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Model order reduction of parameterized nonlinear systems by interpolating input-output behavior

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Abstract In this paper we propose a new approach for model order reduction of parameterized nonlinear systems. Instead of projecting onto the dominant state space, an analog macromodel is constructed for the dominant input-output behavior. This macromodel is suitable for (re)use in analog circuit simulators. The performance of the approach is illustrated for a benchmark nonlinear system.

1 Introduction

Simulation of VLSI chips is becoming CPU and memory intensive, or even infeasible, due to the increasing amount of layout parasitics and devices in analog designs. A popular method for speeding up and/or enabling simulation of large-scale dynamical systems is model order reduction [1]. For linear systems (large parasitic networks), several methods [2–4] have been developed that are now used in industrial circuit simulators.

Well-known methods for nonlinear systems in circuit simulation are Proper Orthogonal Decomposition (POD) based methods [5] and piecewise-linearization (PWL) methods [6]. Both approaches try to obtain reduction by projection on the dominant dynamics. However, both approaches may suffer from difficulties that may limit their practical use [7]. Robust and efficient resimulation of POD models is still a challenge, while PWL based approaches require application-dependent selections strategies for linearization points and weights.

We present a new method for the reduction of large nonlinear systems. The most significant difference with respect to existing methods is that instead of focusing

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on the dominant state dynamics, the proposed method tries to capture the dominant input-output behavior. Another novelty is that the resulting analog model behaves like a circuit element and can easily be used by circuit simulators.

2 Circuit modeling

Complex electrical systems are designed in a modular way, i.e., functional units are designed separately and put together to form the overall system. To enable communication with other circuit blocks, some nodes of each unit act as terminals, or pins. At these, say n_P pins, information in terms of pin voltages and pin currents, $\mathbf{v}_{\text{pin}}, \mathbf{i}_{\text{pin}} \in \mathbb{R}^{n_P}$, respectively, is exchanged. Applying Modified Nodal Analysis (MNA), a block is described by

$$0 = \mathbf{A}_C \frac{d}{dt}\mathbf{q}_C(\mathbf{A}_C^T\mathbf{e}) + \mathbf{A}_R\mathbf{r}(\mathbf{A}_R^T\mathbf{e}) + \mathbf{A}_L\mathbf{j}_L + \mathbf{A}_V\mathbf{j}_V + \mathbf{A}_I\mathbf{i}(t) - \mathbf{A}_{\text{pin}}\mathbf{i}_{\text{pin}}, \tag{1a}$$

$$0 = \frac{d}{dt}\Phi_L(\mathbf{j}_V) - \mathbf{A}_L^T\mathbf{e}, \tag{1b}$$

$$0 = \mathbf{v}(t) - \mathbf{A}_V^T\mathbf{e}, \tag{1c}$$

$$0 = \mathbf{v}_{\text{pin}} - \mathbf{A}_{\text{pin}}^T\mathbf{e}, \tag{1d}$$

where $\mathbf{e}(t) \in \mathbb{R}^{n_e}$, $\mathbf{j}_L(t) \in \mathbb{R}^{n_L}$, $\mathbf{j}_V(t) \in \mathbb{R}^{n_V}$ denote the unknown node voltages and currents through inductors and voltage sources, respectively. The incidence matrices $\mathbf{A}_\Omega \in \{0, \pm 1\}^{n_e \times n_\Omega}$, describe the placement of the basic network elements resistor ($\Omega = \text{R}$), capacitor (C), inductor (L), voltage (V) and current (I) source, respectively. The, in general nonlinear, characteristics of the network elements are represented by $\mathbf{q}_C(\cdot)$, $\Phi(\cdot)$, $\mathbf{r}(\cdot)$, $\mathbf{i}(\cdot)$, $\mathbf{v}(t)$. The incidence matrix $\mathbf{A}_{\text{pin}} \in \{0, \pm 1\}^{n_e \times n_P}$ addresses the circuit nodes acting as pins. Injecting, i.e., prescribing the pin voltages $\mathbf{v}_{\text{pin}}$, the pin currents $\mathbf{j}_{\text{pin}}$ become additional unknowns, meant to be passed back to system the block is embedded in, or vice versa. By this, a circuit unit turns into an input-output system, represented in the compact form

