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A New Method for Stress Testing on Investment Products

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Chapter 1

A New Method for Stress Testing on Investment Products

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This chapter describes the development of so-called stress tests. These methods belong to the concept of risk bearing ability and are used frequently in banks to verify, how potential risks of loosing money are covered by equity capital.

We propose a modification of an existing stress test from the literature to reduce the data requirements and the computational effort and improve the approximation quality. Hence our approach make the stress tests faster and more cost effective for the banks.

1.1. Introduction

The financial crisis of 2007 has shown that possible risks in extreme situations were not appropriately quantified by existing risk estimation methods. If the losses exceed the height of the equity capital, insolvency can be the result, like the case of *Lehman Brothers* shows. To respond this risk, banks are requested to improve their risk management systems continuously. During the past years a change of traditional qualitative technologies to quantitative methods took place, but for more rare risky events, existing measurements are still not sufficient.

Stress tests are understood as all analytical methods, which differ from the procedures of risk quantification in normal market situations and identify loss potentials in extreme situations. Therefore, stress tests include a wide spectrum of methods.

Section 1.2 presents a short introduction to the risk management of German banks. Therefore the essential types of risks, current procedures to measure these risks and their input parameter are discussed. In Section 1.3 we give an overview about various methods of the realization of stress tests and focus on three of these methods. In Section 1.4 we introduce macroeconomic methods to estimate failure probabilities of debtors in extreme situations. Section 1.5 explains how transition matrices, which are important input parameters for risk models, can be adjusted to bad economic years. Finally, in Section 1.6 it is shown how the tails of probability distribution functions can be modeled appropriately by means of the *Peaks over Threshold Method* [1,2].

1.2. Basics of Risk Management

In the financial world, chances and risks are closely connected with each other and have to be considered jointly. Dealing with calculated risks is a central task of banks. Nevertheless the transactions which promise a big chance often accompany with increased risks [3, pp. 7]. The financial crisis has shown once more, that an underestimated risk and opportunities profile may lead to existential consequences. Hence, the establishment of a certified risk management is of paramount importance. The assignment of risk management is to ensure an adequate risk and opportunities profile [3, p. 8]. Every new business can unbalance this profile. To respond this constantly changing risk constellations, banks have to implement a dynamic risk management process. Beside the identification, capture and assessment of risks, the active risk control as well as the monitoring of risk measures are central elements of risk management [3, pp. 62]. The identification of essential risk drivers is a major condition for an effective risk control. For this purpose the potential risks are divided into different types of risks, cf. Table 1.1.

The *Value at Risk* (VaR) is the central measure for the quantification of the loss potential. It is defined as the maximum loss which can occur within a certain observation period, also known as *risk horizon*, with a certain likelihood. The planning horizon of banks is usually one year, so the risk horizon is also measured within this time period. Mathematically, the VaR explains a quantile of the loss distribution. The different types of risks are usually measured by means of independent risk models. Afterwards the

Table 1.1. Definitions of the types of risk according to IBB from [4, pp. 375].

Type of Risk	Definition
Credit Risk	The risk that a counter party respectively a debtor is not able to settle its obligation on the contract terms due to deterioration in creditworthiness or the failure of the counter party.
Market Price Risk	The risk that the value of the portfolio changes on the basis of changes in the market prices or other dimensions observable in the market (e.g. interest rates, spreads, exchange rates).
Operational Risk	The risk of losses as a result of inadequacy and failure of internal procedures, persons and systems as well as the dangers of the entry of losses as a result of external events. The definition encloses legal risks but doesn't contains strategical risks or reputation risks.
Liquidity Risk	The risk that a financial institution is not able to settle its obligations. The liquidity risk also includes the risk of rising refinancing costs.

total loss risk can be calculated by taking the diversification effects between the different risk types into account.

