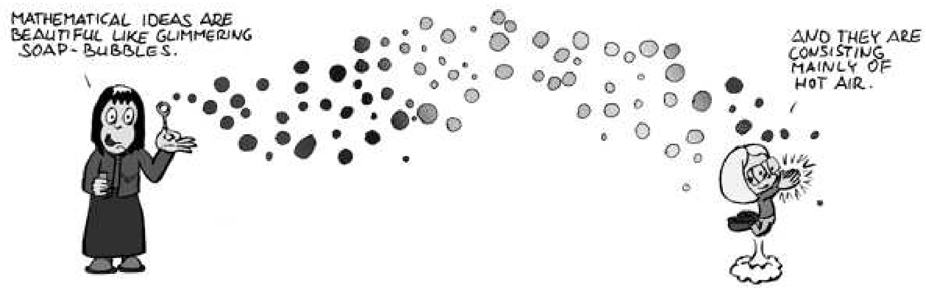
$$\frac{\tan x_1}{\sin x_2} = \frac{\sum_{j=1}^{n} (w_j \sin a_{j1}) / \sin A(x, a_j)}{\sum_{j=1}^{n} (w_j \cos a_{j1} \sin a_{j2}) / \sin A(x, a_j)}$$

from the optimality conditions $\frac{\partial W(x)}{\partial x_1} = \frac{\partial W(x)}{\partial x_2} = 0$ for problems of type $1/S/\bullet/A/\sum$.

Aufgabe 11:
Show that Lemma 3.5 is “tight” in the sense that an example with two or more local minima can be constructed if the radius is $\pi/4 + \varepsilon$, where $\varepsilon > 0$.

Aufgabe 12:
Show that the level curves $L_=(z) := \{x \in \mathbb{R}^2 : W(x) = z\}$ (with $z \geq z^*$) for Weber problems with squared Euclidean distances $1/P/\bullet/l_2^2/\sum$ are circles with center $x^* = (x_1^*, x_2^*)$, where x^* is the optimal location of $1/P/\bullet/l_2^2/\sum$ with objective value z^*. (Recall: $x_k^* = \frac{\sum_{j=1}^{n} w_j a_{jk}}{\sum_{j=1}^{n} w_j}$, $k = 1, 2$.)

Aufgabe 13:
Find an algorithm to solve problems of the type $1/P/R/l_2^2/\sum$ where the forbidden region R is a non-convex polyhedron given by its corner points $(y_1, \ldots, y_L)$. Apply the algorithm to an example problem with existing facility locations $a_1 = (1;1)$, $a_2 = (1;4)$, $a_3 = (2;1)$, $a_4 = (4;1)$, $a_5 = (4;4)$, weights $w_1 = 2$, $w_2 = 1$, $w_3 = 1$, $w_4 = 2$, $w_5 = 4$ and forbidden region R with corner points (in clockwise order) $y_1 = (0;0.4)$, $y_2 = (0;5)$, $y_3 = (7;5)$, $y_4 = (7;3)$, $y_5 = (5;3)$, $y_6 = (5;0.4)$.



Bemerkung: Aktuelle Informationen zur Vorlesung und zu den Übungen finden Sie im Internet unter:

http://www.math.uni-wuppertal.de/opt/location_ss2010/