Systemic Risk and CoVaR in a Gaussian Setting

Manfred Jäger-Ambrożewicz
Institut der deutschen Wirtschaft Köln
www.iwkoeln.de
and
Price-it GmbH
www.priceit.de

Presented at Bergische Universität Wuppertal, FB Mathematik und Naturwissenschaften, Angewandte Mathemtik – Stochastik, 20101115

see the paper at http://ssrn.com/abstract=1675435
Comments are appreciated: mj@finomica.com

The motivation to regulate systemic risk

- Currently banks are regulated on a bank-by-bank base: their own VaR determines their capital requirement.
- ... but a bank that goes south is likely to pull others south too.
- ... it may cause collateral damage.
- ... banks may be incentivated to assume "collective" risks expecting a bail out (support from the taxpayer) in case systemic risk materilizes.
- ... hence have an inefficiently low incentive to search diversification.

The motivation to regulate systemic risk

- Externality 1: a failing bank may trigger spillover effects that the bank does not take into account when fixing its strategy. Indeed, in case of a crisis "liquidity spiral, leading to depressed asset prices and a hostile funding environment, pulling others down and thus leading to further price drops, funding illiquidity, and so on".
- Externality 2: in case of a systemic crisis, there is no private solver – an institution that could for example buy a failing bank – as all private actors are suffering in a systemic crisis. Hence, the cost of bailouts are socialized.

Distortion

- Consider two banks that have the same VaR but the first bank follow strategies that make this bank more systemic.
- It is likely that these strategies create extra returns.
- ... if these extra returns are not properly priced by markets and are not addressed by regulation, assuming systemic risk creates a distorting advantage.
- ... it is likely that the second bank will also assume systemic risk (if competition is severe it might be crowded out otherwise).
- ... a prisoner dilemma.
- ... a race to unwarranted systemic risk.

The setting

- Suppose that i is the name of a typical bank and A refers to the complete banking system without the bank i, i.e. $S = \{i\} \cup A$ is the system under investigation.
- In the following a vector $(X_i, X_A)'$ of a bank i respectively a group A related statistic will be analyzed.
- We assume that X_i, X_A are jointly Gaussian with expected values μ_i, μ_A and variance-covariance matrix

$$\Sigma = \begin{pmatrix} \Sigma_i & \Sigma_{Ai} \\ \Sigma_{Ai} & \Sigma_A \end{pmatrix}.$$

The statistic we focus on

- Conditional Value at Risk

$$\mathrm{CollVaR}^{Ai} := \mathrm{VaR}(X_A|X_i = \mathrm{VaR}(X_i))$$

- Collateral damage at risk

$$\Delta\mathrm{CollVaR}^{Ai} \quad := \quad \mathrm{CoVaR}^{Ai} - \mathrm{VaR}^{A}(X_A|X_i = \mathbf{E}(X_i))$$

... The statistic we focus on

- *The economic meaning of* ΔCollVaR *is the following: We compare A's VaR if i hits its expected value with A's VaR if i hits its* VaR^i*. Hence, we compare a normal VaR of A with a stressed VaR of A.*

... The statistic we focus on

- **Result 1:** Collateral damage at risk

$$
\begin{aligned}
\Delta \mathrm{CollVaR}^{Ai} &= \mathrm{CoVaR}^{Ai} - \mathrm{VaR}^{A}(X_A | X_i = \mathbf{E}(X_i)), \\
\Delta \mathrm{CollVaR}^{Ai} &= -\Phi^{-1}(\alpha)\frac{\Sigma_{Ai}}{\sqrt{\Sigma_i}} = \beta_{Ai}\,\mathrm{VaR}^{\mathrm{mean}}(X_i), \\
\beta_{Ai} &= \frac{\mathrm{cov}(X_i, X_A)}{\mathrm{var}(X_i)}, \\
\mathrm{VaR}^{\mathrm{mean}}(X_i) &= \mathrm{VaR}(X_i) - \mathbf{E}(X_i),
\end{aligned}
$$

At the Margin: it is the beta of i on A whereas usually (CAPM) we would consider the beta of A on i.