Counter party risks can be quantified by means of credit portfolio models. The best known commercial portfolio models are $CreditMetrics^{TM}$, $CreditRisk+^{TM}$ or $CreditPortfolioView^{TM}$. The models $CreditMetrics^{TM}$ respectively $CreditPortfolioView^{TM}$ developed by J.P. Morgan and Co. respectively McKinsey are simulation models. They are determining loss distributions by simulating creditworthiness classes of the different debtors and consider the interdependency between the debtors. The model $CreditRisk+^{TM}$, which was developed by the Credit Suisse First Boston Bank, is based on an analytic model. It determines the loss distribution of a portfolio analytically with procedures from actuarial mathematics [5, p. 4]. Input parameters of these models are e.g. transition matrices, recovery rates in the case of insolvency and market parameters like interest rates and credit spreads of different rest terms, industries and creditworthiness. Transition matrices contain information about the empiric financial loss likelihood of debtors or the likelihood to move into other creditworthiness classes.

Market risks are often measured with a historical simulation. The principle of the historical simulation is to generate potential portfolio value changes under the utilization of historical changes of the market risk determining parameters, e.g. bond exchange rates, yield curves and interest volatilities.

The quantification of the operational risks is often realized with a *Loss Distribution Approach* (LDA). Within the scope of the Loss Distribution Approach empiric observed damage events are described by distribution functions. A loss frequency distribution models the frequency of the damage events which occur within an certain observation period. The modeling of the amount of damages is realized by an amount of damage distribution. By means of a Monte Carlo simulation in a first step a random frequency of damage events is generated with the loss frequency distribution. The value of the loss frequency determines the amount of the random amount of damage events, which are generated with the amount of damage distribution. The sum of these damages yields the total loss for the intended observation period. Therefore, the modelling of the distribution function is crucial for the quantification of the operational risk.

1.3. Stress Tests – an Overview of Typical Methods

Stress tests are the totality of all analytical methods that deviate from standard risk quantification methods and help to identify the loss potentials. Depending on the number of model parameters and risk factors one distinguishes between univariate and multivariate stress tests, cf. Figure 1.1.

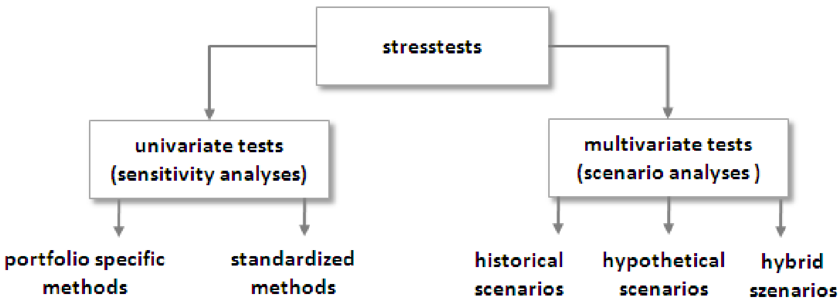

Fig. 1.1. Overview of different groups of stress tests cf. [6, p. 18].

Univariate stress tests, also known as *sensitivity analysis*, determine the influence of the change of single risk factors on the risk of loss. They can help to identify the essential risk factors of a portfolio and the implementation is simple. Contrary, sensitivity analysis do not consider any correlations between the risk factors [6, p. 14]. In reality, simultaneous risk factor changes can be observed, e.g. simultaneous increase of failure probabilities and changing market parameters. Thus sensitivity analysis are only inadequate models of real markets. Nevertheless, they are a useful instrument for an ad-hoc analysis and to identify model uncertainties. Additionally, univariate stress tests can be distinguished in standardized and individual procedures. Standardized analysis are for example the sensitivity analysis suggested from the bank supervision, cf. [4, pp. 71] for further details.

With multivariate stresstests, the effects of several simultaneous risk factor changes can be modeled. These tests permit a clearer picture of the reality, but on the other hand this more complex approach requires additional model acceptances and the interpretation of the results is rather complicated. Multivariate stresstests, also known as *scenario analysis*, are distinguished in historical scenarios, hypothetical scenarios and hybrid scenarios. For further reading we refer the reader to [4, pp. 67].

1.4. Macroeconomic Stress Tests

Besides a huge number of other model parameters, the failure probability of the debtors plays a big role for the quantification of credit risks. Empirical studies, e.g. [7,8] show that the migration behaviour of the debtors in other creditworthiness ratings and therefore also the financial loss probability of the debtors are strongly determined by the economic development. Hence, it is conceivable to describe the financial loss likelihood by means of a regression as a function of macroeconomic factors. Based on this regression, risk considerations under extreme trading conditions can be carried out. The financial loss probability which can be determined with the regression equation under utilization of stressed macroeconomic factors can be applied for the calculation of the Value at Risk.