$$\mathbf{0} = \frac{d}{dt}\mathbf{q}(\mathbf{x}) + \mathbf{j}(\mathbf{x}) + \mathbf{s}(t) + \mathbf{B}\mathbf{u}; \quad \mathbf{y} = \mathbf{B}^T\mathbf{x}, \tag{2}$$

where $\mathbf{u}(t), \mathbf{y}(t) \in \mathbb{R}^{n_P}$ represent the input and output of the system and $\mathbf{x}(t) \in \mathbb{R}^n$ ($n = n_e + n_V + n_L + n_P$) denotes the internal states. Note, that in the following we will omit the excitation $\mathbf{s}(t)$.

Frequently, design parameters, e.g., width and length of transistor channels, are kept variable, in order to optimize them in the design process. We take this into account by including a parameter vector $\rho \in \mathbb{R}^{n_{\text{par}}}$ in the element functions, i.e., by extending $\mathbf{q}(\mathbf{x};\rho)$ and $\mathbf{j}(\mathbf{x};\rho) \in \mathbb{R}^n$ in (2).

For a more detailed description of MNA and the properties of the arising network equations we refer to [8].

2.1 Model order reduction

A compound of subsystems, described by (2), arises e.g., in full system verification and post-layout simulation. The arising overall system is usually of very high dimensionality.

Focusing on the interaction in a compound of systems one is often not interested in the individual internal states $\mathbf{x}(t)$ but merely in the way a subsystem translates $\mathbf{u}(t)$ to $\mathbf{y}(t)$. Classically, Model Order Reduction (MOR) aims at replacing (2) by a dynamical system of reduced dimension $r \ll n$. The idea is, that, given the same input $\mathbf{u}(t)$, the substitute dynamical system with internal states $\mathbf{z}(t) \in \mathbb{R}^r$ produces (almost) the same output $\mathbf{y}(t)$ as the full system (2). Hence, replacing individual blocks by models of reduced order, the dimension of the compound system is kept small, enabling the overall system to be simulated at reasonable computational costs.

MOR for linear systems, arising from parasitic extraction, used in post-layout simulation, reached a high level of maturity. Several methods are now used in industrial circuit simulators. For an overview we refer to [1,9]. MOR for linear problems bases upon the transfer function, i.e., the representation of the dynamical system under consideration in the frequency domain and is usually combined with projecting (2) onto a lower dimensional subspace.

For nonlinear problems the situation is somewhat different. Here, in general no transfer function can be specified and also projection to a lower dimensional subspace may reduce the dimension of the system but not the computational costs evaluating the system.

We propose an approach to reproduce the input-output mapping, starting from time-domain considerations.

2.2 Numerical Time Integration

Systems of type (2) usually can not be solved analytically for $\mathbf{x}(t), \mathbf{y}(t)$. Numerical integration is carried out instead. Both onestep and multistep methods discretize the system. For the backward Euler as a showcase this amounts to

$$\mathbf{0} = \frac{1}{h}\left[\mathbf{q}(x_n) - \mathbf{q}(x_{n-1})\right] + \mathbf{j}(\mathbf{x}_n) + \mathbf{B}\mathbf{u}_n; \quad \mathbf{y}_n = \mathbf{B}^T\mathbf{x}_n. \tag{3}$$