... The statistic we focus on

- The virtues of

$$\Delta\mathrm{CollVaR}^{Ai} = \beta_{Ai}\,\mathrm{VaR}^{\mathrm{mean}}(X_i).$$

- The formula is very simple.
- The statistic $\mathrm{VaR}^{\mathrm{mean}}(X_i)$ has to be calculated anyway.
- The regression coefficient β_{Ai} can easily estimated.

Another statistic we focus on

- Obviously, it is also of interest to study the shape of the complete system S if i is stressed. In other words, we are as much interested to study the stochastic vector $(X_i, X_i + X_A)'$. The variance-covariance matrix of $(X_i, X_i + X_A)' = (X_i, X_S)'$:

$$\Sigma^{iS} = \begin{pmatrix} \Sigma_i & \Sigma_{iA} + \Sigma_i \\ \Sigma_{iA} + \Sigma_i & \Sigma_i + 2\Sigma_{iA} + \Sigma_A \end{pmatrix}.$$

Another statistic we focus on

- **Result 2:** *Delta Conditional Value at Risk equals*

$$\begin{aligned}\Delta \mathrm{CondVaR}^{Si} &= \mathrm{VaR}(S|X_i = \mathrm{VaR}(X_i)) - \mathrm{VaR}(S|X_i = \mathbf{E}(X_i)) \\ &= -\varphi \frac{\Sigma_{iA} + \Sigma_i}{\sqrt{\Sigma_i}} = \beta_{Ai} \mathrm{VaR}^{\mathrm{mean}}(X_i) + \mathrm{VaR}^{\mathrm{mean}}(X_i).\end{aligned}$$

Yet another statistic we focus on

- **Result 3:** *Delta Contributed Value at Risk*

$$
\begin{aligned}
\Delta\mathrm{ContrVaR}^{iS} &:= \mathrm{VaR}(X_i|X_S = \mathrm{VaR}(X_S)) - \mathrm{VaR}(X_i|X_S = \mathrm{E}(X_S)) \\
&= -\varphi \frac{\Sigma_{iA} + \Sigma_i}{\sqrt{\Sigma_i + 2\Sigma_{iA} + \Sigma_A}} \\
&= \frac{\Sigma_{iA} + \Sigma_i}{\Sigma_S}(-\varphi)\sqrt{\Sigma_S} = \beta_{iS}\mathrm{VaR}^{\mathrm{mean}}(X_S).
\end{aligned}
$$

... Yet another statistic we focus on

- Note that

$$\frac{\Sigma_{iA}+\Sigma_i}{\Sigma_S}+\frac{\Sigma_{iA}+\Sigma_A}{\Sigma_S}=\frac{\Sigma_i+2\Sigma_{iA}+\Sigma_A}{\Sigma_S}=\frac{\Sigma_S}{\Sigma_S}=1.$$

  Hence

$$\Delta\text{ContrVaR}^{iS}+\Delta\text{ContrVaR}^{AS}=\text{VaR}^{\text{mean}}(X_S).$$