Equation (1.1) is a linear regression modelling the relation of the dependent variable y_t and one or several independent variables $x_{i,t}$, the *macroeconomic factors*. The variable ε_t summarizes all influences which cannot be covered by the regression model and t denotes the time:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_n x_{n,t} + \varepsilon_t. \tag{1.1}$$

Stress tests can be generated by using extreme values of the macroeconomic variables as input parameters for the already estimated regression equation. It is possible, that the results of the stressed financial loss probabilities are beyond the accepted range between zero and one. To fulfill this condition first the financial loss probability p_t is transformed by an logistic function. Afterwards the connection to the macroeconomic factors is given by the transformed variable y_t:

$$y_t = \ln\left(\frac{1-p_t}{p_t}\right), \tag{1.2}$$

where y_t is called *macroeconomic index*. With the inverse of the logistic function, values of the macroeconomic index can be inverse transformed in stressed financial loss probabilities.

While the values of the dependent and independent variables are given, the weights $\beta_0, \ldots, \beta_n$ must be estimated with a suitable procedure, e.g. the least square method or the maximum likelihood method.

The historical time series of financial loss probabilities and the macroeconomic variables, e.g. the gross domestic product, the rate of unemployment, the commercial climate index, share indices or interest rates can be obtained from statistical offices or financial news and data services, e.g. Bloomberg. If there is no financial loss probability available, these data can also be estimated from ratings. This later approach is often used in practice [9, pp. 110].

To estimate the parameters of the regression, certain assumptions about the stochastic conditions of the error terms have to be met, e.g. average like zero, homoscedasticity, no autocorrelation, independence of the independent variables, cf. [10]. These assumptions have to be checked and the quality of the regression equation has to be analyzed by means of the coefficient of determination and hypothesis tests [10, pp. 240].

1.5. Shifting Migration Matrices

For the calculation of credit risks, usually so-called *transition matrices* are necessary. They contain empirically observed migration probabilities of the debtors to transfer into other creditworthiness classes, for the loss of the debtors and the probability to stay in the actual creditworthiness class within a certain observation period.

In practice, average migration matrices are used for the parametrization of credit portfolio models. Average migration matrices contain the average values of (as a rule one-year-old) migration probabilities which have been observed over a period of several business cycles. They are more stable and more prestigious than one-year-old migration matrices. The migration probabilities are generated just on the basis of the migration behaviour of single calendar years. Nevertheless, they are not suitable, to illustrate the actual economic situation of single years. The parametrization with average migration matrices leads to distorted risk indicators [11, p. 1]. In average transition matrices, the probability to migrate in better creditworthiness classes are too pessimistically in times of economic recoveries, while this probabilities are overestimated in times of economic recessions.

Below we introduce the *One-Parameter Representation*, a method developed by Forest, Belkin and Suchower [11] that can be used for the generation of more stable, from the economic situation influenced, transition matrices. Then our new *modification of the One-Parameter Representation method* will be presented that reduces the costly data requirements.

1.5.1. *One-Parameter Representation*

In $CreditMetrics^{TM}$ the yield of a debtor is described by means of a standard normal random variable $X \sim \mathcal{N}(0, 1)$, see [12, p. 92]. Forest, Belkin and Suchower [11] describe this yield as a sum of a systematic and for a debtors specific component:

$$X = \sqrt{1-\rho^2}Y + \sqrt{\rho}Z. \tag{1.3}$$

Here, Y describes the debtors specific component and Z describes the systematic, the economic influenced, part of the yield of a debtor and ρ denotes a weighting factor of both components and a measure for the correlation between the yield X and the systematic component Z. While the specific component of a debtor varies within one year, the economic component Z is a, within one year, stable quantity.

In the sequel the variable $Z(t)$ describes the value of the systematic component of the year t. During an economic recovery $Z(t)$ takes positive values. Contrary, negative Z-values results from times of economic recessions. As standard normal random variable, Z has the mathematical expectation value zero. Thus the economic situation has in average no influence on the yields of the debtors.

