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Abstract

A set of 54 test examples is presented for a very general class of feasibility problems that involve expensive function evaluations, e.g. simulation runs. The presence of expensive functions is artificially introduced by defining parts or complete problems as expensive. These examples are adapted from the CUTEr test set [1].

Keywords: feasibility problem, nonlinear system, derivative-free, expensive function, test set

1 Introduction

Simulations have become a fundamental tool in the academic world as well as in industry over the past years. Especially engineering applications benefit from the rising number of simulation tools for a large variety of applications. One of the major goals of these simulations is the improvement of the simulated processes, which leads to several kinds of simulation-based problems in optimization. In this paper, we consider feasibility problems, i.e. nonlinear systems of equations and inequalities.

As simulations are in many applications very time consuming and do usually not provide derivative information, they can be considered as so called expensive functions. Specialized solution algorithms for such problems have to use as few evaluations of the expensive functions, e.g. simulation runs, as possible. Usually the cost of one evaluation of the expensive function, i.e. the computation time or the cost for a single simulation run or experiment, is dominating the total cost of the algorithm. Hence it is reasonable to compare different algorithms for problems with expensive functions in terms of the number of evaluations of the expensive functions.

In this paper we present a set of 54 test problems of a very general class of feasibility problems involving one or several expensive functions of the form

$$
\begin{aligned}
c_{\mathcal{E}}(x, u(x)) &= 0 \\
c_{\mathcal{I}}(x, u(x)) &\leq 0,
\end{aligned}
\tag{1}
$$

where $u : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is an expensive function, which is assumed to be smooth, and $c_{\mathcal{E}} : \mathbb{R}^{n+m} \rightarrow \mathbb{R}^p$ and $c_{\mathcal{I}} : \mathbb{R}^{n+m} \rightarrow \mathbb{R}^q$ are smooth "cheap" functions, i.e. $c_{\mathcal{E}}$ and $c_{\mathcal{I}}$ are explicitly given or fast and cheap to evaluate. This nested function formulation provides the possibility of modeling as much known information of a given process as available within the outer functions $c_{\mathcal{E}}$ and $c_{\mathcal{I}}$. Numerical tests have shown that, in general, the more information is provided in the outer functions, the less function evaluations of the expensive function are necessary [3].

2   Set of test examples

The following set of 54 test problems is based on a subset of the test problems in the CUTEr collection [1] and is inspired by the selection of test problems for FILTRANE, a nonlinear feasibility problem solver [2]. The test problems are selected to represent different function types and reasonable problem sizes. Since many engineering applications only have a small number of variables (often called design variables) and a small number of outcomes of the expensive function, we have mainly chosen test problems with less than 10 variables, but also problems up to 100 variables are included. Some problems are scalable and can therefore easily be transformed to have a larger number of variables. Some reasonable values are included in the concerned problem descriptions depending on the suggestions in [1].

None of the available test problems in the CUTEr collection with an appropriate problem size contains inequalities. Thus, we have redefined some equations as inequalities to obtain a well-balanced and significant test set for problems of the form (1).

To simulate the presence of one or more expensive functions, some parts of the nonlinear systems of equations are defined to be expensive. The advantage of simple functions as being artificially expensive instead of using 'real' expensive functions is the much faster solution time of the test examples. Thus, larger test series like parameter tests can be executed to a greater extend. Counting the number of evaluations of the expensive function then provides information on the expected execution time for practical applications.

In Table 1, we give an overview over the test problems which were taken from the CUTEr collection and the corresponding modifications. The problems names that are listed below are adaped from the corresponding problems in the CUTEr test set in [1]. Besides the problem dimensions and the number of equations and inequalities, the table also contains information about the specific choice of the expensive function(s) we use in the corresponding problems. This information is provided in the rows 'Ex.func.' in Table 1 in the following way:

- The *first* character gives the number of expensive functions in the particular problem.

- The *second* character specifies the type of the expensive function. The expensive function can be:

E	exponential
H	hyperbolic
L	logarithmic
P	polynomial
T	trigonometric
X	of type x^x

If more than one of these types are involved, all of them are mentioned.

- The *third* character provides information on the occurrence of the expensive function in the overall system:

A	at least one part in every equation or inequality is considered as expensive
C	the complete system is considered as expensive
E	entire equations or inequalities are considered as expensive
O	only parts in some equations or inequalities are considered as expensive

In addition to the descriptions of the feasibility problems, a starting point is given for all problems as well as suggestions for parameter choices, if necessary. These information are also based on those for the corresponding problems in [1].

