Standort-Optimierung

Handout 3

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Bergische Universität Wuppertal
Fachbereich C – Angewandte Mathematik / Optimierung und Approximation
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Algorithm 4.3: Approximation Algorithm for $m/P/\bullet/l_p/\sum$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought; nonnegative weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$ and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$.

Step 1: For $i = 1, \dots, m$ do:
Determine an (approximate) optimal solution $\tilde{x}_i$ for $1/P/\bullet/l_p/\sum$ with existing facility locations $a_1, \dots, a_n$ and weights $w_{1,i1}, \dots, w_{1,in}$.

Step 2: Set $\tilde{x} := (\tilde{x}_1, \dots, \tilde{x}_m)$ and determine

$$\Delta(\tilde{x}) := \frac{WM_{new}(\tilde{x})}{WM_{ex.}(\tilde{x})} = \frac{\sum_{i=1}^{m-1} \sum_{r=i+1}^{m} w_{2,ir} l_p(\tilde{x}_i, \tilde{x}_r)}{\sum_{i=1}^{m} \sum_{j=1}^{n} w_{1,ij} l_p(\tilde{x}_i, a_j)}$$

Output: Approximate solution $\tilde{x}$ of $m/P/\bullet/l_p/\sum$ and error bound $\Delta(\tilde{x})$.

Algorithm 4.4: Improved Approximation Algorithm for $m/P/\bullet/l_p/\sum$

Input: Existing facilities $a_1, \dots, a_n \in \mathbb{R}^2$; number m of new facilities sought; nonnegative weights $w_{1,ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$ and $w_{2,ir}$, $i = 1, \dots, m-1$, $r = i+1, \dots, m$, desired accuracy $\varepsilon > 0$.

Step 1: For $i = 1, \dots, m$ do:
Determine an (approximate) optimal solution $\tilde{x}_i$ for $1/P/\bullet/l_p/\sum$ with existing facility locations $a_1, \dots, a_n$ and weights $w_{1,i1}, \dots, w_{1,in}$.

Step 2: Set $\tilde{x} := (\tilde{x}_1, \dots, \tilde{x}_m)$ and determine

$$\Delta(\tilde{x}) := \frac{WM_{new}(\tilde{x})}{WM_{ex.}(\tilde{x})} = \frac{\sum_{i=1}^{m-1} \sum_{r=i+1}^{m} w_{2,ir} l_p(\tilde{x}_i, \tilde{x}_r)}{\sum_{i=1}^{m} \sum_{j=1}^{n} w_{1,ij} l_p(\tilde{x}_i, a_j)}$$

Step 3: If $\Delta(\tilde{x}) < \varepsilon$, STOP.

Step 4: For $i = 1, \dots, m$ do:
Determine an optimal solution $\hat{x}_i$ of $1/P/\bullet/l_p/\sum$ with existing facility locations $\{a_1, \dots, a_n\} \cup \{\tilde{x}_k : k \in \{1, \dots, m\} \setminus \{i\}\}$ and weights $w_{1,i1}, \dots, w_{1,in}, w_{2,1i}, \dots, w_{2,(i-1)i}, w_{2,i(i+1)}, \dots, w_{2,im}$.

Step 5: Set $\hat{x} := (\hat{x}_1, \dots, \hat{x}_m)$.
If $\hat{x} = \tilde{x}$, STOP.
Otherwise, set $\tilde{x} := \hat{x}$ and goto Step 3.

Output: Approximate solution $\tilde{x}$ of $m/P/\bullet/l_p/\sum$ and error bound $\Delta(\tilde{x})$.