Universität Erlangen-Nürnberg
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Prof. Dr. K. Klamroth
Barbara Pfeiffer

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# Integer and Nonlinear Optimization
## Exercise 9

**Problem 1**
Solve the problem

$$\begin{array}{lrcr} \max & 7x_1 + 2x_2 & & \\ \text{s.t.} & -x_1 + 2x_2 & \leq & 4 \\ & 5x_1 + x_2 & \leq & 20 \\ & -2x_1 - 2x_2 & \leq & -7 \\ & \underline{x} & \in & \mathbb{Z}_+^2 \end{array}$$

by the branch and bound method. Choose as branching variable the variable that violates integrality most.

**Problem 2**
Consider the integer knapsack problem

$$\begin{array}{llll} \max & \sum\limits_{i=1}^{n} & c_i x_i & \\ \text{s.t.} & \sum\limits_{i=1}^{n} & a_i x_i & \leq \; b \\ & & x_i & \in \; \mathbb{Z}_+^n \quad , \quad i = 1, \ldots, n. \end{array}$$

(a) Transfer the dynamic programming algorithm for the 0-1-knapsack problem to the integer case. What is the order of complexity of this algorithm?

(b) Is it possible to design a different recursion such that the same order of complexity as for the 0-1-knapsack problem is obtained?

(c) Apply your (best) algorithm to solve the following problem:

$$\begin{array}{lll} \max & 7x_1 + 9x_2 + 2x_3 + 15x_4 & \\ \text{s.t.} & 3x_1 + 4x_2 + x_3 + 7x_4 & \leq 10 \\ & x_1, \ldots, x_4 \in \mathbb{Z}_+^4 & \end{array}$$

**Problem 3**
Consider the following nonlinear programming problem:

$$\begin{array}{lrcl} \min & f(x_1, x_2) & = & -x_1 - x_2 \\ \text{s.t.} & x_1^2 + x_2^2 & \leq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

**See back side!**

(a) Verify that this is a convex programming problem.

(b) Solve this problem graphically.

**Problem 4**

Consider the problem of locating a curve of a specified type as 'closely' as possible to a given set of points in $\mathbb{R}^2$. Assume that the objective is to minimize the sum of squared deviations ('least squares method'). Furthermore assume that the curve to be fitted is of exponential type which occurs for example in growth models:

$$y = \alpha_1 + \alpha_2 e^{\beta x}$$

(nonlinear parametric regression). The parameters to be estimated are $\alpha_1, \alpha_2$ and $\beta$. Suppose that $n$ pairs of values $(x_1, y_1), \ldots, (x_n, y_n)$ have been observed, and formulate a nonlinear program describing this problem.