Example	Var.	Eq.	Ineq.	Ex.func.	Example	Var.	Eq.	Ineq.	Ex.func.
Aircrfta_exp	5	5	0	4-P-O	Eigena_exp	110	55	55	55-P-O
Argauss_exp	3	15	0	15-E-A	Eigenb_exp	6	3	3	3-P-O
Arglale_exp	10	10	10	20-P-C	Eigenc_exp	30	30	0	1-P-O
Arglble_exp	50	1	99	100-P-C	Gottfr_exp	2	2	0	2-P-E
Arglcle_exp	10	19	0	18-P-C	Growth_exp	3	1	11	2-L-O
Argtrig_exp	10	10	0	10-T-A	Hatfldf_exp	3	3	0	3-EP-E
Artif_exp	10	0	10	10-T-A	Hatfldg_exp	25	25	0	10-P-E
Arwhdne_exp	10	9	9	9-P-E	Himmelba_exp	2	2	0	2-P-C
Bdvalue_exp	10	10	0	6-P-O	Himmelbc_exp	2	2	0	2-P-O
Bdvalues_exp	10	10	0	3-P-O	Himmelbd_exp	2	2	0	2-P-O
Booth_exp	2	2	0	2-P-C	Himmelbe_exp	3	3	0	3-P-C
Bratu2d_exp	25	25	0	25-E-A	Hypcir_exp	2	2	0	2-P-C
Bratu2dt_exp	25	25	0	25-E-A	Integreq_exp	10	10	0	3-P-O
Bratu3d_exp	27	27	0	27-E-A	Msqrta_exp	4	4	0	4-P-O
Brownale_exp	10	10	0	1-P-E	Msqrtb_exp	9	9	0	9-P-O
Broydn3d_exp	10	10	0	8-P-O	Pfit1_exp	3	2	1	1-X-O
Broydnbd_exp	10	10	0	3-P-O	Pfit2_exp	3	3	0	1-X-O
Cbratu2d_exp	8	8	0	4-ET-O	Pfit3_exp	3	3	0	1-P-O
Cbratu3d_exp	16	16	0	8-ET-O	Pfit4_exp	3	3	0	1-X-O
Chandheq_exp	10	10	0	2-P-E	Powellbs_exp	2	2	0	2-PX-C
Chemrcta_exp	10	10	0	4-P-E	Powellsq_exp	2	2	0	2-PH-C
Chemrctb_exp	10	6	4	8-E-O	Recipe_exp	3	3	0	3-PH-C
Chnrsbne_exp	10	18	0	5-P-E	Rsnbrne_exp	2	2	0	2-P-C
Cluster_exp	2	2	0	2-PT-E	Semicon1_exp	10	10	0	9-E-O
Coolhans_exp	9	9	0	5-P-E	Semicon2_exp	10	10	0	9-E-O
Cubene_exp	2	2	0	2-P-C	Sinvalne_exp	2	2	0	2-PT-C
Deconvne_exp	61	10	30	5-P-O	Zangwil3_exp	3	3	0	3-P-C

Table 1: Overview and characterization of the test problems

Problem 1: Aircrfta_exp

$c_1 : -0.727x_2x_3 + 8.39x_3x_4 - u_1x_5 + 63.5x_2x_4 - 3.933x_1 + 0.107x_2 + 0.126x_3 - 9.99x_5 = 0$

$c_2 : u_2 - 0.987x_2 - 22.95x_4 - 2.837 = 0$

$c_3 : u_3 + 1.132x_2x_4 + 0.002x_1 - 0.235x_3 + 5.67x_5 = 0$

$c_4 : -x_1x_5 + x_2 - x_4 - 0.1168 = 0$

$c_5 : u_4 - 0.1168 = 0$

with the expensive functions

$$u_1 = 684.4x_4$$
$$u_2 = 0.949x_1x_3 + 0.173x_1x_5$$
$$u_3 = -0.716x_1x_2 - 1.578x_1x_4$$

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$$u_4 = x_1 x_2 + x_3$$

and an initial point $\bar{x} = (0, 0, 0, 0, 0)$.

Problem 2: Argauss_exp

$$c_i : x_1 + u_i - k_i = 0 \qquad \text{for} \ \ i = 1, \ldots, 15$$

with

$$k = (0.0009, 0.0044, 0.0175, 0.0540, 0.1295, 0.2420, 0.3521, 0.3989, 0.3521, 0.2420, 0.1295,$$
$$0.0540, 0.0175, 0.0044, 0.0009)^\top,$$

the expensive functions

$$u_i = e^{\frac{x_2(4 - \frac{i}{2} - x_3)^2}{-2}} \qquad \text{for} \ \ i = 1, \ldots, 15$$

and an initial point $\bar{x} = (0.4, 1, 0)$.

Problem 3: Arglale_exp

$$c_i : u_i \leq 0 \qquad \text{for} \ \ i = 1, \ldots, n$$
$$c_i : u_i = 0 \qquad \text{for} \ \ i = n + 1, \ldots, 2n$$

with the expensive functions

$$u_i = \frac{n-1}{n} x_i - 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{x_j}{n} \qquad \text{for} \ \ i = 1, \ldots, n$$

$$u_i = -1 - \sum_{j=1}^{n} \frac{x_j}{n} \qquad \text{for} \ \ i = n + 1, \ldots, 2n,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 50, 100 or 200.

Problem 4: Arglble_exp

$$c_i : u_i \leq 0 \qquad \qquad \text{for} \ \ i = 1, \ldots, n - 1$$
$$c_n : n \sum_{j=1}^{\frac{n}{2}} j \cdot x_j - 1 = 0$$

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with the expensive functions

$$u_i = i \sum_{j=1}^{\frac{n}{2}} j \cdot x_j - 1 \qquad \text{for } i = 1, \ldots, n-1,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 100$. Alternative choices for n may be 2, 10, 50 or 200.

Problem 5: Arglcle_exp

$$c_i : u_i = 0 \qquad \text{for } i = 1, \ldots, n-2$$

with the expensive functions

$$u_i = i \sum_{j=1}^{\frac{n}{2}-2} (j+1) \cdot x_j - 1 \qquad \text{for } i = 1, \ldots, n-2,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 20$. Alternative choices for n may be 50, 100 or 200.

Problem 6: Argtrig_exp

$$c_i : u_i - n + \sum_{j=1}^{n} \cos(x_j) = 0 \qquad \text{for } i = 1, \ldots, n$$

with the expensive functions

$$u_i = i(\sin(x_i) + \cos(x_i) - 1) \qquad \text{for } i = 1, \ldots, n,$$

an initial point $\bar{x} = \frac{1}{n}(1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 50, 100 or 200.