Given $\mathbf{u}_n$ and $\mathbf{x}_{n-1}$, (3) defines $\mathbf{x}_n$ and $\mathbf{y}_n$, i.e., approximations to $\mathbf{x}(t_n)$ and $\mathbf{y}(t_n)$ at $t_n = t_{n-1} + h$. Applying a Newton-Raphson technique to solve this problem, a series of linear equations have to be solved. The main ingredients for setting up the corresponding linear system are

$$\alpha\mathbf{C}(\bar{\mathbf{x}}) + \mathbf{G}(\bar{\mathbf{x}}); \quad \alpha\mathbf{q}(\bar{\mathbf{x}}) + \mathbf{j}(\bar{\mathbf{x}}); \quad \mathbf{q}(\mathbf{x}_{n-1}), \tag{4}$$

with $\mathbf{C}(\cdot) = \frac{d}{dx}\mathbf{q}(\cdot)$, $\mathbf{G}(\cdot) = \frac{d}{dx}\mathbf{j}(\cdot)$, evaluated at some intermediate points $\bar{\mathbf{x}}$. For the backward Euler we have $\alpha = h^{-1}$. The term $\mathbf{q}(\mathbf{x}_{n-1})$ reflects the history of the dynamic elements.

Note, that for didactic reasons only we stick to the Euler discretisation during the rest of this paper. For an overview of schemes applicable to DAEs we refer to [10].

3 Input-output behavior macromodeling

Being interested in the translation of the input to the output reads, in terms of the discretised problem (3): we are interested in $\mathbf{y}_n$ and $\mathbf{x}_n$ is an auxiliary quantity only. Hence, ideally we are able to replace the system (3) by an input-output mapping

$$\tau : \mathbb{R}^{n_P} \to \mathbb{R}^{n_P}, \quad \mathbf{u}_n \mapsto \mathbf{y}_n = \tau(\mathbf{u}_n). \tag{5a}$$

At first glance it seems that this is not realizable. From (4), not only a combined evaluation of $\{\mathbf{q},\mathbf{j}\}$ and $\{\mathbf{C},\mathbf{G}\}$ is needed but also the dynamics' history $\mathbf{q}(\mathbf{x}_{n-1})$.

However, for homogeneous structures, i.e., blocks comprising only resistive (R), capacitive (C) or inductive (L) elements, the mapping τ_Ω ($\Omega = R,C,L$), can be derived. Still, in general, no analytic expression can be specified. The idea is to replace function evaluation with interpolation from tabulated data (see Table 1).

Table 1 Macromodel - tabulated data

$\mathbf{u}$	$\mathbf{u}^{(1)}$	$\cdots$	$\mathbf{u}^{(k)}$
τ_Ω	$\tau_\Omega^{(1)}$	$\cdots$	$\tau_\Omega^{(k)}$
$\mathbf{T}_\Omega$	$\mathbf{T}_\Omega^{(1)}$	$\cdots$	$\mathbf{T}_\Omega^{(k)}$

This table includes also the derivative of τ w.r.t. the input, i.e.,

$$\mathbf{T} : \mathbb{R}^{n_P} \to \mathbb{R}^{n_P \times n_P} : \mathbf{u}_n \mapsto \mathbf{T}(\mathbf{u}_n) = \left.\frac{\partial \tau(\mathbf{u})}{\partial \mathbf{u}}\right|_{\mathbf{u}=\mathbf{u}_n}. \tag{5b}$$

Later, we will see why this is necessary.

The basic concept is to replace homogeneous structures by a macromodel or macroelement with the same characteristics the combination of elements has. Resistors turn voltages to currents, capacitors answer with charges when a voltage is applied and inductors show a current-flux relation. These facts are to be preserved by the macromodel.

In the following we give some details for purely resistive and purely capacitive structures, i.e., for static and dynamic blocks.

3.1 Models for resistive structures

A circuit block consisting of resistors only is described by

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_R \mathbf{r}(\mathbf{A}_R^T \mathbf{e}) - \mathbf{A}_{\text{pin}} \mathbf{j}_{\text{pin}}, \\ \mathbf{0} &= \mathbf{v}_{\text{pin}} - \mathbf{A}_{\text{pin}}^T \mathbf{e}. \end{aligned} \tag{6a}$$

We choose the pin voltages $\mathbf{v}_{\text{pin}}$ as input parameters. Assuming sufficient regularity of the conductance matrix $\mathbf{G}_r(\mathbf{w}) := \frac{\partial}{\partial \mathbf{w}} \mathbf{r}(\mathbf{w})$, (6a) implicitly defines the node voltages and pin currents as functions of the pin voltages, i.e., $\mathbf{e} = \mathbf{e}(\mathbf{v}_{\text{pin}})$ and $\mathbf{j} = \mathbf{j}(\mathbf{v}_{\text{pin}})$, respectively. We differentiate (6a) with respect to $\mathbf{v}_{\text{pin}}$ to get:

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_R \mathbf{G}_r(\mathbf{A}_R^T \mathbf{e}) \mathbf{A}_R^T \frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}} - \mathbf{A}_{\text{pin}} \frac{\partial \mathbf{j}_{\text{pin}}}{\partial \mathbf{v}_{\text{pin}}}, \\ \mathbf{0} &= \mathbf{I}_{n_P} - \mathbf{A}_{\text{pin}}^T \frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}} \end{aligned} \tag{6b}$$

where $\mathbf{I}_{n_P}$ is the $n_P \times n_P$ identity matrix.

For purely resistive structures we construct Table 1, describing the mapping "pin voltages" to "pin currents" in the following way:

1. Choose a discrete set of $k \in \mathbb{N}$ terminal voltages $\mathbf{v}_{p,1}, \ldots, \mathbf{v}_{p,k}$ with $\mathbf{v}_{p,i} \in \mathbb{R}^{n_P}$
2. For each $i \in \{1, \ldots, k\}$
   a. compute $\mathbf{e}_i = \mathbf{e}(\mathbf{v}_{p,i})$ and $\mathbf{j}_{p,i} = \mathbf{j}_{\text{pin}}(\mathbf{v}_{p,i})$ by solving (6a) for $\mathbf{v}_{\text{pin}} = \mathbf{v}_{p,i}$
   b. solve the linear system (6b) for $\frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}}|_{\mathbf{v}_{p,i}}$ and $\frac{\partial \mathbf{j}_{\text{pin}}}{\partial \mathbf{v}_{\text{pin}}}|_{\mathbf{v}_{p,i}} =: \mathbf{J}_{p,i}$. Here, $\mathbf{G}_r(\cdot)$ is evaluated at $\mathbf{A}_R^T \mathbf{e}_i$. This amounts to computing the Schur complement

$$\mathbf{J}_{p,i} = \frac{\partial \mathbf{j}_{\text{pin}}}{\partial \mathbf{v}_{\text{pin}}} = \left(\mathbf{A}_{\text{pin}}^T \left(\mathbf{A}_R \mathbf{G}_r(\mathbf{A}_R^T \mathbf{e}_i) \mathbf{A}_R^T \right)^{-1} \mathbf{A}_{\text{pin}} \right)^{-1}. \tag{6c}$$

3. The parameters for the resistive macromodel from Table 1 are

$$\mathbf{u}^{(i)} = \mathbf{v}_{p,i}\,, \quad \tau_R^{(i)} = \mathbf{j}_{p,i}\,, \quad \mathbf{T}_R^{(i)} = \mathbf{J}_{p,i}$$

for $i = 1, \ldots, k$ where $\mathbf{v}_{p,i} \in \mathbb{R}^{n_P}$, $\mathbf{j}_{p,i} \in \mathbb{R}^{n_P}$, $\mathbf{J}_{p,i} \in \mathbb{R}^{n_P \times n_P}$

3.2 Models for capacitive structures

The distribution of charges and voltages in a network of capacitors is described by

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_C \mathbf{q}(\mathbf{A}_C^T \mathbf{e}) - \mathbf{A}_{\text{pin}} \mathbf{q}_{\text{pin}}, \\ \mathbf{0} &= \mathbf{v}_{\text{pin}} - \mathbf{A}_{\text{pin}}^T \mathbf{e}, \end{aligned} \tag{7a}$$

where $\mathbf{q}_{\text{pin}}$ are point charges at the structure's pins. In other words: we map a large number of charges $\mathbf{q}(\cdot)$ to n_P point charges $\mathbf{q}_{\text{pin}}$. Here the voltage $\mathbf{v}_{\text{pin}} \in \mathbb{R}^{n_P}$ is prescribed at the pins.