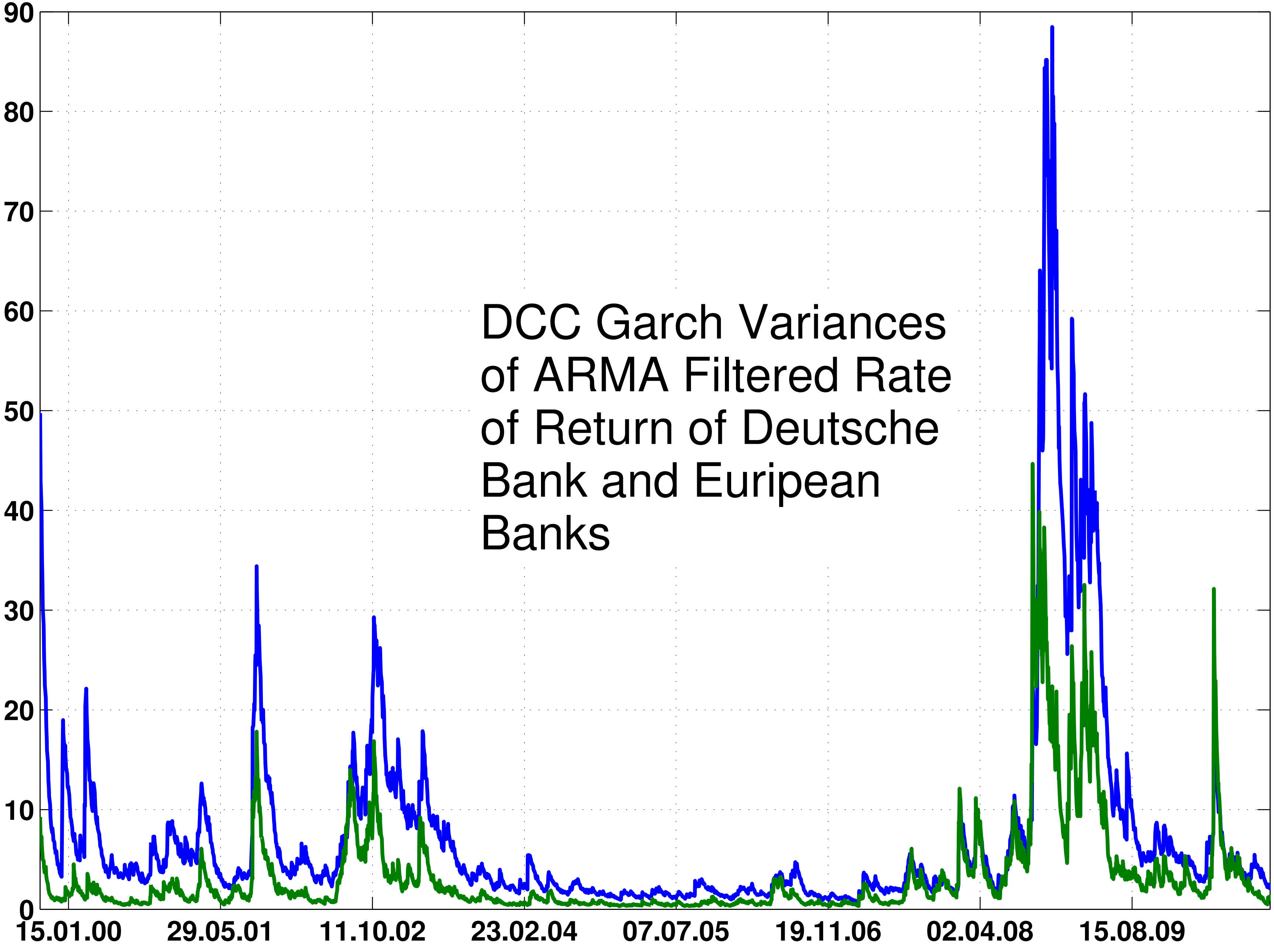
- The sum of the contributions equals aggregate systemic risk.
- Indeed, (a relative of) contributed Value at Risk is a standard statistic in portfolio management.