Problem 7: Artif_exp

$$c_i : u_i - 0.05(x_i + x_{i+1} + x_{i+2}) = 0 \qquad \text{for } i = 1, \ldots, n$$

with the expensive functions

$$u_i = \arctan(\sin(ix_{i+1})) \qquad \text{for } i = 1, \ldots, n,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 8: Arwhdne_exp

$$c_i : u_i = 0 \qquad \text{for } i = 1, \ldots, n-1$$
$$c_i : 4x_{i-(n-1)} - 3 \leq 0 \qquad \text{for } i = n, \ldots, 2(n-1)$$

with the expensive functions

$$u_i = x_i^2 + x_n^2 \qquad \text{for } i = 1, \ldots, n-1,$$

an initial point $\bar{x} = (5, \ldots, 5, 1)$ and $n = 10$. Alternative choices for n may be 2, 100, 500 or 1000.

Problem 9: Bdvalue_exp

$$c_1 : 2x_i - x_{i+1} + u_i = 0$$
$$c_i : 2x_i - x_{i-1} - x_{i+1} + u_i = 0 \qquad \text{for } i = 2, \ldots, n-1$$
$$c_n : 2x_i - x_{i-1} + u_i = 0$$

with the expensive functions

$$u_i = \frac{1}{2(n+1)^2} \left(x_i + \frac{i+n+2}{n+1} \right)^3 \qquad \text{for } i = 1, \ldots, n,$$

an initial point $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ with $\bar{x}_i = \left(\frac{i-1}{n+1}\right)^2 - \frac{i-1}{n+1}$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 10: Bdvalues_exp

$$c_i : 2x_{i+1} - x_i - x_{i+2} + u_i = 0 \qquad \text{for } i = 1, \ldots, k$$
$$c_i : 2x_{i+1} - x_i - x_{i+2} + \frac{1}{2(n+1)^2} \left(x_i + \frac{i+n+2}{n+1} \right)^3 = 0 \qquad \text{for } i = k+1, \ldots, n$$

with the expensive functions

$$u_i = \frac{1}{2(n+1)^2} \left(x_i + \frac{i+n+2}{n+1} \right)^3 \qquad \text{for } i = 1, \ldots, k,$$

an initial point $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ with $\bar{x}_i = s \cdot \left(\left(\frac{i-1}{n+1}\right)^2 - \frac{i-1}{n+1} \right)$, $n = 10$, $k = 3$ and $s = 1000$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 11: Booth_exp

$$c_i : u_i = 0 \text{ for } i = 1, 2$$

with the expensive functions

$$u_1 = x_1 + 2x_2 - 7$$
$$u_2 = 2x_1 + x_2 - 5$$

and an initial point $\bar{x} = (0,0)$.

Problem 12: Bratu2d_exp

$$c_{i,j} : 4x_{i,j} - (x_{i+1,j} + x_{i-1,j} + x_{i,j-1} + x_{i,j+1}) - 4u_{i,j} = 0 \qquad \text{for} \ \ i,j = 2, \ldots, n-1$$

with

$$x_{1,j} = x_{n,j} = x_{i,1} = x_{i,n} := 0 \qquad \text{for} \ \ i,j = 2, \ldots, n-1,$$

the expensive functions

$$u_{i,j} = \frac{e^{x_{i,j}}}{(n-1)^2} \qquad \text{for} \ \ i,j = 2, \ldots, n-1,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 7$. Alternative choices for n may be 10, 22, 32 or 72.

Problem 13: Bratu2dt_exp

$$c_{i,j} : 4x_{i,j} - (x_{i+1,j} + x_{i-1,j} + x_{i,j-1} + x_{i,j+1}) - 6.80812u_{i,j} = 0 \qquad \text{for} \ \ i,j = 2, \ldots, n-1$$

with

$$x_{1,j} = x_{n,j} = x_{i,1} = x_{i,n} := 0 \qquad \text{for} \ \ i,j = 2, \ldots, n-1,$$

the expensive functions

$$u_{i,j} = \frac{e^{x_{i,j}}}{(n-1)^2} \qquad \text{for} \ \ i,j = 2, \ldots, n-1,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 7$. Alternative choices for n may be 10, 22, 32 or 72.

Problem 14: Bratu3d_exp

$$c_{i,j,k} : 6x_{i,j,k} - (x_{i+1,j,k} + x_{i-1,j,k} + x_{i,j-1,k} + x_{i,j+1,k} + x_{i,j,k-1} + x_{i,j,k+1}) - 6.80812u_{i,j,k} = 0$$

for $i, j, k = 2, \ldots, n-1$ with

$$x_{1,j,k} = x_{n,j,k} = x_{i,1,k} = x_{i,n,k} = x_{i,j,1} = x_{i,j,n} := 0 \qquad \text{for} \ \ i,j,k = 2, \ldots, n-1,$$

the expensive functions

$$u_{i,j,k} = \frac{e^{x_{i,j,k}}}{(n-1)^2} \qquad \text{for} \ \ i,j,k = 2, \ldots, n-1,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 5$. Alternative choices for n may be 3, 8, 10 or 17.

Problem 15: Brownale_exp

$$c_i : \sum_{j=1}^{n} x_j + x_i - (n+1) = 0 \qquad \text{for } i = 1, \ldots, n-1$$

$$c_n : u - 1$$

with the expensive function

$$u = \prod_{j=1}^{n} x_j,$$

an initial point $\bar{x} = \frac{1}{2}(1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 100, 200 or 1000.