Analog to the procedure for resistive structures (Sect. 3.1) we construct Table 1 for purely capacitive structures. For different voltages $\mathbf{v}_{\text{pin}} \in \{\mathbf{v}_{p,1}, \ldots, \mathbf{v}_{p,k}\}$ we solve the nonlinear system (7a) for $\mathbf{q}_{\text{pin}}(\mathbf{v}_{p,i}) =: \mathbf{q}_{p,i}$ and $\mathbf{e}(\mathbf{v}_{p,i})$.

The column in Table 1 reflecting the charge replies is made up of $\{\mathbf{q}_{p,1}, \ldots, \mathbf{q}_{p,k}\}$. Items for the column in Table 1 describing the Jacobians $\mathbf{T}_C$ are found by solving

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_C \mathbf{C}_q(\mathbf{A}_C^T \mathbf{e}) \mathbf{A}_C^T \frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}} - \mathbf{A}_{\text{pin}} \frac{\partial \mathbf{q}_{\text{pin}}}{\partial \mathbf{v}_{\text{pin}}}, \\ \mathbf{0} &= \mathbf{I}_{n_P} - \mathbf{A}_{\text{pin}}^T \frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}} \end{aligned} \tag{7b}$$

for $\frac{\partial \mathbf{e}}{\partial \mathbf{v}_{\text{pin}}}|_{v_{p,i}}$ and $\frac{\partial \mathbf{q}_{\text{pin}}}{\partial \mathbf{v}_{\text{pin}}}|_{v_{p,i}} =: \mathbf{Q}_{p,i} =: \mathbf{T}_C^{(i)}$, i.e., from the Schur complement

$$\mathbf{Q}_{p,i} = \mathbf{T}_C^{(i)} = \left(\mathbf{A}_{\text{pin}}^T \left(\mathbf{A}_C \mathbf{C}_q(\mathbf{A}_C^T \mathbf{e}_i) \mathbf{A}_C^T \right)^{-1} \mathbf{A}_{\text{pin}} \right)^{-1}. \tag{7c}$$

Subblocks containing only inductive elements can be treated in a very similar way. For the sake of space-saving we skip the description here.

3.3 Parameterized structures

A purely resistive structure that contains parameterized elements is modeled by

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_R \mathbf{r}(\mathbf{A}_R^T \mathbf{e}; \rho) - \mathbf{A}_{\text{pin}} \mathbf{j}_{\text{pin}}, \\ \mathbf{0} &= \mathbf{v}_{\text{pin}} - \mathbf{A}_{\text{pin}}^T \mathbf{e}, \end{aligned} \tag{8}$$

with the vector $\rho \in \mathbb{R}^{n_{\text{par}}}$ of parameters. The task is now to not only cover a range of terminal voltages $\mathbf{v}_{\text{pin}}$ but also a parameters ρ in a reasonable range.

Therefore, the procedure from Sect. 3.1 has to be adapted: besides sweeping over a range of pin voltages $\mathbf{v}_{\text{pin}} \in \{\mathbf{v}_{p,1}, \ldots, \mathbf{v}_{p,k}\}$ we scan in parallel the input-output behavior for different parameters $\rho \in \{\rho_1, \ldots, \rho_l\}$. This leads to an extended macromodel

$$\tau : \mathbb{R}^{n_P} \times \mathbb{R}^{n_{\text{par}}} \to \mathbb{R}^{n_P}, \quad (\mathbf{u}_n, \rho) \mapsto \mathbf{y}_n = \tau(\mathbf{u}_n, \rho), \tag{9}$$

realized by table with datapoints $\left[(\rho^{(\mu)}, \mathbf{u}^{(\nu)}), (\tau^{(\mu,\nu)}, \mathbf{T}^{(\mu,\nu)}) \right]$ for $\mu = 1, \ldots, l$ and $\nu = 1, \ldots, k$.