To determine the systematic component $Z(t)$ of one certain year, the observed one-year-old migration probability $\tilde{p}_{i,j}(t)$ in year t will be compared with the average migration probability $p_{i,j}$, which is not influenced by the economic situation. For every combination of an initial rating i and a final rating j, these probabilities can be taken from one-year-old and average transition matrices. Figures 1.2 and 1.3 show the transitions matrices published by the rating agency Standard and Poor's (S&P) [13].

		endrating							
		AAA	AA	A	BBB	BB	B	CCC/C	default
initial rating	AAA	87,10	6,45	3,23	0,00	0,00	1,08	2,15	0,00
	AA	0,00	80,87	17,94	0,59	0,00	0,00	0,20	0,40
	A	0,00	1,67	92,27	5,18	0,47	0,00	0,00	0,40
	BBB	0,00	0,00	2,74	92,44	3,82	0,29	0,21	0,50
	BB	0,00	0,10	0,00	5,35	83,65	8,95	1,13	0,82
	B	0,00	0,00	0,00	0,16	4,14	82,31	9,09	4,30
	CCC/C	0,00	0,00	0,00	0,00	0,00	14,10	52,57	33,33

Fig. 1.2. One-year-old transition matrix of 2008, data taken from S&P [13].

		endrating							
		AAA	AA	A	BBB	BB	B	CCC/C	default
initial rating	AAA	91,33	7,88	0,55	0,06	0,08	0,03	0,06	0,00
	AA	0,60	90,51	8,10	0,56	0,06	0,09	0,03	0,03
	A	0,04	2,14	91,50	5,61	0,42	0,17	0,03	0,08
	BBB	0,01	0,16	4,14	90,24	4,28	0,74	0,17	0,26
	BB	0,02	0,06	0,21	5,87	83,87	7,99	0,89	1,10
	B	0,00	0,06	0,17	0,30	6,45	82,97	4,93	5,12
	CCC/C	0,00	0,00	0,27	0,40	1,13	13,77	54,60	29,85

Fig. 1.3. Averaged one-year transition matrix of 1981–2008, data taken from S&P [13].

Following the $CreditMetrics^{TM}$ approach, the density of the standard normal deviation is, for every initial rating, partitioned into transition thresholds $x_{i,j}$, cf. Figure 1.4. These thresholds were estimated by means of the probability of the transition matrices as follows:

$$\begin{aligned} x_{i,\text{default}} &= \phi^{-1}(p_{i,\text{default}}) \\ x_{i,\text{CCC}} &= \phi^{-1}(p_{i,\text{default}} + p_{i,\text{CCC}}) \\ &\vdots \end{aligned}$$

where ϕ is the standard normal deviation. $Z(t)$ must be chosen such that the cyclical one-year-old migration probabilities $p(t)$ are well approximated by:

$$\Delta\left(x_{i,j}, Z(t)\right) = \phi\left(\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right) - \phi\left(\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right), \tag{1.4}$$

cf. Figure 1.5.

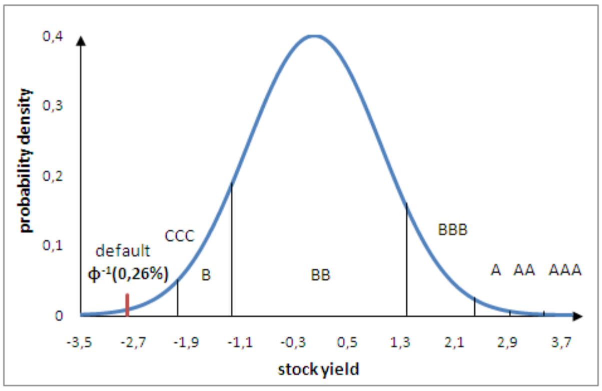

Fig. 1.4. Transition thresholds of standard normal distributed stock yield, cf. [12, p. 88].

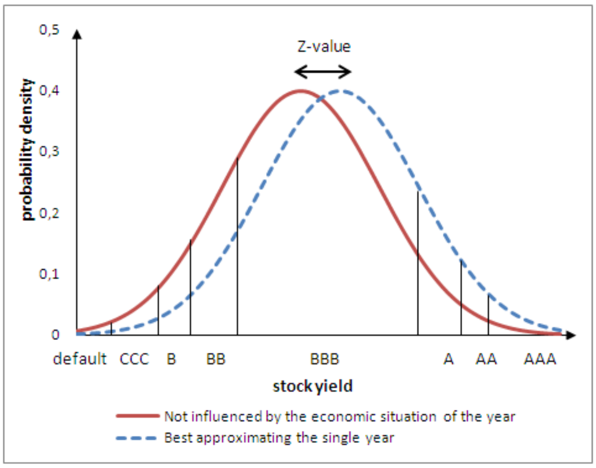

Fig. 1.5. Approximation of the one-year-old migration probabilities $p(t)$, cf. [11, p. 49].