Problem 16: Broydn3d_exp

$$c_1 : 3x_1 - 2x_2 - 2x_1^2 + 1 = 0$$
$$c_i : u_i + 1 = 0 \qquad \text{for } i = 2, \ldots, n-1$$
$$c_n : 3x_n - x_{n-1} - 2x_n^2 + 1 = 0$$

with the expensive functions

$$u_i = 3x_i - x_{i-1} - 2x_{i+1} - 2x_i^2 \qquad \text{for } i = 2, \ldots, n-1,$$

an initial point $\bar{x} = (-1, \ldots, -1)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 17: Broydnbd_exp

$$c_i : 2x_i + 5x_i^2 - \sum_{\substack{j=1 \\ j \neq i}}^{5} (x_j + x_j^2) = 0 \qquad \text{for } i = 1, \ldots, 5$$

$$c_i : 2x_i + 5x_i^2 - u_i = 0 \qquad \text{for } i = 6, \ldots, n-2$$

$$c_i : 2x_i + 5x_i^2 - \sum_{\substack{j=i-4 \\ j \neq i}}^{n} (x_j + x_j^2) = 0 \qquad \text{for } i = n-1, n$$

with the expensive functions

$$u_i = \sum_{\substack{j=i-4 \\ j \neq i}}^{i+1} (x_j + x_j^2) \qquad \text{for } i = 6, \ldots, n-2,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 18: Cbratu2d_exp

$$c_{1,i,j} : 4x_{1,i,j} - (x_{1,i+1,j} + x_{1,i-1,j} + x_{1,i,j-1} + x_{1,i,j+1}) - 4u_{i,j} = 0 \qquad \text{for } i, j = 2, \ldots, n-1$$

$$c_{2,i,j} : 4x_{2,i,j} - (x_{2,i+1,j} + x_{2,i-1,j} + x_{2,i,j-1} + x_{2,i,j+1}) - 4\frac{e^{x_{2,i,j}}}{(n-1)^2} \sin(x_{1,i,j}) = 0$$

$$\text{for } i, j = 2, \ldots, n-1$$

with

$$x_{k,1,j} = x_{k,n,j} = x_{k,i,1} = x_{k,i,n} := 0 \qquad \text{for } i, j = 2, \ldots, n-1, \ k = 1, 2,$$

the expensive functions

$$u_{i,j} = \frac{e^{x_{1,i,j}} \cos(x_{2,i,j})}{(n-1)^2} \qquad \text{for } i, j = 2, \ldots, n-1,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 4$. Alternative choices for n may be 7, 16, 23 or 40.

Problem 19: Cbratu3d_exp

$$c_{1,i,j,k} : 6x_{1,i,j,k} - (x_{1,i+1,j,k} + x_{1,i-1,j,k} + x_{1,i,j-1,k} + x_{1,i,j+1,k} + x_{1,i,j,k-1} + x_{1,i,j,k+1})$$
$$- 6.80812u_{i,j,k} = 0 \qquad \text{for } i, j, k = 2, \ldots, n-1$$

$$c_{2,i,j,k} : 6x_{2,i,j,k} - (x_{2,i+1,j,k} + x_{2,i-1,j,k} + x_{2,i,j-1,k} + x_{2,i,j+1,k} + x_{2,i,j,k-1} + x_{2,i,j,k+1})$$
$$- 6.80812\frac{e^{x_{1,i,j,k}}}{(n-1)^2} \sin(x_{2,i,j,k}) = 0 \ \text{ for } i, j, k = 2, \ldots, n-1$$

with

$$x_{h,1,j,k} = x_{h,n,j,k} = x_{h,i,1,k} = x_{h,i,n,k} = x_{h,i,j,1} = x_{h,i,j,n} := 0$$

for $i, j, k = 2, \ldots, n-1, \ h = 1, 2$, the expensive functions

$$u_{i,j,k} = \frac{e^{x_{i,j,k} \cos(x_{2,i,j,k})}}{(n-1)^2} \quad \text{for } i, j, k = 2, \ldots, n-1,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 4$. Alternative choices for n may be 3, 7, 10 or 12.

Problem 20: Chandheq_exp

$$c_i : x_i - x_i \sum_{j=1}^{n} \frac{ix_j}{2n(i+j)} - 1 = 0 \qquad \text{for } i = 1, \ldots, n-2$$

$$c_i : u_i = 0 \qquad\qquad \text{for } i = n-1, n$$

with the expensive functions

$$u_i = x_i - x_i \sum_{j=1}^{n} \frac{ix_j}{2n(i+j)} - 1 \qquad \text{for } i = n-1, n,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 10$. Alternative choices for n may be 50 or 100.

Problem 21: Chemrcta_exp

$c_{1,1} : u_1 = 0$

$c_{2,1} : u_2 = 0$

$$c_{1,i} : \left(\frac{(n-1)^2}{5} + n - 1 \right)(x_{1,i-1} - 2x_{1,i}) + \frac{(n-1)^2}{5} x_{1,i+1} + 0.0675 x_{2,i} e^{25 - \frac{25}{x_{1,i}}} = 0$$
$$\text{for } i = 2, \ldots, n-1$$

$$c_{2,i} : \left((n-1)^2 + n - 1 \right)(x_{2,i-1} - 2x_{2,i}) + (n-1)^2 x_{2,i+1} - 0.135 x_{2,i} e^{25 - \frac{25}{x_{1,i}}} = 0$$
$$\text{for } i = 2, \ldots, n-1$$

$c_{1,n} : u_3 = 0$

$c_{2,n} : u_4 = 0$

with the expensive functions

$$u_1 = \frac{5}{n-1} - x_{1,1} - \frac{5}{n-1} x_{1,2}$$
$$u_2 = \frac{1}{n-1} - x_{2,1} - \frac{1}{n-1} x_{2,2}$$
$$u_3 = x_{1,n} - x_{1,n-1}$$
$$u_4 = x_{2,n} - x_{2,n-1},$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 5$. Alternative choices for n may be 25, 50, 250 or 500.