3.4 Using the macromodels

A system containing subblocks, of purely resistive and capacitive nature can basically be modeled by

$$\mathbf{0} = \frac{d}{dt}\mathbf{q}(\mathbf{x}) + \mathbf{j}(\mathbf{x}) + \mathbf{s}(t) + \mathbf{B}_R \tau_R(\mathbf{B}_R^T \mathbf{x}) + \mathbf{B}_C \frac{d}{dt}\tau_C(\mathbf{B}_C^T \mathbf{x}), \tag{10}$$

with incidence matrices $\mathbf{B}_R, \mathbf{B}_C$ describing the interfaces. In this way, we accommodate the characteristics of a subblock being reactive or nonreactive. Macromodels of inductive nature are added to (10) very similar to the inclusion of capacitor-like macroelements.

Applying any numerical time integration technique to (10), we see, that the basic ingredients for the systems to be solved in this process are (cf. (4))

$$\begin{aligned} &\alpha\left[\mathbf{C} + \widetilde{\mathbf{T}}_C\right](\bar{\mathbf{x}}_n) + \left[\mathbf{G} + \widetilde{\mathbf{T}}_R\right](\bar{\mathbf{x}}_n) \text{ and } \alpha[\mathbf{q} + \widetilde{\tau}_C](\bar{\mathbf{x}}_n) + [\mathbf{j} + \widetilde{\tau}_R](\bar{\mathbf{x}}_n); \\ &\mathbf{q}(\mathbf{x}_{n-1}) \text{ and } \widetilde{\tau}_C(\mathbf{x}_{n-1}), \end{aligned} \tag{11}$$

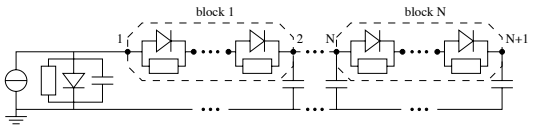
where $\widetilde{\tau}_\Omega(\cdot) = \mathbf{B}_\Omega \tau_\Omega(\mathbf{B}_\Omega^T \cdot)$ and $\widetilde{T}_\Omega(\cdot) = \mathbf{B}_\Omega T_\Omega(\mathbf{B}_\Omega^T \cdot)$ for $\Omega \in \{R, C\}$. We clearly see that the Jacobians (5b) are necessary as well.

Recall, that evaluation of the macromodel-functions and the corresponding Jacobians are realized by interpolation from the corresponding Table 1.

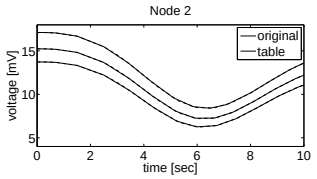
4 Numerical Experiments

An extended, parameterized version of the transmission line [6] serves as a test example. This circuit, displayed in Fig. 1, consists of a series of N blocks, the M diodes in each block are modeled by $i_d(u) = \exp(\rho \cdot u) - 1$. For the resistive block

Fig. 1 Transmission line



(with $M = 100$) a compact model is derived by sweeping $v_{\text{pin}} = \{0.0, \pm 0.01, \pm 0.02\}$ and $\rho = \{35, 55\}$, i.e., by solving (6a,6b) $5 \cdot 2 = 10$ times for each combination of (v_{pin}, ρ). For testing, the block was instantiated $N = 10$ times and a current source $i(t) = 0.5(\cos(2\pi \cdot 0.1 \cdot t) + 1)$ was chosen. To test the accuracy of the reduced model, each of the $N = 10$ blocks was replaced by a tablemodel. From Fig. 4 a speedup of about 10 for each choice of the parameter ρ and an almost perfect match with full system simulation can be observed.



	full system	reduced system extraction time: 5.0
$p = 40$	116.1	9.4
$p = 45$	100.0	10.6
$p = 50$	102.1	12.0

Fig. 2 Parameterized transmission line: $p = \{40, 45, 50\}$

5 Conclusion

We have presented a method that directly approximates the input-output behavior of large parameterized nonlinear circuits by interpolating precomputed contributions to the network equations. Numerical results confirm that significant speedups can be obtained while maintaining accuracy. Extensions to mixed static/dynamic circuits and advanced interpolation methods are subject to future research.

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