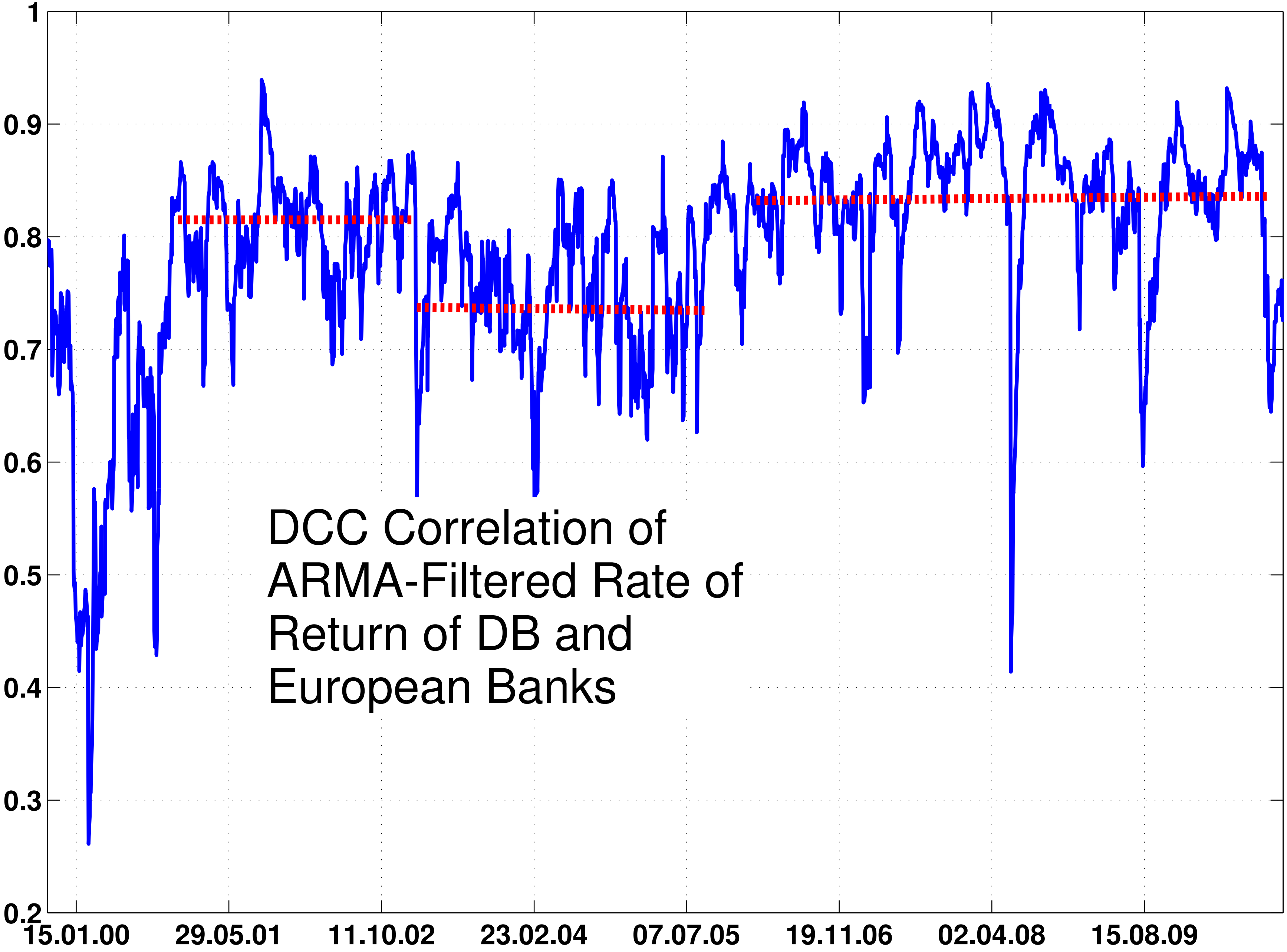
Gaussian setting: Closed formulas are nice but ... static

- The setting considered so far is static.
- Consider

$$\begin{aligned}\Delta \mathrm{CollVaR}^{Ai} &= \beta_{Ai}\,\mathrm{VaR}^{\mathrm{mean}}(X_i) = \rho_{Ai}\frac{\Sigma_A}{\Sigma_i}\mathrm{VaR}^{\mathrm{mean}}(X_i) \\ &= \rho_{Ai}\frac{\Sigma_A}{\sqrt{\Sigma_i}}(-\varphi)\sqrt{\Sigma_i}.\end{aligned}$$

- Three sources of dynamics:
  1. Market volatility may fluctuate $(\Sigma_A)_t$
  2. Banks i's volatility may fluctuate $(\Sigma_i)_t(,\mathrm{VaR}^{\mathrm{mean}}(X_i)_t)$
  3. Correlation may fluctuate $(\rho_{Ai})_t$
- Indeed, positive correlation is likely to arise when it is least needed!