Problem 22: Chemrctb_exp

$$c_1 : x_2 - x_1 + \frac{5}{(n-1)} = 0$$

$$c_i : \left(\frac{(n-1)^2}{5} + n - 1 \right)(x_{i-1} - 2x_i) + \frac{(n-1)^2}{5} x_{i+1} - u_{i-1} = 0 \quad \text{for } i = 2, \ldots, n-3$$

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$$c_i : \left(\frac{(n-1)^2}{5} + n - 1\right)(x_{i-1} - 2x_i) + \frac{(n-1)^2}{5}x_{i+1} - u_{i-1} \leq 0 \quad \text{for} \ i = n-2, n-1$$

$$c_n : x_n - x_{n-1} = 0$$

with the expensive functions

$$u_i = 3.1725e^{25 - \frac{25}{x_{i+1}}} \qquad \text{for} \ i = 1, \ldots, n-2,$$

an initial point $\bar{x} = (1, \ldots, 1)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 23: Chnrsbne_exp

$$c_i : u_i = 0 \qquad\qquad \text{for} \ i = 1, \ldots, \frac{n+1}{2}$$

$$c_i : 4\alpha_i(x_i - x_{i+1}^2) = 0 \qquad \text{for} \ i = \frac{n+1}{2} + 1, \ldots, n-1$$

$$c_i : x_{i-n} - 1 = 0 \qquad\qquad \text{for} \ i = n, \ldots, 2(n-1)$$

with the expensive functions

$$u_i = 4\alpha_i(x_i - x_{i+1}^2) \qquad \text{for} \ i = 1, \ldots, \frac{n+1}{2},$$

an initial point $\bar{x} = (-1, \ldots, -1)$, $n = 10$ and

$\alpha = [1.4, 2.4, 1.4, 1.75, 1.2, 2.25, 1.2, 1, 1.1, 1.5, 1.6, 1.25, 1.25, 1.2, 1.2, 1.4, 0.5, 0.5, 1.25, 1.8,$
$\quad 0.75, 1.25, 1.4, 1.6, 2, 1, 1.6, 1.25, 2.75, 1.25, 1.25, 1.25, 3, 1.5, 2, 1.25, 1.4, 1.8, 1.5, 2.2,$
$\quad 1.4, 1.5, 1.25, 2, 1.5, 1.25, 1.4, 0.6, 1.5].$

Alternative choices for n may be 25 or 50.

Problem 24: Cluster_exp

$$c_1 : u = 0$$
$$c_2 : (\cos(x_2) - x_1)(x_2 - \cos(x_1)) = 0$$

with the expensive functions

$$u = (x_1 - x_2^2)(x_1 - \sin(x_2))$$

and an initial point $\bar{x} = (0, 0)$.

Problem 25: Coolhans_exp

$c_i : u_i = 0$ for $i = 1, \ldots, 5$

$c_6 : (1.3725 \cdot 10^{-7} x_1 + 937.62 x_4 - 42.207 x_7) x_2 + (1.3725 \cdot 10^{-7} x_2 + 937.62 x_5 - 42.207 x_8) x_5$
$\quad + (1.3725 \cdot 10^{-7} x_3 + 937.62 x_6 - 42.207 x_9) x_8 + 0.13880 \cdot 10^{-6} x_2 - 1886 x_5 + 42.362 x_8$
$\quad + 948.21 = 0$

$c_7 : (1.3725 \cdot 10^{-7} x_1 + 937.62 x_4 - 42.207 x_7) x_3 + (1.3725 \cdot 10^{-7} x_2 + 937.62 x_5 - 42.207 x_8) x_6$
$\quad + (1.3725 \cdot 10^{-7} x_3 + 937.62 x_6 - 42.207 x_9) x_9 + 0.13880 \cdot 10^{-6} x_3 - 1886 x_6 + 42.362 x_9$
$\quad = 0$

$c_8 : -0.13877 \cdot 10^{-6} x_1 + 42.362 x_4 - 2.0705 x_7 = 0$

$c_9 : -0.13877 \cdot 10^{-6} x_3 + 42.362 x_6 - 2.0705 x_9 = 0$

with the expensive functions

$u_1 = 0.0060893 x_1 - 44.292 x_4 + 2.0011 x_7$

$u_2 = 0.0060893 x_2 - 44.292 x_5 + 2.0011 x_8 + 948.21$

$u_3 = 0.0060893 x_3 - 44.292 x_6 + 2.0011 x_9$

$u_4 = (1.3725 \cdot 10^{-7} x_1 + 937.62 x_4 - 42.207 x_7) x_1 + (1.3725 \cdot 10^{-7} x_2 + 937.62 x_5 - 42.207 x_8) x_4$
$\quad + (1.3725 \cdot 10^{-7} x_3 + 937.62 x_6 - 42.207 x_9) x_7 + 0.13880 \cdot 10^{-6} x_1 - 1886 x_4 + 42.362 x_7$

$u_5 = -0.13877 \cdot 10^{-6} x_2 + 42.362 x_5 - 2.0705 x_8 - 42.684$

and an initial point $\bar{x} = (1, \ldots, 1)$.

Problem 26: Cubene_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = x_1 - 1$$
$$u_2 = 10 x_2 - x_1^3$$

and an initial point $\bar{x} = (-1, 2)$.