Forest, Belkin and Suchower [11] solve this optimization problem by using a modified least square method:

$$\min_{Z(t)} \sum_i \sum_j n_i(t) \frac{\left(\tilde{p}_{i,j}(t) - \Delta\left(x_{i,j}, Z(t)\right)\right)^2}{\Delta\left(x_{i,j}, Z(t)\right)\left(1 - \Delta\left(x_{i,j}, Z(t)\right)\right)}. \tag{1.5}$$

The weighting factor ρ for the calculation of $\Delta\left(x_{i,j}, Z(t)\right)$ is a-priori unknown. It will be estimated by means of random values of ρ. For every ρ all values $Z(t)$ of the observation period will be estimated. ρ will be set on the value, for which the variance of $Z(t)$ is next to value one. Based on the assumption, that the mathematical expectation value of $Z(t)$ is zero, $Z(t)$ and as a consequence X is also a standard normal random variable and the model assumption is fulfilled. Transition matrices describe the economic situation of a special year can be generated by means of the transition thresholds $x_{i,j}$, the value of $Z(t)$ and the weighting factor ρ. The transition probability $P(i, j, Z(t))$ of the year t is, for every initial rating i and final rating j, given by:

$$P(i,j,Z(t)) = \phi\Bigg(\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\Bigg) - \phi\Bigg(\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\Bigg). \qquad (1.6)$$

1.5.2. *Modification of the One-Parameter Representation*

Average transition matrices do not consider any systematic effects and hence the risks are underestimated in extreme situations. Within the scope of stress tests economic adjusted transition matrices should be generated and used for risk quantification.

As described in [12, p. 92], the stock yield X is standard normally distributed in $CreditMetrics^{TM}$ i.e. in average, it will not be influenced by the economic situation. Nevertheless, the distribution of the stock yield differs from the standard normal distribution in single years. For the sake of convenience we regard the case, that the stock yield X_t of the special year t is normal distributed with mathematical expectation value μ_t and variance σ_t^2.

During an economic recession the level of the stock yields are lower than in normal economic periods. Additionally, the insecurity on the capital markets lead to larger fluctuations of the stock yields. The mathematical expectation of the share yields during weak economic years should become smaller and the variance should be larger than in positive economic phases. To determine the parameters μ_t and σ_t of the distribution of the stock yields in a certain year t, it is necessary to estimate the threshold values $x_{i,j}$ of the average migration probabilities, cf. *One-Parameter Representation* [11]. Afterwards it is necessary to approximate the one-year-old transition probability $p_{i,j}(t)$ best possible by means of the average transition probability, adjusted by μ_t and σ_t^2.

$$\Delta\left(x_{i,j}, \mu_t, \sigma_t^2\right) = \phi\left(\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right) - \phi\left(\frac{x_{i,j} - \mu_t}{\sigma_t}\right).$$

The approximation of the one-year-old transition probability by means of the adjusted average transition probability corresponds to the original method of Forest, Belkin and Suchower [11] can be interpreted analogously. The calculation of μ_t and σ_t^2 is done with the help of the least-square method:

$$\min_{\mu,\sigma^2} \sum_i \sum_j n_i(t) \frac{\left(\tilde{p}_{i,j}(t) - \Delta\left(x_{i,j},\mu,\sigma^2\right)\right)^2}{\Delta\left(x_{i,j},\mu,\sigma^2\right)\left(1-\Delta\left(x_{i,j},\mu,\sigma^2\right)\right)}. \tag{1.7}$$

In contrast to the original method, the economic influence is not only modeled by shifting the parameter Z but rather by shifting the mathematical expectation value and the variance. The migration probability $P(i,j,\mu_t,\sigma_t^2)$ of a year which is influenced by the economic situation, can be derived for every combination of initial rating i and final rating j as follows:

$$P(i,j,\mu_t,\sigma_t^2) = \phi\left(\frac{x_{i,j+1}-\mu_t}{\sigma_t}\right) - \phi\left(\frac{x_{i,j}-\mu_t}{\sigma_t}\right). \tag{1.8}$$

Compared to the original method of Forest, Belkin and Suchower [11] our new method possess the following advantages:

Lower expenditure for data mining: Instead of a whole history of transition matrices, only an average and a single year transition matrix is needed to estimate the parameter ρ.

