

# Standort-Optimierung

## Handout 1

Sommersemester 2010



Bergische Universität Wuppertal

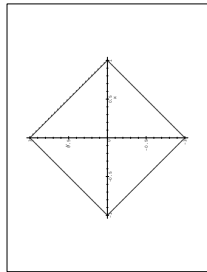
Fachbereich C – Angewandte Mathematik / Optimierung und Approximation

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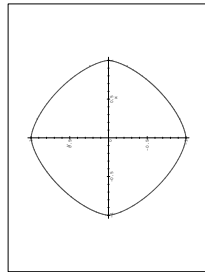
### $l_p$ Distances:

$$l_p(x, y) = (|x_1 - y_1|^p + |x_2 - y_2|^p)^{\frac{1}{p}}, \quad x, y \in \mathbb{R}^2, \quad 1 \leq p < \infty$$

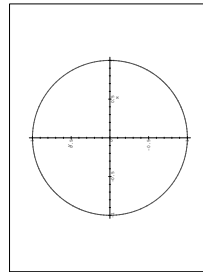
$$l_\infty(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}, \quad x, y \in \mathbb{R}^2$$



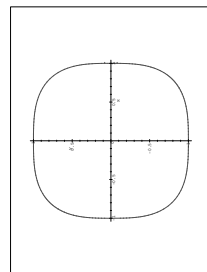
$$l_1(x, 0) = 1$$



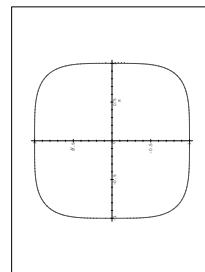
$$l_{1.5}(x, 0) = 1$$



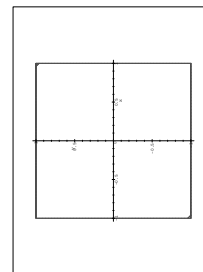
$$l_2(x, 0) = 1$$



$$l_3(x, 0) = 1$$



$$l_4(x, 0) = 1$$



$$l_\infty(x, 0) = 1$$

**Property 1:**  $l_p(x, y)$  decreases as  $p$  increases.

**Property 2:** As  $p \rightarrow \infty$ ,  $l_p(x, y)$  becomes the larger of  $|x_1 - y_1|$  and  $|x_2 - y_2|$ .

### Weiszfeld Algorithm for $1/P/\bullet/l_2/\Sigma$

- Input: Existing facilities  $a_1, \dots, a_n \in \mathbb{R}^2$ ; weights  $w_1, \dots, w_n > 0$ .
- Step 1: If  $CR_r \leq w_r$  for  $r \in \{1, \dots, n\}$ , set  $x^* := a_r$ , STOP.
- Step 2: Select a starting solution  $x^{(0)} = (x_1^{(0)}, x_2^{(0)})$ ; set  $l := 0$ .  
( $x^{(0)}$  could be chosen, for example, as the optimal solution of  $1/P/\bullet/l_2^2/\Sigma$ .)
- Step 3: For  $k = 1, 2$  do

$$x_k^{(l+1)} := \frac{\sum_{j=1}^n \frac{w_j a_{jk}}{l_2(x^{(l)}, a_j)}}{\sum_{j=1}^n \frac{w_j}{l_2(x^{(l)}, a_j)}}$$

- Step 4: If  $x^{(l+1)}$  satisfies a stopping criterion, set  $x^* := x^{(l+1)}$ , STOP.  
Otherwise, set  $l := l + 1$  and goto Step 3.

Output: Approximation  $x^*$  of an optimal solution for  $1/P/\bullet/l_2/\Sigma$ .

### Hyperbolic Approximation Algorithm for $1/P/\bullet/l_p/\Sigma$ , $1 < p < \infty$

- Input: Existing facilities  $a_1, \dots, a_n \in \mathbb{R}^2$ ; weights  $w_1, \dots, w_n > 0$ ;  $p \in \mathbb{R}$  with  $1 < p < \infty$ .
- Step 1: If  $CRP_r \leq w_r$  for  $r \in \{1, \dots, n\}$ , set  $x^* := a_r$ , STOP.
- Step 2: Select a starting solution  $x^{(0)} = (x_1^{(0)}, x_2^{(0)})$ ; set  $l := 0$ .
- Step 3: For  $k = 1, 2$  do

$$x_k^{(l+1)} := \frac{\sum_{j=1}^n \frac{w_j a_{jk}}{d'(x^{(l)}, a_j) \cdot d''(x_k^{(l)}, a_{jk})}}{\sum_{j=1}^n \frac{w_j}{d'(x^{(l)}, a_j) \cdot d''(x_k^{(l)}, a_{jk})}}$$

- Step 4: If  $x^{(l+1)}$  satisfies a stopping criterion, set  $x^* := x^{(l+1)}$ , STOP.  
Otherwise, set  $l := l + 1$  and goto Step 3.

Output: Approximation  $x^*$  of an optimal solution for  $1/P/\bullet/l_p/\Sigma$ .

$$CR_r = \left[ \left( \sum_{\substack{j=1 \\ j \neq r}}^n \frac{w_j (a_{r1} - a_{j1})}{l_2(a_r, a_j)} \right)^2 + \left( \sum_{\substack{j=1 \\ j \neq r}}^n \frac{w_j (a_{r2} - a_{j2})}{l_2(a_r, a_j)} \right)^2 \right]^{\frac{1}{2}}$$

$$CRP_r = \left[ \sum_{\substack{j=1 \\ j \neq r}}^n \frac{w_j \operatorname{sign}(a_{r1} - a_{j1}) |a_{r1} - a_{j1}|^{p-1}}{(l_p(a_r, a_j))^{p-1}} \right]^{\frac{p}{p-1}} + \left[ \sum_{\substack{j=1 \\ j \neq r}}^n \frac{w_j \operatorname{sign}(a_{r2} - a_{j2}) |a_{r2} - a_{j2}|^{p-1}}{(l_p(a_r, a_j))^{p-1}} \right]^{\frac{p}{p-1}} \right]^{\frac{p-1}{p}}$$

$$d'(x, a_j) = \left( ((x_1 - a_{j1})^2 + \epsilon)^{\frac{p}{2}} + ((x_2 - a_{j2})^2 + \epsilon)^{\frac{p}{2}} \right)^{1 - \frac{1}{p}}$$

$$d''(x_k, a_{jk}) = ((x_k - a_{jk})^2 + \epsilon)^{1 - \frac{p}{2}}, \quad k = 1, 2.$$