Problem 27: Deconvne_exp

$$c_i : \sum_{j=1}^{\min\{i+1, 11\}} x_{i-j+1} x_{40+j} - t_i = 0 \qquad \text{for } i = 1, \ldots, 10$$

$$c_i : \sum_{j=1}^{\min\{i+1,11\}} x_{i-j+1} x_{40+j} - t_i \le 0 \qquad \text{for } i \in \{11, \ldots, 40\} \setminus \{20, 25, 30, 35, 40\}$$

$$c_i : u_{\frac{i}{5}-3} - t_i \le 0 \qquad\qquad \text{for } i = 20, 25, 30, 35, 40$$

with the expensive functions

$$u_i = \sum_{j=1}^{\min\{i+1,11\}} x_{i-j+1} x_{40+j} \qquad \text{for } i = 20, 25, 30, 35, 40,$$

an initial point

$$\bar{x} = (\bar{x}_1, \ldots, \bar{x}_{40}, 0.01, 0.02, 0.4, 0.6, 0.8, 3, 0.8, 0.6, 0.44, 0.01, 0.01)$$

with $\bar{x}_1 = \ldots = \bar{x}_{40} = 0$ and

$$t = [0, 0, 0.0016, 0.0054, 0.0702, 0.1876, 0.332, 0.764, 0.932, 0.812, 0.3464, 0.2064, 0.083,$$
$$0.034, 0.0618, 1.2, 1.8, 2.4, 9, 2.4, 1.801, 1.325, 0.0762, 0.2104, 0.268, 0.552, 0.996, 0.36,$$
$$0.24, 0.151, 0.0248, 0.2432, 0.3602, 0.48, 1.8, 0.48, 0.36, 0.264, 0.003, 0.003].$$

Problem 28: Eigena_exp

$$c_{1,i,j} : \sum_{k=1}^{n} x_{i,k} x_{k,j} x_{n+1,k} - a_{i,j} \le 0 \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

$$c_{2,i,j} : \sum_{k=1}^{n-1} x_{i,k} x_{k,j} - 1 + u_{i,j} = 0 \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

with the expensive functions

$$u_{i,j} = x_{i,n} x_{n,j} \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

an initial point $\bar{x}$ with $\bar{x}_{i,i} = 1$ for $i = 1, \ldots, n$ and $\bar{x}_{i,j} = 0$ if $i \ne j$, constants $a_{i,i} = 1$ for $i = 1, \ldots, n$ and $a_{i,j} = 0$ if $i \ne j$ and $n = 10$. Alternative choices for n may be 2 or 50.

Problem 29: Eigenb_exp

$$c_{1,i,j} : \sum_{k=1}^{n} x_{i,k} x_{k,j} x_{n+1,k} - a_{i,j} \le 0 \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

$$c_{2,i,j} : \sum_{k=1}^{n-1} x_{i,k} x_{k,j} - 1 + u_{i,j} = 0 \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

with the expensive functions

$$u_{i,j} = x_{i,n} x_{n,j} \qquad \text{for } j = 1, \ldots, n, \ i = 1, \ldots, j$$

an initial point $\bar{x}$ with $\bar{x}_{i,i} = 1$ for $i = 1, \ldots, n$, $\bar{x}_{i,j} = 0$ if $i \neq j$, constants $a_{i,i} = 2$ for $i = 1, \ldots, n$, $a_{i-1,i} = -1$ for $i = 2, \ldots, n$ and $a_{i,j} = 0$ if $i-1 \neq j \neq i$ and $n = 2$. Alternative choices for n may be 10 or 50.

Problem 30: Eigenc_exp

$$c_1 : x_5 u_1 + x_6 x_3^2 - 2 = 0$$
$$c_2 : x_5 x_2 x_1 + x_6 x_4 x_3 + 1 = 0$$
$$c_3 : x_5 u_2 + x_6 x_4^2 - 1 = 0$$
$$c_4 : u_1 + x_3^2 - 1 = 0$$
$$c_5 : x_2 x_1 + x_4 x_3 - 1 = 0$$
$$c_6 : u_2 + x_4^2 - 1 = 0$$

with the expensive functions

$$u_1 = x_1^2$$
$$u_2 = x_2^2$$

and an initial point $\bar{x} = (1, 0, 0, 1, 1, 1)$.

Problem 31: Gottfr_exp

$$c_1 : u_1 = 0$$
$$c_2 : x_2 + 7.5 u_2 = 0$$

with the expensive functions

$$u_1 = x_1 - 0.1136(x_1 + 3x_2)(1 - x_1)$$
$$u_2 = (2x_1 - x_2)(1 - x_2)$$

and an initial point $\bar{x} = (2, 2)$.

Problem 32: Growth_exp

$$c_1 : x_1 a_1^{x_2 + \ln(a_1)x_3} - b_1 = 0$$
$$c_i : x_1 a_i^{u_{i-1}} - b_i \leq 0 \qquad \text{for } i = 2, 3$$
$$c_i : x_1 a_i^{x_2 + \ln(a_i)x_3} - b_i \leq 0 \qquad \text{for } i = 4, \dots, 12$$

with the expensive functions

$$u_{i-1} = x_2 + \ln(a_{i+1})x_3 \qquad \text{for } i = 2, 3,$$

an initial point $\bar{x} = (100, 0, 1)$ and constants $a = (14, 15, 12, 8, 16, 20, 11, 13, 18, 25, 9, 10)$ as well as

$$b = (14.5949, 16.1078, 12, 8, 18.0596, 24.25, 10.4627, 13.0205, 20.4569, 32.9863, 8.4305, 9.5294).$$

Problem 33: Hatfldf_exp

$$c_1 : u_1 = 0$$
$$c_2 : x_1 - 0.056 + x_2 e^{u_2} = 0$$
$$c_3 : u_3 + x_2 e^{3x_3} = 0$$

with the expensive functions

$$u_1 = x_1 - 0.032 + x_2 e^{x_3}$$
$$u_2 = 2x_3$$
$$u_3 = x_1 - 0.099$$

and an initial point $\bar{x} = (0.1, 0.1, 0.1)$.