Less computing time: The computing time decreases considerably because, only one least square estimation is needed to estimate the parameter μ_t and σ_t.

Better approximation: An additional degree of freedom decreases the sum of the errors. Hence, a better approximation of the single year transition matrix is possible.

1.6. Stress Tests by means of the Peaks over Threshold Method

The operational risk is an other essential type of risk beside the credit risk and market risk. In the beginning of the year 2008, the case of the french major bank Société Générale showed that a single trader can cause a damage in the high of over 5 billion Euro that can lead, in the worst case, to the insolvency of a bank.

This section shows how to use methods of extreme value theory (EVT) for validating the Loss Distribution Approach in the context of stress testing.

1.6.1. *The Loss Distribution Approach*

Within the scope of the quantification of the operational risk by means of the Loss Distribution Approach, empirical, real observed damage data will be described by the help of distribution functions. A *damage frequency distribution* models the frequency of incoming damages within a certain period. The frequency should correspond to the forecast period (or risk horizon). To represent the height of the damages, a damage height distribution will be generated. In a Monte Carlo simulation a value from the damage frequency distribution and afterwards accordingly values from the damage height distribution are generated. The sum of these damages is the *total loss for the forecast period*, cf. Figure 1.6. Many of such simulations of annual damages generating an empiric distribution function of annual damages. Finally, this procedure allows the calculation of the Value at Risk.

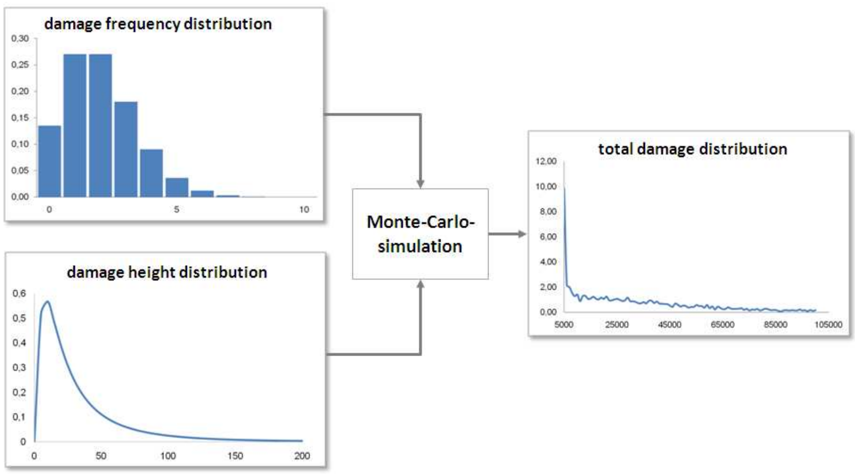

Fig. 1.6. Loss Distribution Approach, cf. [14, p. 68].

The damage frequency distribution is often modeled with a Poisson distribution which is a discrete distribution and suitable for modelling integers [14, p. 50]. Contrary, the damage height distribution is often modeled with a logarithm normal distribution. It fits the character of the distribution of operational risks particularly well, e.g. the skewness and leptokurtosis [14, p. 56].

1.6.2. *Stress Testing*

The observed damages, which are used for the generation of the damage height distribution, represent predominantly damages of regular scope. Hence, the damage height distribution works especially well around the distribution center. Because of the insufficient number of observed extreme damages, the tails of the damage height distribution does not describe well the observations. But it is within this scope, where most of the existence-threatening damages occur. Hence a stress test should figure out the influence of this weak point. For this purpose a tail distribution should be estimated by means of the Peak over Threshold method.

1.6.3. *The Peaks over Threshold method*

The *Peaks over Threshold* method (POT method) is used to model distribution tails in cases where only a few observations are available. Because it is assumed that observations of the distribution center do not deliver any pieces of information about level and probability of extreme events, solely the observation above a certain threshold of the distribution will be used zu model the tail distribution [15, p. 488]. The basic idea is to estimate the left and the right tail by fitting a generalized Pareto distribution (GPD) to the observation lying above a certain threshold marking the beginning of the tail regin.