90
80
70
60
50
40
30
20
10
0
15.01.00
29.05.01
11.10.02
23.02.04
07.07.05
19.11.06
02.04.08
15.08.09
DCC Garch Variances of ARMA Filtered Rate of Return of Deutsche Bank and Euripean Banks


DCC Correlation of ARMA-Filtered Rate of Return of DB and European Banks
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
15.01.00
29.05.01
11.10.02
23.02.04
07.07.05
19.11.06
02.04.08
15.08.09

DCC Garch

- Consider $\mathbf{X}_t = \mathbf{\Sigma}_t^{1/2}\mathbf{Z}_t$, where $\mathbf{Z}_t$ strict white noise and define the dynamics of $\mathbf{\Sigma}_t$ (the covariance of X_t).
- Engle and Sheppard suggested to split this into (1) volatility of the margins and (2) correlation.
- One uses

$$\mathbf{\Sigma}_t = (\mathbf{\Delta}_t)\mathbf{P}_t(\mathbf{\Delta}_t),$$

where $\Delta_t = \text{diag}(\sigma_{t,1}...\sigma_{t,N})$ is a diagonal matrix of std and $\mathbf{P}_t$ is the matrix of correlations.

... DCC Garch

- univariate Garch of the diagonal of $\mathbf{\Sigma}_t$, i.e. GARCH(1,1)

$$\sigma_{t,k}^2 = \alpha_{k0} + \alpha_{ki} X_{t,k}^2 + \beta_{ki} \sigma_{t-1,k}^2.$$

- Devolatized time series $\mathbf{Y}_t = \Delta_t^{-1} \mathbf{X}_t$,
- Correlation Dynamics

$$\mathbf{P}_t = \mathscr{P}\left[(1-\alpha-\beta)\bar{\mathbf{P}} + \alpha \mathbf{Y}_t \mathbf{Y}_t^T + \beta \mathbf{Q}_{t-1}\right],$$

where $\mathscr{P}(A_t) = (\mathrm{diag}(\sqrt{A_{t,ii}}))^{-1} A_t (\mathrm{diag}(\sqrt{A_{t,ii}}))^{-1}$.

Gaussian setting: Closed formulae are nice but ... the tail

- Tail dependency $\lambda = \lim_{q \to 0+} \text{Prob}(X_2 \leq F_2(q) | X_1 \leq F_1(q))$.
- ... using the Copula $\lambda = \lim_{q \to 0+} \frac{C(q,q)}{q}$.
- Problem: $\lambda_{\text{Gaussian}} = 0$.
- ... does the unconditional distribution of Garch with Gaussian innovation have tail dependency?

Agenda

- Develop a simple robust model to calculate regulatory requirements that tax systemic risk.
- the rule (formula) should generalize

$$\begin{aligned} \Delta\text{CollVaR}^{Ai} &= \beta_{Ai}\,\text{VaR}^{\text{mean}}(X_i), \\ \Delta\text{ContriVaR}^{iS} &= \beta_{iS}\,\text{VaR}^{\text{mean}}(X_S), \end{aligned}$$

  while “keeping” the structure.
- model the difference to the Gaussian setting:

$$\Delta\text{ContriVaR}^{iS} = \Lambda(\beta_{iS}) \cdot \text{VaR}^{\text{mean}}(X_S)$$

  for some robust transformation Λ.