Problem 34: Hatfldg_exp

$$c_i : u_i = 0 \qquad \text{for } i = 1, \dots, 3$$
$$c_i : x_i - x_{13} - u_i + 1 = 0 \qquad \text{for } i = 4, \dots, 10$$
$$c_i : x_i - x_{13} - x_i(x_{i-1} - x_{i+1}) + 1 = 0 \qquad \text{for } i = 11, \dots, 24$$
$$c_{25} : x_{25} - x_{13} - x_{24}x_{25} + 1 = 0$$

with the expensive functions

$$u_1 : x_1 - x_{13} - x_1 x_2 + 1$$
$$u_i : x_i - x_{13} - x_i(x_{i-1} - x_{i+1}) + 1 \qquad \text{for } i = 2, 3$$

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$$u_i : x_i(x_{i-1} - x_{i+1}) \qquad\qquad \text{for } i = 4, \ldots, 10$$

and an initial point $\bar{x} = (1, \ldots, 1)$.

Problem 35: Himmelba_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = 4(x_1 - 5)$$
$$u_2 = x_2 - 6$$

and an initial point $\bar{x} = (8, 9)$.

Problem 36: Himmelbc_exp

$$c_1 : u_1 + x_1^2 = 0$$
$$c_2 : u_2 - 7 = 0$$

with the expensive functions

$$u_1 = x_2 - 11$$
$$u_2 = x_1 + x_2^2$$

and an initial point $\bar{x} = (1, 1)$.

Problem 37: Himmelbd_exp

$$c_1 : u_1 - 1 = 0$$
$$c_2 : 2314x_2 + u_2 + 49x_1^2 + 49x_2^2 - 681 = 0$$

with the expensive functions

$$u_1 = 12x_2 + x_1^2$$
$$u_2 = 84x_1$$

and an initial point $\bar{x} = (1, 1)$.

Problem 38: Himmelbe_exp

$$c_1 : u_1 = 0$$

$$c_2 : u_2 = 0$$
$$c_3 : u_3 = 0$$

with the expensive functions

$$u_1 = -x_1 + 1$$
$$u_2 = -x_2 + 1$$
$$u_3 = -x_3 + \frac{1}{4}(x_1 + x_2)$$

and an initial point $\bar{x} = (-1.2, 2, 0)$.

Problem 39: Hypcir_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = x_1 x_2 - 1$$
$$u_2 = x_1^2 + x_2^2 - 4$$

and an initial point $\bar{x} = (0, 1)$.

Problem 40: Integreq_exp

$$c_i : x_i + \frac{n+1-i}{2(n+1)^2} \sum_{j=1}^{i} j a_j + \frac{i}{2(n+1)^3} \sum_{j=i+1}^{n} (n+1-j) a_j = 0 \qquad \text{for } i = 1, \ldots, n$$

with

$$a_j = \left(x_i + \frac{n+j+1}{n+1} \right)^3 \qquad \text{for } i = 3, 5, 6, \ldots, n$$
$$a_i = u_i \qquad \text{for } i = 1, 2, 4,$$

the expensive functions

$$u_i = \left(x_i + \frac{n+j+1}{n+1} \right)^3 \qquad \text{for } i = 1, 2, 4,$$

an initial point $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ with $\bar{x}_i = \frac{i(i-n-1)}{(n+1)^2}$ and $n = 10$. Alternative choices for n may be 50, 100 or 500.

Problem 41: Msqrta_exp

$$c_{i,j} : u_{i,j} + \sum_{t=2}^{n} x_{i,t} x_{t,j} - n \sin^2(1) = 0 \qquad \text{for } i, j = 1, \ldots, n$$

with the expensive functions

$$u_{i,j} = x_{i,1} x_{1,j} \qquad \text{for } i, j = 1, \ldots, n,$$

an initial point $\bar{x}$ with $\bar{x}_{i,j} = \sin(1) - \frac{4 \sin(1)}{5}$ and $n = 2$. Alternative choices for n may be 7, 10, 23, 32 or 70.

Problem 42: Msqrtb_exp

$$c_{i,j} : u_{i,j} + \sum_{t=2}^{n} x_{i,t} x_{t,j} - n \sin^2(1) = 0 \qquad \text{for } i = 1, \ldots, n-1, \; j = 2, \ldots, n$$

$$c_{i,j} : u_{i,j} + \sum_{t=2}^{n} x_{i,t} x_{t,j} - (n-1) \sin^2(1) = 0 \qquad \text{if either } i = n \text{ or } j = 1$$

$$c_{i,j} : u_{i,j} + \sum_{t=2}^{n} x_{i,t} x_{t,j} - (n-2) \sin^2(1) = 0 \qquad \text{if } i = n \text{ and } j = 1$$

with the expensive functions

$$u_{i,j} = x_{i,1} x_{1,j} \qquad \text{for } i, j = 1, \ldots, n,$$

an initial point $\bar{x}$ with $\bar{x}_{i,j} = \sin(1) - \frac{4 \sin(1)}{5}$ and $n = 3$. Alternative choices for n may be 7, 10, 23, 32 or 70.

Problem 43: Pfit1_exp

$$c_1 : 18\frac{2}{3} - x_1(x_1 + 1)x_2 x_3^2 + x_1 x_2 x_3(1 - (1 + x_3)^{-x_1 - 1}) \leq 0$$

$$c_2 : 8 - \frac{1}{2}x_1 x_2(x_1 + 1)x_3^2 + x_1 x_2 x_3 - x_2(1 - (1 + x_3)^{-x_1}) = 0$$

$$c_3 : 23\frac{1}{9} - x_2 x_1(x_1 + 1)x_3^2 u = 0$$

with the expensive functions

$$u = 1 - (1 + x_3)^{-x_1 - 2}$$

and an initial point $\bar{x} = (1, 0, 1)$.