Let $X_1, \ldots, X_n$ denote a series of independent and identically distributed (i.i.d.) random variables with distribution function F. Also, assume that the distribution belongs to the maximum domain of attraction (MDA): $F \in MDA(H_\xi)$ for a $\xi \in \mathbb{R}$. For one threshold value u let $Y_1, \ldots, Y_{N_u}$ be the series of the excesses over the threshold u, where

$$N_u = \sum_{i=1}^{n} 1_{X_i > u}$$

counts the number of these excesses, resulting from n random variables $X_1, \ldots, X_n$. Here, $1_{X_i > u}$ denotes an indicator function, i.e.

$$1_{X_i > u} = \begin{cases} 1, & \text{if } X_i > u \\ 0, & \text{else} \end{cases} .$$

Hence, the excess distribution of X is given by

$$\begin{aligned} F_u(y) &= \mathbb{P}(Y \leq y | X > u) = \mathbb{P}(X - u \leq y | X > u) \\ &= \mathbb{P}(X \leq y + u | X > u) = \frac{F(y+u) - F(u)}{1 - F(u)}, \quad y \geq 0. \end{aligned}$$

The tail of this distribution can be obtained by

$$F(y+u) = (1 - F(u))F_u(y) + F(u). \tag{1.9}$$

Due to the Pickands-Balkema-de-Haan-Theorem [16, p. 135] the excess distribution $F_u(y)$ can be approximated for large threshold values u by a generalized Pareto distribution given by

$$G_{\xi,\beta(u)}(x) = \begin{cases} 1 - (1 + \frac{\xi}{\beta(u)} x)^{-\frac{1}{\xi}}, & \text{if } \xi \neq 0, \\ 1 - \exp(-\frac{x}{\beta(u)}), & \text{if } \xi = 0, \end{cases},$$

provided the underlying distribution function F_u satisfies the Fisher-Tippett Theorem [16, p. 130] (which is true for all common continuous distributions used in finance).

An estimator for the frequency of these excesses reads

$$1 - F(u) = \frac{1}{n} \sum_{i=1}^{n} 1_{X_i > u} = \frac{N_u}{n},$$

cf. [1, p. 354]. Thus the equation (1.9) can be rewritten as

$$F(y+u) = \frac{N_u}{n} G_{\xi,\beta(u)} + (1 - \frac{N_u}{n}).$$

1.6.4. *Parameter estimation*

To estimate the tail distribution of a given random variable it is necessary to choose a suitable threshold, which divides the whole distribution in a distribution body and a distribution tail. Based on this threshold, the excesses can be identified and the parameter of the tail distribution can be estimated. The parameter of the generalized Pareto distribution depends on this threshold u [1, pp. 356]. The quality of the chosen threshold influences the results of all following investigations.

In the literature there are different approaches to identify a suitable threshold value. Beside graphical methods, for example the *Mean-Excess* (ME) plot [2,17] or the *Hill graph method* [18, Section 3.3], also heuristic methods are used, e.g. an arbitrary threshold level of 90% confidence level [19] or 1.645 times the unconditional variance of the data [20] that corresponds to 5% of the extreme observations in case of normally distributed data. In practice it is usual to take a certain quantile or a constant value of the threshold [14, p. 100]. More complex methods, like the Bias-variance bootstrap, are based on simulations and aiming to optimize the bias-variance trade off [14, p. 100].

After the distribution tail is defined by the choice of a suitable threshold, the parameter of the generalized Pareto distribution can be estimated by means of excesses. Common approaches for the estimation of the generalized Pareto distribution are for example the maximum likelihood method or the weighted moment method [14, pp. 356].

Conclusion

In this chapter we gave an introduction to the stress test method in extrem value theory that is a tool to deal with probabilities related to extreme and hence rare events. We proposed a modification to the established One-Parameter Representation method of Forest, Belkin and Suchower [11] that overcomes several shortcomings. The new method has a small data requirements, needs only a computational effort and intensive tests with real market data from the IBB [21] showed that also the approximation quality is improved significantly.

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