Problem 44: Pfit2_exp

$$c_1 : 60\frac{4}{9} - x_1(x_1 + 1)x_2x_3^2 + x_1x_2x_3(1 - (1 + x_3)^{-x_1-1}) = 0$$

$$c_2 : 26\frac{2}{3} - \frac{1}{2}x_1x_2(x_1 + 1)x_3^2 + x_1x_2x_3 - u = 0$$

$$c_3 : 71\frac{1}{9} - x_2x_1(x_1 + 1)x_3^2(1 - (1 + x_3)^{-x_1-2}) = 0$$

with the expensive functions

$$u = x_2(1 - (1 + x_3)^{-x_1})$$

and an initial point $\bar{x} = (1, 0, 1)$.

Problem 45: Pfit3_exp

$$c_1 : 126\frac{2}{9} - x_1(x_1 + 1)x_2x_3^2 + x_1x_2x_3(1 - (1 + x_3)^{-x_1-1}) = 0$$

$$c_2 : 56\frac{8}{9} - u + x_1x_2x_3 - x_2(1 - (1 + x_3)^{-x_1}) = 0$$

$$c_3 : 143\frac{11}{27} - x_2x_1(x_1 + 1)x_3^2(1 - (1 + x_3)^{-x_1-2}) = 0$$

with the expensive functions

$$u = \frac{1}{2}x_1x_2(x_1 + 1)x_3^2$$

and an initial point $\bar{x} = (1, 0, 1)$.

Problem 46: Pfit4_exp

$$c_1 : 216\frac{8}{81} - x_1(x_1 + 1)x_2x_3^2 + x_1x_2x_3(1 - (1 + x_3)^{-x_1-1}) = 0$$

$$c_2 : 98\frac{26}{27} - \frac{1}{2}x_1x_2(x_1 + 1)x_3^2 + x_1x_2x_3 - x_2(1 - (1 + x_3)^{-x_1}) = 0$$

$$c_3 : 239\frac{163}{243} - x_2x_1(x_1 + 1)u = 0$$

with the expensive functions

$$u = x_3^2(1 - (1 + x_3)^{-x_1-2})$$

and an initial point $\bar{x} = (1, 0, 1)$.

Problem 47: Powellbs_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = 1 + 10000x_1x_2$$
$$u_2 = 1.0001 + e^{-x_1} + e^{-x_2}$$

and an initial point $\bar{x} = (0, 100)$.

Problem 48: Powellsq_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = x_2^2 + \frac{20x_1}{x_1 + 0.1}$$
$$u_2 = x_1^2$$

and an initial point $\bar{x} = (3, 1)$.

Problem 49: Recipe_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$
$$c_3 : u_3 = 0$$

with the expensive functions

$$u_1 = x_1 - 5$$
$$u_2 = x_2^2$$
$$u_3 = \frac{x_3}{x_2 - x_1}$$

and an initial point $\bar{x} = (2, 5, 1)$.

Problem 50: Rsnbrne_exp

$$c_1 : u_1 = 0$$

$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = 10(x_2 - x_1^2)$$
$$u_2 = x_1 - 1$$

and an initial point $\bar{x} = (-10, 10)$.

Problem 51: Semicon1_exp

$$c_1 : x_2 - 2x_1 + \frac{100}{n+1}e^{-40x_1} - u_1 - \frac{100}{n+1} = 0$$

$$c_i : x_{i-1} + x_{i+1} - 2x_i + \frac{100}{n+1}e^{-40x_i} - u_i - \frac{100}{n+1} = 0 \qquad \text{for } i = 2, \ldots, 0.9n$$

$$c_i : x_{i-1} + x_{i+1} - 2x_i + \frac{100}{n+1}e^{-40x_i} - \frac{1000}{n+1}e^{40(x_i-700)} + \frac{1000}{n+1} = 0 \text{ for } i = 0.9n+1, \ldots, n$$

$$c_n : x_{n-1} - 2x_n + \frac{100}{n+1}e^{-40x_n} - \frac{1000}{n+1}e^{40(x_n-700)} - \frac{100}{n+1} = 0$$

with the expensive functions

$$u_i = \frac{1000}{n+1}e^{40(x_i-700)} \qquad \text{for } i = 1, \ldots, 0.9n,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 52: Semicon2_exp

$$c_1 : x_2 - 2x_1 + \frac{20}{n+1}e^{-8x_1} - u_1 - \frac{20}{n+1} = 0$$

$$c_i : x_{i-1} + x_{i+1} - 2x_i + \frac{20}{n+1}e^{-8x_i} - u_i - \frac{20}{n+1} = 0 \qquad \text{for } i = 2, \ldots, 0.9n$$

$$c_i : x_{i-1} + x_{i+1} - 2x_i + \frac{20}{n+1}e^{-8x_i} - \frac{200}{n+1}e^{8(x_i-140)} + \frac{200}{n+1} = 0 \text{ for } i = 0.9n+1, \ldots, n$$

$$c_n : x_{n-1} - 2x_n + \frac{20}{n+1}e^{-8x_n} - \frac{200}{n+1}e^{8(x_n-140)} - \frac{20}{n+1} = 0$$

with the expensive functions

$$u_i = \frac{200}{n+1}e^{8(x_i-140)} \qquad \text{for } i = 1, \ldots, 0.9n,$$

an initial point $\bar{x} = (0, \ldots, 0)$ and $n = 10$. Alternative choices for n may be 50, 100, 500 or 1000.

Problem 53: Sinvalne_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$

with the expensive functions

$$u_1 = 100(x_2 - \sin(x_1))$$
$$u_2 = \frac{x_1}{2}$$

and an initial point $\bar{x} = (-1, 4.712389)$.

Problem 54: Zangwil3_exp

$$c_1 : u_1 = 0$$
$$c_2 : u_2 = 0$$
$$c_3 : u_3 = 0$$

with the expensive functions

$$u_1 = x_1 - x_2 + x_3$$
$$u_2 = -x_1 + x_2 + x_3$$
$$u_3 = x_1 + x_2 - x_3$$

and an initial point $\bar{x} = (100, -1, 2.5)$.

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